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BENDING STRESS ANALYSIS OF A SPUR GEAR FOR MATERIAL STEEL 15NI2CR1MO28

S. Prabhakaran, D. S. Balaji and R. Praveen Kumar
Department of Mechanical Engineering, Chennai, India
E-Mail: prabhakaran.sk1980@gmail.com

ABSTRACT

The main factors that cause the failure of gears are the bending stress and contact stress of the gear tooth. Stress analysis has been a key area of research to minimize failure and optimize design. This paper gives a finite element model for investigation of the stresses in the tooth during the meshing of gears for material steel 15Ni2Cr1Mo28. The model involves the involute profile of a spur gear. The geometrical parameters, such as the face width and module, are considered important for the variation of stresses in the design of gears. Using modeling software, 3-D models for different modules in spur gears were generated, and the simulation was performed using ANSYS to estimate the bending stress. The Lewis formula is used to calculate the bending stress. The results of the theoretical stress values are compared with the stress values from the finite element analysis.

Keywords: gearing, transmission system, root bending stress, surface contact stress, finite element analysis.

INTRODUCTION

Gearing is one of the most effective methods for transmitting power from one shaft to another with or without changing the speed. Spur gears are the most common type of gears. Spur gears have straight teeth, are mounted on parallel shafts and are mainly used to create very large gear reductions.

Modern mechanical design involves complicated shapes, which are sometimes made of different materials that as a whole cannot be modeled by existing mathematical tools. Engineers need the FEA to evaluate their design. A complex problem is divided into smaller and simpler problems that can be solved using the existing knowledge of the mechanics of materials and mathematical tools. The finite element method can proficiently supply this information, but the generation of a proper model is time consuming. Therefore, a pre-processor method that builds up the geometry required for finite element analysis may be used to reduce the modeling time, such as Pro/Engineer. Pro/Engineer can generate three-dimensional models of gears. In Pro/Engineer, the generated model geometry is opened in ANSYS for analysis. The application of finite element analysis allows the formation of bearing contacts during the cycle of meshing to be investigated and a stress analysis to be performed. The design of finite element models and the settings of boundary conditions are automated [1]. The theory of gearing and the modifications of the gear geometry are necessary to improve the conditions of meshing [2]

LITERATURE REFERENCES

The calculation of the tooth bending strength and surface durability of normal and high contact ratios may be sufficient for preliminary designs or standardized purposes, but the stresses calculated using these simple equations derived from the linear theory of elasticity and the Hertzian contact model are not in good agreement with experimental results [2, 3]. The contact stress between two gear teeth was analyzed for different contact positions, which represented a pair of mating gears during rotation. Each case was represented by a sequence position of contacts between these two teeth. Finite element models were generated for these cases, and the stress was analyzed. The results were presented and finite element analysis results were compared with theoretical calculations [1]. The contact stress and bending stress of a helical gear set with localized bearing contact were predicted using finite element analysis (FEA). The gear stress distribution was investigated using the commercial FEA package, ABAQUS/Standard. Furthermore, several examples have been presented to demonstrate the influences of the gear's design parameters and the contact positions on the stress distribution [4]. Researchers have analyzed the contact stresses between spur gear teeth using a plane model and validated the Hertz stress and AGMA contact stress with the finite element contact stress [5, 6]. The comparative reasoning of AGMA standards with FEA is main driver of the recent design and manufacturing of gears [7]. The quasi-static characteristic of finite element analysis allows the model to accurately simulate the distribution of equivalent stress and displacement change in the process of teeth meshing. The results agree well with the actual meshing [8]. The stress calculated for a pair of gears using the Lewis formula, Hertz equation and AGMA standards is comparable with FEA, and the PRO-E software and finite element software are good tools to define a safe design [9, 10].

Bending stress (Lewis equation)

In 1893, Wilfred Lewis provided a formula to estimate the bending stress in a tooth. He modeled a gear tooth that takes the full load at its tip as a simple cantilever beam. If we substitute a gear tooth for the rectangular beam, we can find the critical point in the root fillet of the gear by inscribing a parabola, as illustrated in Figure-1.

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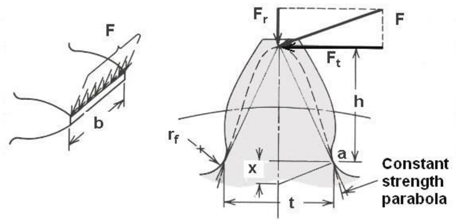

Figure-1. Force acting on gear tooth.

Lewis considered gear teeth as a cantilever beams with a static normal force, F, applied at the tip.

The following assumptions are made in the derivation:

The full load is applied to the tip of a single tooth at the static condition.

The radial component is negligible.

The load is distributed uniformly across the full face width.

Forces due to tooth sliding friction are negligible.

The stress concentration in the tooth fillet is negligible.

$$\sigma_b = \frac{F_t}{b * m * Y} \qquad [1]$$

F_t=(P/v)*K_v [2]

The Lewis equation indicates that the tooth bending stress varies with the following:

Directly with the load, Ft,
Inversely with the tooth width, b,
Inversely with the tooth module, m,
Inversely with the tooth form factor, y.

When teeth mesh, the load is delivered to the teeth with a certain impact. If we simply calculate the bending stress, the velocity factor should be used in the calculation [16]. Thus, the Lewis equation takes the following form:

$$\sigma_b = \frac{Ft}{bm_nY_J}(Kv * Ko * Ks(0.93Km) \qquad [3]$$

Input parameters of spur gear

Table-1. Geometric input parameters for spur gear.

Description	Gear	Pinion
Material	Steel 15Ni2Cr1Mo28	Steel 15Ni2Cr1Mo28
Number of teeth(Z)	63	18
Young's Modulus(E)	$2.08*10^5$ N/mm^2	$2.08*10^5$ N/mm^2
Speed (N)	228 rpm	800 rpm
Power (P), kW	45	45
Poisson Ratio	0.3	0.3
Normal Module (m), mm	2,3,4,5,6,7	2,3,4,5,6,7
Normal Pressure Angle	20^o	20^o

Table-2. Results obtained for different modules of spur gear.

		Module (m)					
Description	**Formula used**	**2**	**3**	**4**	**5**	**6**	**7**
Pitch Diameter (d) mm	m*Z_1	36	54	72	90	108	126
Circular Pitch (P_c) mm	$\pi d_1/Z_1$	6.28	9.42	12.56	15.7	18.84	21.98
Diameter Pitch (P_d)	Z_1/d_1	36	54	72	90	108	126
Centre Distance (a) mm	m(Z_1+Z_2)/2	6.28	9.42	12.56	15.7	18.84	21.98
Velocity	(3.14*d_1N)/60	0.50	0.33	0.25	0.20	0.17	0.14
Velocity factor K_v	$(6+v)/6$	81	121.5	162	202.5	243	283.5

Effect of modules in spur gear

In spur gears, the bending stresses are calculated by varying the modules (2, 3, 4, 5, 6, and7 mm) for a constant load as well as by keeping the other parameters, such as the number of teeth, pressure angle and face width, constant. The comparison of the theoretical bending stress

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values obtained from Lewis equation and AGMA for different modules is shown in Table-3. The table clearly shows that the bending stress values obtained from the Lewis equation and AGMA are very close to each other and that the bending stress decreases with an increase in the module.

Bending stress calculation using the Lewis equation from equation-1& 2

Tangential load Ft	=7462.68N
Face width b	=50mm
Module m	=5mm
Lewis form factor Y	=0.3

$$\sigma_b = \frac{5617}{51 * 5 * 0.3}$$
$$\sigma_b = 73.42\ \mathrm{N/mm^2}$$

Bending stress calculation using AGMA from equation-3

Geometric factor Y_j	=0.5
Dynamic Velocity factor K_v	=1.89
Overload factor K_o	=1
Size factor K_s	=0.75
Load distribution factor K_m	=1.4

$$\sigma_b = \frac{7462.68}{50 * 5 * 0.5}(1.89 * 1 * 0.75(0.93 * 1.4)$$
$$\sigma_b = 71.42\ \mathrm{N/mm^2}$$

Table-3. Comparison of maximum bending stress values by Lewis equation and AGMA for different module in spur gear.

S. No	**Module** (mm)	**Bending stress by Lewis formula** ($\mathrm{N/mm^2}$)	**Bending stress by AGMA** ($\mathrm{N/mm^2}$)
	2	2488	2746
	3	737.18	813.85
	4	311	343.32
	5	159.22	175.78
	6	92.14	101.73
	7	58.02	64.06

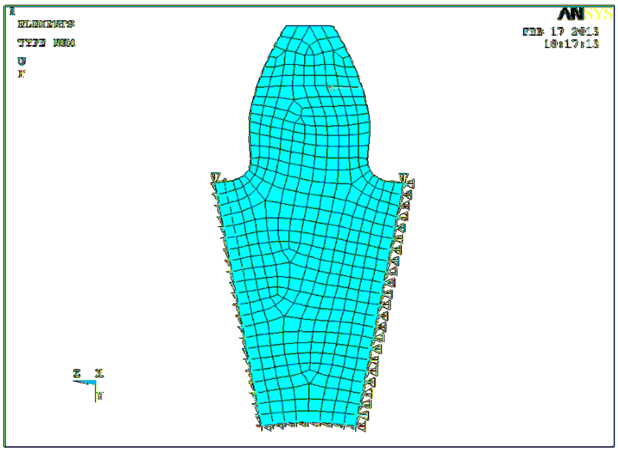

Figure-2. Spur gear tooth profile with mesh for module 5.

The two-dimensional tooth profile with mesh for module 5 in Finite Element Analysis (FEA) is shown in Figure-2. During stress analysis by ANSYS, the desired constraints and loads were applied to obtain bending stress values for the above profile.

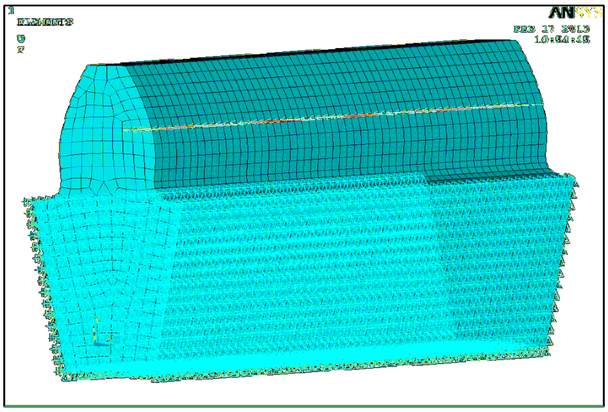

Figure-3. Bending stress model with mesh for spur gear of module 5.

The bending stress model with mesh in FEA for module 5 for three-dimensional tooth profile is shown in Figure-3. During stress analysis by ANSYS, the desired constraints and loads were applied to obtain the distribution of bending stresses along the contact line of action.

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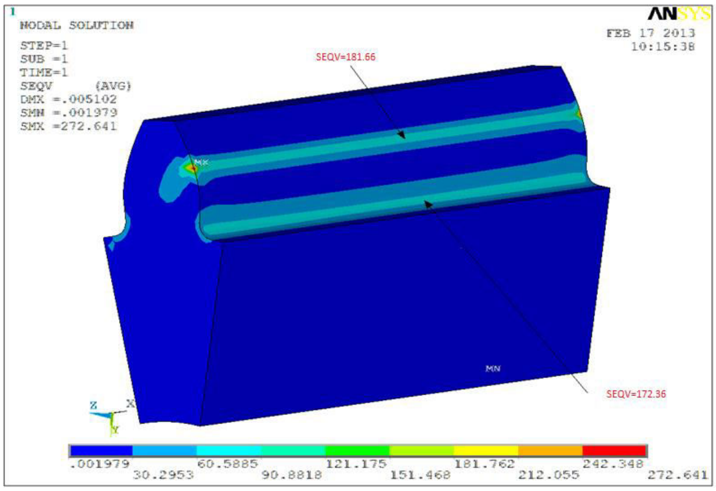

Figure-4. Bending stress distribution plots of spur gear for module 5.

The bending stress distribution plot is shown in Figure-4 for module 5 in which the maximum bending stress is 74.14 N/mm^2. This maximum bending stress value is almost identical with the values obtained by AGMA and Lewis equations.

Table-4. Comparison of maximum bending stress values for different modules in spur gear.

S. No.	Module, m (mm)	Bending stress value, σ_b (N/mm^2)			Difference (%)
		Lewis formula	AGMA	ANSYS	
	2	2488	2746	2796.14	1.79
	3	737.18	813.85	823.25	1.14
	4	311	343.32	352.12	2.50
	5	159.22	175.78	181.66	3.24
	6	92.14	101.73	103.36	1.58
	7	58.02	64.06	65.14	1.66

Table-4 shows the comparison of the bending stress values obtained by Lewis equation, AGMA, and ANSYS for different modules. The differences of the bending stress values between the Lewis equation and ANSYS are also tabulated.

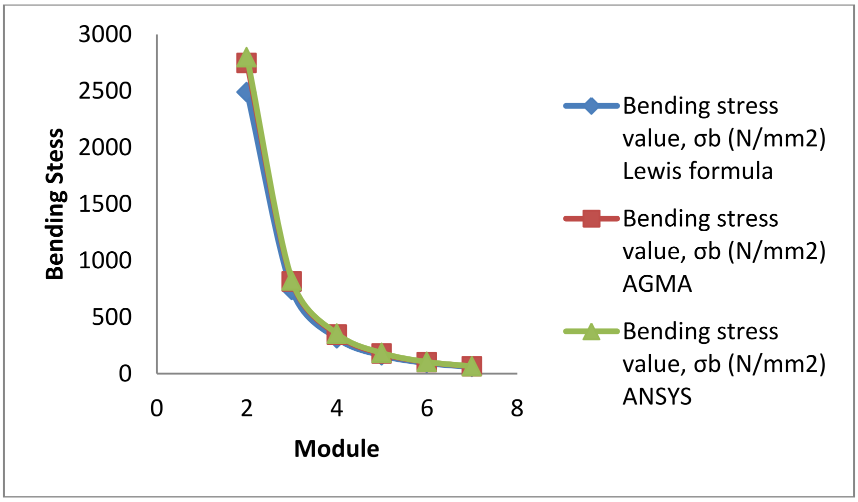

Figure-5. Comparison of bending stress for different modules in spur gear.

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Figure-5 clearly shows that the bending stress values obtained by using all three methods are very close to each other and well within the limit. The bending stress value negatively correlates with the module. Higher modules are preferred for larger power transmission with minimum bending stress values.

Result and Discussion

The findings in spur gear clearly show that the bending stress negatively correlates from 2796.14 N/mm^2 to 65.14 N/mm^2 with the module ranging from 2 mm to 7 mm respectively, with a maximum difference of 3.24 % and minimum difference of 1.14 % between AGMA and ANSYS results for material steel 15Ni2Cr1Mo28 with 63 teeth in gear and 18 teeth in pinion.

As a result, module with a larger face width is preferred in order to determine the material strength during the manufacture of gears for both materials.

CONCLUSIONS

In spur gear, the design of the teeth is purely based on bending and contact stresses. The bending stress using AGMA for different modules in spur gear were calculated for steel 15Ni2Cr1Mo28 materials. The bending stresses were also calculated for spur gear using the Lewis and equation. The results obtained for the bending stress by AGMA and Lewis equation are validated using the FEA approach.

The spur gear tooth profile are geometrically modeled by applying constraints and suitable loads for steel 15Ni2Cr1Mo28 material. Meshing was performed using the finite element method. The analysis results yielded by ANSYS were compared with the AGMA and Lewis equation.

The results of spur gear for steel 15Ni2Cr1Mo28 clearly show that the bending stresse decrease with an increase in the module. Hence, higher modules can be preferred for larger power transmission with minimum bending stress values.

Nomenclature

AGMA	-	American Gear Manufacturers Association
a	-	Center distance between shafts in mm
P_c	-	Circular Pitch
CAD	-	Computer Aided Design
P_d	-	Diametric pitch
b	-	Face width in mm
n	-	Factor of safety
FEA	-	Finite Element Analysis
FEM	-	Finite Element Method
i	-	Gear (or) transmission ratio
Y_j	-	Geometry factor
σ_b	-	Induced bending stress in N/mm^2
Y	-	Lewis Form factor
K_m	-	Load distribution factor
m_n	-	Normal module in mm
α	-	Normal pressure angle
Z_1, Z_2	-	Number of teeth in pinion, gear
Ko	-	Overload factor
d_p, d_g	-	Pitch circle diameter of pinion, gear
V_1	-	Pitch Line Velocity
v_1,v_2	-	Poisson's ratio of pinion, gear
P	-	Power transmitted in kw
r_{g1},r_{g2}	-	Radii of curvature of gear
r_{p1},r_{p2}	-	Radii of curvature of pinion
Ks	-	Size factor
N_1,N_2	-	Speed of pinion, gear in rpm
C_f	-	Surface condition factor
Ft	-	Tangential force
E_1,E_2	-	Young’s modulus of pinion, gear N/mm^2

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