



## FORWARD KINEMATIC MODELING BY SCREW THEORY OF A 3 DOF EXOSKELETON FOR HUMAN UPPER LIMB

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### ABSTRACT

This article shows the method to calculate the forward kinematics of an exoskeleton for human upper limb, using the screws theory. The mathematical calculations are shown and are compare with the real values. This forward kinematics is used to calculate the orientation and position of the final effector of the exoskeleton, and send this data to a robotic arm, to replicate the movements of the final effector of the exoskeleton with the robot. The movements that the robot must do are calculated using inverse kinematics, but it is not included in this document.

**Keywords:** screw theory, biomechanics, upper limb exoskeleton, forward kinematics.

### INTRODUCTION

The study of the movements of the human body and how they can be represented in a computer allows researchers an understanding of the functions of the joints, to develop machines to help people, an example is the development of forward kinematics of an arm, in order to control an exoskeleton for two tasks, the first is worn on the shoulder rehabilitation process [1], and the second for use as a haptic interface for virtual training [2], they took the ranges of motion of each joint parameterized studies in [3], to validate their models. In [4, 5, 6] and [7] also work with the development of an upper limb exoskeleton and get a kinematic model that allows them to represent a human arm. In [8] performed a study of the forces that must be minimized in the process of developing a robotic rehabilitation as they had the previous articles. All previous authors developed his kinematic models by the classical theory of Denavit-Hartenberg (D-H), and this is intended to address the calculation by the theory of screws, as described in Murray, it has certain advantages.

### SCREW THEORY

Screw theory provides an alternative to the traditional method of DH for the direct and inverse kinematics, from algebraic point of view. The way of representing the matrix as Twist and Screw has two advantages; the first is that they allow a comprehensive description of the motion of a rigid body which does not suffer from the singularities that arise when using local coordinates. The second is that a geometric description of the movements is obtained, which greatly simplifies the analysis of mechanisms, in [9] are described the steps to get the forward kinematics. Here are the steps to make the forward kinematics:

- Define the placement of the coordinate systems  $S$  and  $G$ , where the first is the global reference system, and the later one the final effector system.
- Locate the  $\omega_i$  on each joint, those are  $\mathbb{R}^3$  unitary vectors that are over the rotation or displacement axis depending the kind of joint.

- Select the  $q_i$  points needed, those must be placed over one or more axis, in order to let the Twist creation.
- With the  $\omega_i$  and  $q_i$  the *Twists* are calculated, represented by the Greek letter  $\xi_i$ , which can be in two ways depending the joint's kind, as in the next equation.

$$\xi_i = \begin{bmatrix} -\omega_i \times q_i \\ \omega_i \end{bmatrix} : Rotacional$$

$$\xi_i = \begin{bmatrix} v_i \\ 0 \end{bmatrix} : Traslacional$$

$$\therefore v_i = \omega_i$$

- The  $g_s^G(0)$  homogeneous matrix is obtained which represent the position and orientation of the final effector  $G$  seen from the reference system  $S$ , where all values of the articular variables are default.
- The matrix's exponentials  $e^{\xi_i \theta_i}$  are generated from the *Twists*, which represent the homogeneous transformation matrix equivalent for the joint.
- At last calculate the forward kinematics, by multiplying all the  $e^{\xi_i \theta_i}$  and the matrix  $g_s^G(0)$ , getting the  $g_s^G(\theta)$  as shown in the next equation where  $n$  is the number of joins.

$$g_s^G(\theta) = e^{\xi_1 \omega_1} e^{\xi_2 \omega_2} \dots e^{\xi_n \omega_n} g_s^G(0)$$

$$i = 1, 2, \dots, n$$

### HUMAN UPPER LIMB EXOSKELETON MODELLING

This section will decompose the upper limb in three sets, respect to the joint that causes the movement to analyse what would be the mechanically equivalent joint, and which is it kinematic model. To move the upper limbs are involved these bones: scapula, clavicle, humerus, ulna, radius and carpal bones, which are connected together with tendons and ligaments, forming the shoulder, elbow and wrist joints. The aim is for each of the joints in the human body get a mechanism which can be used to replace it when modeling. The human upper limb has seven joints, also known as DOF (Degrees Of Freedom). The human joints, and the movement that they provide, are shown (Figure-1) [3].

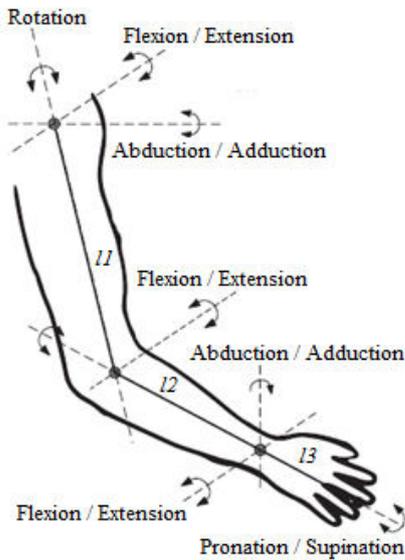


Figure-1. Human arm.

From the above it can be inferred that the shoulder has three DOF, the elbow joint has one DOF, and the wrist has three DOF. The exoskeleton designed will be used as a teleoperate device to control the robot VALI 2.0 [11, 12] (Figure-2), which is used as a military robot to handle explosive devices. This robot is teleoperated using a gamepad, device that is usually used as a teleoperate device in a large number of robots [13, 14]. Due that the robot has five DOF; the exoskeleton needs equal number of them. To teleoperate the robot, the exoskeleton must have two DOF in the shoulder, one DOF in the elbow, and two in the wrist. The movement of each joint in the exoskeleton is measured by a sensor located in the joint, and the data of all the sensor are sent to the computer, in order to calculate the forward kinematics of the exoskeleton.



Figure-2. Robot VALI 2.0.

The idea is to copy the movements of the exoskeleton with the robot (Figure-3), but, as their dimensions and range of movements are not the same; the movement of the joints cannot be sent directly to the robot. It is necessary to calculate kinematics in order to move the robot in the right way.

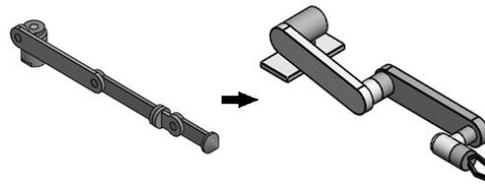


Figure-3. Simplified models of the exoskeleton and the robot.

The final design of the exoskeleton (Figure-4) includes all the DOF, the back support (to put it on the user), and the hand support (with an independent joystick to move the mobile platform of the robot).



Figure-4. Isometric view of the exoskeleton.

**FORWARD KINEMATICS**

The complete process to calculate the forward kinematics in the exoskeleton starts with the following assumptions: the exoskeleton has five DOF, but two of them are not included in calculus, due that their values are the same in the inverse kinematics of the robot.

The simplified model of the exoskeleton is used to locate the coordinate systems and dimensions necessary for the forward kinematics. The initial step is locate the S and G systems (Figure-3).

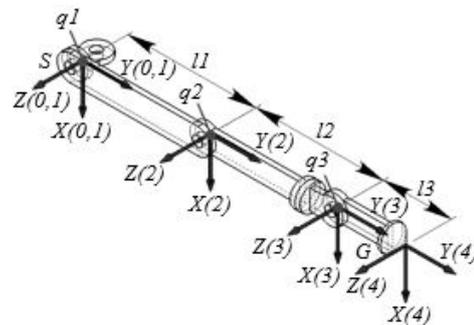


Figure-5. Place of axis and points for screw theory.

With the coordinate systems located, and following the steps described in the Screw Theory, the  $\omega_i$  are located, and their mathematical representations are show in (1).



$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (1)$$

The  $q_i$  points are located at the origin of each  $\omega_i$  and the coordinates are given respect to the  $S$  system.

$$q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} q_2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix} q_3 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 1 \end{bmatrix} \quad (2)$$

The Twists are calculated with (1) and (2), according to step number 4 of the Screw Theory.

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \xi_3 = \begin{bmatrix} l_1 + l_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (3)$$

In the simplified model of the exoskeleton where located the final effector system  $G$ , the rotation and location of this system referred to global reference system is easily calculated and it is described in (4).

$$g_s^T(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -(l_1 + l_2 + l_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The assembly of matrix's exponentials  $e^{\xi_i \theta_i}$  is made according to (5). It is necessary to calculate the  $e^{\hat{\omega} \theta_i}$  and  $p_i$ . The first one is a 3x3 matrix, and the last one is a 3x1 vector. For the calculus of  $p_i$ , the  $v$  represent the 3 initial values of the corresponding Twist.

$$e^{\xi_i \theta_i} = \begin{bmatrix} e^{\hat{\omega} \theta_i} & p_i \\ 0 & 1 \end{bmatrix} \quad (5)$$

$$\therefore e^{\hat{\omega} \theta_i} = I + \hat{\omega} s \theta_i + \hat{\omega}^2 (1 - c \theta_i)$$

$$\therefore p_i = (I - e^{\hat{\omega} \theta_i})(\omega \times v) + \omega \omega^T v \theta_i$$

Where  $e^{\hat{\omega} \theta_i}$  for each  $\theta$  is equal to:

$$e^{\hat{\omega} \theta_1} = \begin{bmatrix} c \theta_1 & -s \theta_1 & 0 \\ s \theta_1 & c \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} e^{\hat{\omega} \theta_2} = \begin{bmatrix} c \theta_2 & -s \theta_2 & 0 \\ s \theta_2 & c \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$e^{\hat{\omega} \theta_3} = \begin{bmatrix} c \theta_3 & -s \theta_3 & 0 \\ s \theta_3 & c \theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And  $p_i$  correspond to:

$$p_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} p_2 = \begin{bmatrix} l_1 s \theta_2 \\ l_1 (1 - c \theta_2) \\ 0 \end{bmatrix} \quad (7)$$

$$p_3 = \begin{bmatrix} (l_1 + l_2) s \theta_3 \\ (l_1 + l_2)(1 - c \theta_3) \\ 0 \end{bmatrix}$$

With (6) and (7), and according to (5), the matrix's exponential finally are expressed in (8), (9) and (10).

$$e^{\xi_1 \theta_1} = \begin{bmatrix} c \theta_1 & -s \theta_1 & 0 & 0 \\ s \theta_1 & c \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$e^{\xi_2 \theta_2} = \begin{bmatrix} c \theta_2 & -s \theta_2 & 0 & l_1 s \theta_2 \\ s \theta_2 & c \theta_2 & 0 & l_1 (1 - c \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

$$e^{\xi_3 \theta_3} = \begin{bmatrix} c \theta_3 & -s \theta_3 & 0 & s \theta_3 (l_1 + l_2) \\ s \theta_3 & c \theta_3 & 0 & 0 \\ 0 & 0 & 1 & (1 - c \theta_3)(l_1 + l_2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Getting the  $g_s^G(\theta)$  is easy, according to the last step in forward kinematics, as follows:

$$g_s^G(\theta) = e^{\xi_1 \omega_1} e^{\xi_2 \omega_2} e^{\xi_3 \omega_3} g_s^T(0)$$

$$= \begin{bmatrix} c \theta_{123} & -s \theta_{123} & 0 & -l_3 s \theta_{123} - l_2 s \theta_{12} - l_1 s \theta_1 \\ s \theta_{123} & c \theta_{123} & 0 & l_3 c \theta_{123} + l_2 c \theta_{12} + l_1 c \theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

$$\therefore c \theta_{123} = c(\theta_1 + \theta_2 + \theta_3) ; c \theta_{12} = c(\theta_1 + \theta_2)$$

$$\therefore s \theta_{123} = s(\theta_1 + \theta_2 + \theta_3) ; s \theta_{12} = s(\theta_1 + \theta_2)$$

The matrix (11) represents the forward kinematics of the exoskeleton, and to verify it, three cases are studied with the data taken from the simplified planar model. Using (11) the forward kinematics of the examples (Figure 6, 7 and 8) was calculated. In the examples are given the values of each angle  $\theta$ , and the position and orientation of the final effector of the planar robot. The dimensions in the examples are given in millimeters and radians. The lengths of the links are:

$$l_1 = 397 ; l_2 = 450 ; l_3 = 212$$

Each example has its own result. These results are showed (13, 14 and 15). The values of these matrixes correspond with the values measured directly in the planar robot, according with the matrix (12), which represents the position and orientation of an element in the space [10].

$$g_s^G(\theta) = \begin{bmatrix} R & p \\ 0^T & 1 \end{bmatrix} \quad (12)$$

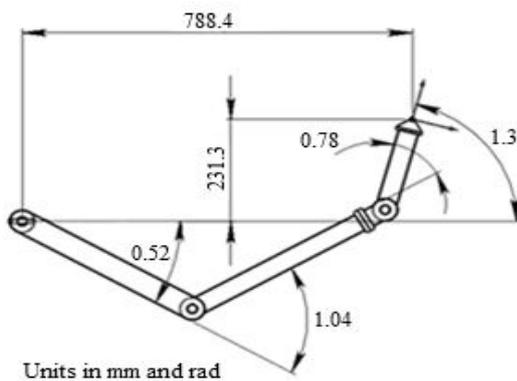


Figure-6. Planar robot example 1.

$$g_S^G(\theta) = \begin{bmatrix} 0.259 & -0.965 & 0 & -230.975 \\ 0.965 & 0.2596 & 0 & 788.7946 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

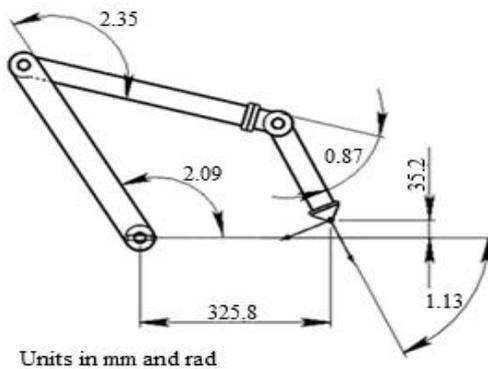


Figure-7. Planar robot example 2.

$$g_S^G(\theta) = \begin{bmatrix} 0.4209 & 0.907 & 0 & -34.545 \\ -0.907 & 0.4209 & 0 & 325.881 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (14)$$

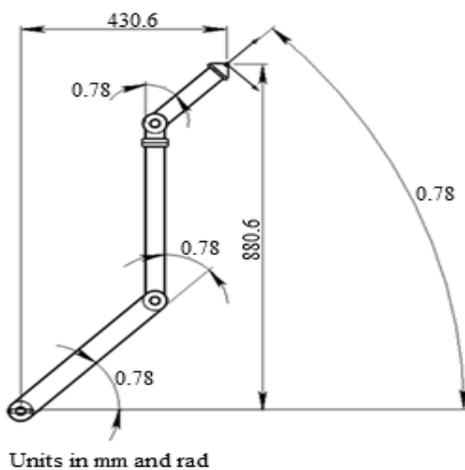


Figure-8. Planar robot example 3.

$$g_S^G(\theta) = \begin{bmatrix} 0.7071 & -0.7071 & 0 & -880.628 \\ 0.7071 & 0.7071 & 0 & 430.628 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

**CONCLUSIONS**

With the screw theory is possible to obtain kinematics models from complex joints configurations, as the ones in the human upper limbs with a minor quantity of parameters, such as those generated with the D-H methodology, with two for each joint using screws against fourth parameters in D-H.

Since each joint have more than one DOF, the creation of twists is made with less points than axis, this will help with the inverse kinematic calculation, being that the Paden-Kahan sub problems are best when two or more axis intersect over the same point.

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