



# COMPARATIVE PERFORMANCE EVALUATION OF M-ARY QAM MODULATION SCHEMES USING SIMULINK AND BERTool

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## ABSTRACT

With the fast development of modern communication techniques, the demand for reliable high data rate transmission is increased significantly. Different modulation techniques allow researchers to send different bits per symbol achieving different and higher throughputs or efficiencies. Because of its efficiency in power and bandwidth, M-ary Quadrature Amplitude Modulation (M-QAM) is one of widely used modulation techniques in the practice. Therefore, a need of studying and evaluating the performance of QAM modulation schemes is an important task for designers. In this paper, M-QAM modulation schemes for even number of bits per symbol (32- and 128-QAM) and for odd number of bits per symbol (16- and 64-QAM), over Additive White Gaussian Noise (AWGN) channel, are studied. A Simulink-based simulation model for M-ary QAM is designed. Theoretical and simulation results for Bit Error Ratio (BER) performance of QAM modulation schemes are obtained using Matlab/Simulink and Matlab/BERTool. The results are evaluated and compared.

**Keywords:** AWGN, BER, BERTool, QAM, simulink.

## 1. INTRODUCTION

In the last years, the major transition from analog to digital is occurred in all areas of communications. The main reason for this is that digital communication system is more reliable than an analog one [1], [2]. The main factor that differentiates both the analog and digital communications is the modulation technique used with them. The digital modulation provides more information capacity and quicker system availability with great quality communication, that's why, the applications of various digital modulation techniques continue to grow, with the growth and evolvement of the digital communications industry.

Modern modulation techniques exploit the fact that digital baseband data may be sent by varying both envelope and phase/frequency of a carrier wave. Because the envelope and the phase offer two degrees of freedom, such modulation techniques map baseband data into four or more possible carrier signals. Such modulation techniques are called M-ary modulation, since they can represent more signals than if just the amplitude or phase were varied alone [1], [3], [4]. In an M-ary signaling scheme, may be sent one of  $M$  possible signals  $s_1(t), s_2(t), \dots, s_m(t)$  during each signaling interval of duration  $T_s$ . For almost all applications, the numbers of possible signals are  $M=2^m$ , where  $m$  is an integer [5] which corresponds to the number of bits per symbol. The symbol duration  $T_s=mT_b$ , where  $T_b$  is the bit duration. Depending on whether the amplitude, phase, or frequency is varied, the modulation technique is called M-ASK (M-ary Amplitude Shift Keying), M-PSK (M-ary Phase Shift Keying) or M-FSK (M-ary FSK Frequency Shift Keying) [2], [6]. Modulation which alters both amplitude and phase is M-QAM (M-ary Quadrature Amplitude Modulation) [7], [8]. Different bandwidth efficiency at the expense of power efficiency can be achieved using M-ary modulation techniques.

Nowadays, QAM is one of the most common modulation schemes used in communication systems. It is very widely used in cable TV, Wi-Fi, wireless local-area networks (LANs), wireless sensor networks (WSN), satellites and cellular telephone systems to produce maximum data rate in limited bandwidth. Moreover, some specific variants of QAM are used in some specific applications and standards. The wide use of QAM modulation schemes requires continuously studying and evaluating of their performance under different conditions. There is also a need of seeking automated methods of designing digital modulation models using the latest software including Simulink and BERTool in Matlab [9].

The objective of this paper is to study, evaluate and compare the bit error rate (BER) performance of M-QAM with an even number of bits per symbol (16-QAM and 64-QAM) and with an odd number of bits per symbol (32-QAM and 128-QAM) under/over Additive White Gaussian Noise (AWGN) channel, using Matlab/Simulink and Matlab/BERTool.

## 2. M-ary QAM

QAM is a modulation technique where the amplitude is allowed to vary with phase. QAM signaling is viewed as a combination of ASK and PSK. Also, it can be viewed as an ASK in two dimension. It is such a class of non-constant envelope schemes that can achieve higher bandwidth efficiency than M-PSK with the same average signal power [2], [7].

### 2.1 Signal model

In practice, the information symbols are Gray coded in order to restrict the erroneous symbol decisions to single bit error, i.e., the adjacent symbols in the transmitter constellation should not differ more than one bit. Usually the Gray coded symbols are separated into in-phase (I) and quadrature (Q) bits and then are mapped to M-QAM constellation.



It is well known, that square QAMs are the typically used constellations when the number of bits in a symbol,  $m = \log_2(M)$  is even. If there is a requirement for the transmission of an odd number of bits per symbol, non-square QAM constellations, such as rectangular and cross shape constellations are used [10]. Rectangular QAM constellations are, in general, sub-optimal in the sense that they do not maximally space the constellation points for a given energy. However, the rectangular QAMs are much easier to modulate and demodulate than other non-rectangular QAMs. But in terms of an energy efficiency, cross QAM is usually the better choice, as it has lesser average and peak energy as compared to rectangular QAM.

For the case of  $I \times J$  rectangular QAM (odd bit constellations),  $M = I \times J$ ,  $I = 2^{(m-1)/2}$  and  $J = 2^{(m+1)/2}$  [11]. Assume that  $I \times J$  rectangular QAM consists of two independent one-dimensional amplitude modulation signals, as well, that all the transmit symbols are equally likely. Under these conditions, M-QAM modulated signal with Gray coding can be represented mathematically in terms of its in-phase and quadrature components as shown in [12]

$$s(t) = A_I \cos(2\pi f_c t) - A_Q \sin(2\pi f_c t), 0 \leq t \leq T_s \quad (1)$$

where  $f_c$  is the carrier frequency,  $A_I \in \{\pm d, \pm 3d, \dots, (I-1)d\}$ ,  $A_Q \in \{\pm d, \pm 3d, \dots, (J-1)d\}$  as  $A_I$  and  $A_Q$  are the amplitudes of the in-phase and quadrature components. These amplitudes correspond to  $M$  possible symbols in the two-dimensional space. Let  $2d$  is the Euclidean distance between two adjacent signal points [4]. Denoting  $E_b$  as the bit energy,  $d$  can be represented in terms of  $E_b$ ,  $I$  and  $J$  as follows [12]

$$d = \sqrt{\frac{3E_b \log_2(I \times J)}{I^2 + J^2 - 2}} \quad (2)$$

Constellation structure of cross and rectangular QAM are slightly different as a constellation shape of cross QAM can be constructed from rectangular QAM by shifting  $\sqrt{2M}/8$  columns on either side to top and bottom positions in the manner shown in [13].

For the case of M-ary square QAM ( $I = J$  and  $M = I \times I$ ), Equation (2) becomes

$$d = \sqrt{\frac{3E_b \log_2(M)}{2(M-1)}} \quad (3)$$

## 2.2 Bit error rate for M-QAM in AWGN channel

The BER is an important factor in determining and evaluating the performance and usefulness of modulation schemes. The BER is the number of bits in error divided by the total number of transferred bits during a studied time interval [14], i.e.

$$BER = \frac{\text{Number of bits in error}}{\text{Total number of transferred bits}} \quad (4)$$

BER can also be defined in terms of the probability of error ( $POE$  or  $P_b$ ) [15]. The probability of error  $P_b$  and BER are somewhat different concepts. But since their numerical values are quite similar, whenever a reference is made about  $P_b$  it is implied BER. The probability of error is used to measure the performance of each modulation scheme with assumption that systems are operating with additional white Gaussian noise.

The probability of bit error for M-QAMs is given mathematically in a different manner depending on whether the number of bits per symbol is even or odd [16]. For square M-QAM, where  $m = \log_2(M)$  is even, the  $P_b$  can be described as follows

$$P_b = \frac{2P_0 - P_0^2}{\log_2(M)} \quad (5)$$

Where

$$P_0 = \frac{2(\sqrt{M}-1)}{\sqrt{M}} \text{erf} \left( \sqrt{\frac{3 \log_2(M)}{M-1} \frac{E_b}{N_0}} \right) \quad (6)$$

For rectangular QAM, where  $m = \log_2(M)$  is odd, the  $P_b$  can be written as

$$P_b \leq \frac{1}{\log_2(M)} \left[ 1 - \left( 1 - 2 \text{erf} \left( \sqrt{\frac{3 \log_2(M)}{M-1} \frac{E_b}{N_0}} \right) \right)^2 \right] \quad (7)$$

In the above equations, erf is the error function [11],  $E_b/N_0$  which is a form of signal-to-noise ratio (SNR) where  $E_b$  is the bit energy and  $N_0$  is the noise power spectral density. The error function is different for the each of the various modulation methods [11], [15]. Equations (5) and (7) can be transformed to a specific type depending on the modulation order [17]. For example, to calculate the theoretical BER for 16-QAM, 32-QAM, 64-QAM and 128-QAM, the expressions that are given in Table-1 may be used.

**Table-1.** Probability of bit error for M-QAM modulation schemes.

M	m	Probability of Bit Error (BER): $P_b$	
16	4	$P_b \cong 0.375 \operatorname{erfc} \left( \sqrt{\frac{2 E_b}{5 N_0}} \right)$	(8)
32	5	$P_b \cong 0.33 \operatorname{erfc} \left( \sqrt{\frac{15 E_b}{62 N_0}} \right)$	(9)
64	6	$P_b \cong 0.292 \operatorname{erfc} \left( \sqrt{\frac{1 E_b}{7 N_0}} \right)$	(10)
128	7	$P_b \cong 0.26 \operatorname{erfc} \left( \sqrt{\frac{21 E_b}{254 N_0}} \right)$	(11)
In Equations. (8)-(11), $\operatorname{erfc}$ is the complementary error function, defined as $\operatorname{erfc} = 1 - \operatorname{erf}$ .			

### 3. M-QAM SIMULATION MODEL

To evaluate the BER performance of M-ary QAM modulation, a baseband simulation model, using Matlab/Simulink environment [9], [18] is designed. The model is shown in Figure-1. It allows implementing of comparative BER performance evaluation of M-ary QAM modulation schemes for different modulation orders (different values of M) over AWGN channel. The model block functions are as follows:

- The Random Integer Generator block is used to generate uniformly distributed random integers in the range of  $[0, M-1]$ , where M is the M-ary number.
- The Integer to Bit Converter block maps each integer to a group of bits.
- The Rectangular QAM Modulator Baseband block modulates the input signal using the rectangular quadrature amplitude modulation. If the Constellation ordering parameter of this block is set to Gray and m is odd, the block codes the constellation so that pairs of nearest points differ in one or two bits. The constellation is cross-shaped.
- The AWGN Channel block models a noisy channel by adding white Gaussian noise to the modulated signal.

The variance of the noise added per sample affecting the final error rate is given by equation

$$\text{Noise variance} = \frac{\text{Signal Power} \times \text{Symbol Period}}{\text{Sample Time} \times 10^{(E_s/N_0)/10}} \quad (12)$$

where,

*Signal Power* is the actual power of the symbols.

*Symbol Period* is the duration of a channel symbol, in seconds.

*Sample Time* is the sampling time, in seconds.

$E_s/N_0$  is the ratio of signal energy per symbol to noise power spectral density, in decibels.

- The Rectangular QAM Demodulator Baseband block demodulates the input signal using the rectangular quadrature amplitude modulation.
- Bit to Integer Converter block maps groups of bits to integers.
- The Error Rate Calculation block counts the bits that differ between the received signal and the transmitted signal.
- The Display block (Display/BER) displays the bit error rate (BER), the total number of errors and the total number of bits processed during the simulation.
- The "To Workspace" block writes the simulated BER values into an array or structure in the main Matlab workspace.

The simulation model shown above is used in conjunction with the BERTool under Matlab. BERTool supports three ways of evaluating the BER performance of M-QAMs that are [19]: theoretical, semi-analytic and Monte Carlo.

### 4. RESULTS AND COMPARATIVE EVALUATION

In this paper, the BER performance of M-QAM (M=16, 32, 64 and 128) over AWGN channel is evaluated, applying a theoretical and a Monte Carlo simulation approach. The theoretical method is based on Equation (5), Equation (6) and Equation (7). To implement Monte Carlo simulations, the Simulink model, which is shown in Figure-1, is loaded into BERTool.

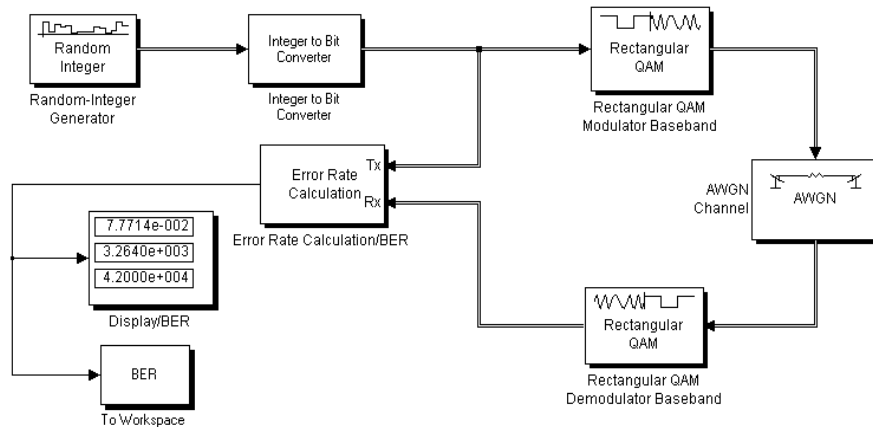
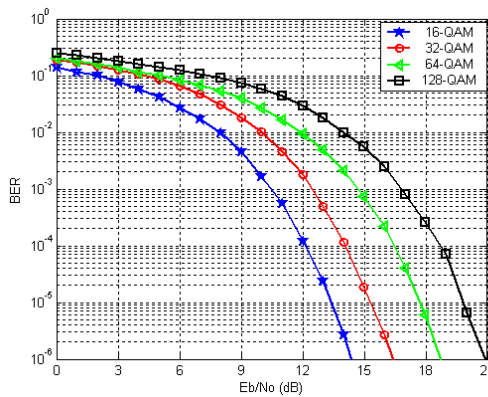


Figure-1. Simulink simulation model of M-QAM.

The results of BER performance of M-QAM for  $M=16, 32, 64$  and  $128$ , obtained after simulating the model from Figure-1 in Monte Carlo, are shown in Figure-2.

Figure-2. Simulated BER performance of M-QAM for  $M=16, 32, 64, 128$  over AWGN channel.

From Figure-2, it is clear that when for a fixed value of  $E_b/N_0$  the order of the modulation increases from 16 to 128 the BER also increases resulting to the BER performance reduction. From the results, it is also observed, that the increase of the  $E_b/N_0$  value for a given modulation scheme leads to the BER decrease namely to the BER performance improvement. For example, at  $E_b/N_0 = 6$  dB the BER of 16-QAM is 4.6 times smaller than that of 128-QAM. It can therefore, be concluded from the simulated BER curves that for a fixed  $E_b/N_0$  value the 16-QAM has a better BER performance than 32-QAM, 64-QAM and 128-QAM.

The results show that with  $E_b/N_0$  increase, the BER decreases exponentially with respect to  $E_b/N_0$ , for all four modulation schemes. The numerical results shown in Table-2, are the extracted from Figure-2 values of  $E_b/N_0$ , where for the different QAMs is achieved  $BER = 10^{-4}$  and  $BER = 10^{-6}$ .

Table-2. Required  $E_b/N_0$  value to achieve  $BER=10^{-4}$  and  $BER=10^{-6}$ .

Modulation Format	Required $E_b/N_0$ (dB)	
	$BER=10^{-4}$	$BER=10^{-6}$
16-QAM	12.1	14.3
32-QAM	14.2	16.4
64-QAM	16.5	18.7
128-QAM	18.8	21.0

Analyzing the results at a BER of  $10^{-4}$ , it can be seen that for each increase in the number of bits per symbol, an additional  $\sim (2.1 - 2.3)$  dB of  $E_b/N_0$  is required to achieve that same performance. From the results in Table-2, the same conclusion for the case of  $BER=10^{-6}$  can be done. That means, in this case also an additional  $\sim (2.1 - 2.3)$  dB of  $E_b/N_0$  per each increase in the number of bits per symbol is required in order to maintain performance.

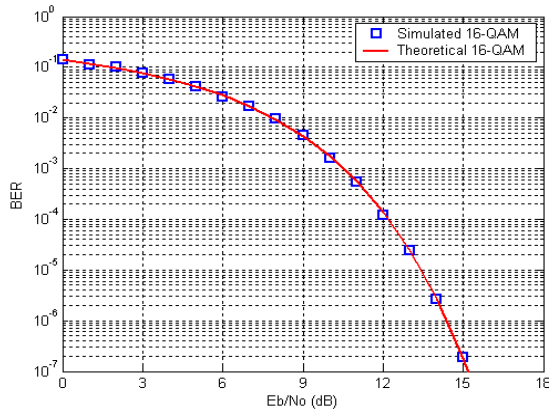
The simulated and theoretical results for BER as a function of  $E_b/N_0$  for the various QAM orders (16-QAM, 32-QAM, 64-QAM and 128-QAM) in the presence of Additive White Gaussian Noise are given in Figures 3-6.

The comparison of simulated results with the theoretical results shows that they both have their BER's increasing as the QAM order increases. From Figure-3 and Figure-5, it can be seen that for even-bit QAM constellations, such as 16-QAM and 64-QAM, the simulated results are almost identical with the theoretical results. But for case of QAM with odd bits per symbol, such as 32-QAM and 128-QAM there is a slight difference between the simulated and the theoretical curve for BER vs  $E_b/N_0$ . The curves are identical till 6 dB for 32-QAM (see Figure-4) and till 9 dB, respectively for 128-QAM (see Figure-6), after which they differ. Interestingly, as the QAM order increases, the  $E_b/N_0$  difference between theoretical and simulated curve, increases.

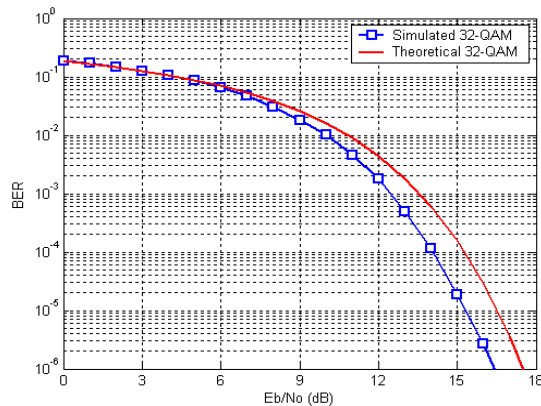
For example, from Figure-4 and Figure-6, it can be seen that, at a BER of  $10^{-4}$ , the  $E_b/N_0$  difference between theoretical and simulated graph is 1.12 dB for 32-QAM and 1.2 dB for 128-QAM. In other words, the



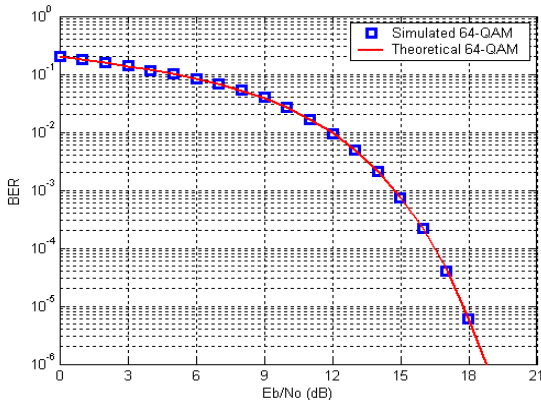
simulation results provide a better performance over the theoretical results when BER is less than 0.01 in an AWGN channel.



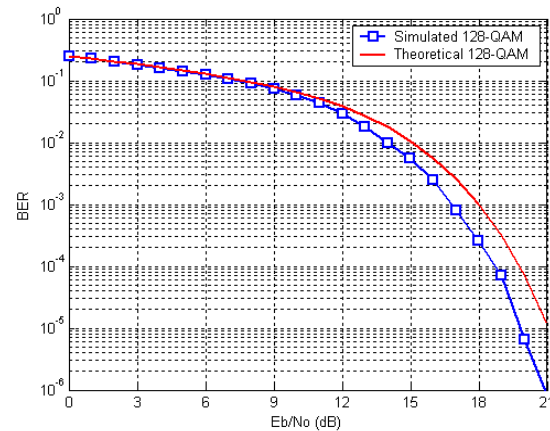
**Figure-3.** Comparison of simulated and theoretical BER performance of 16-QAM in the presence of additive white gaussian noise.



**Figure-4.** Comparison of simulated and theoretical BER performance of 32-QAM in the presence of additive white gaussian noise.



**Figure-5.** Comparison of simulated and theoretical BER performance of 64-QAM in the presence of additive white gaussian noise.



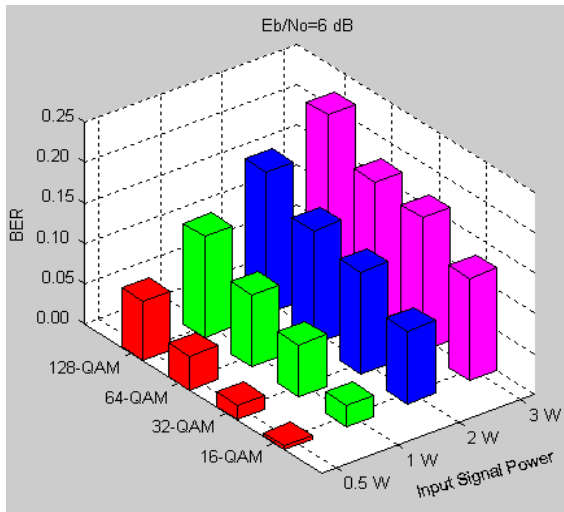
**Figure-6.** Comparison of simulated and theoretical BER performance of 128-QAM in the presence of additive white gaussian noise.

The difference between the theoretical and simulation results is because in case of odd bits per symbol, the theoretical approach is based on the BER expression of rectangular QAM modulation (see Equation (7)) whilst the Monte Carlo simulation is performed, provided that the modulator in the model presented in Figure-1, generates cross-shaped QAM constellations.

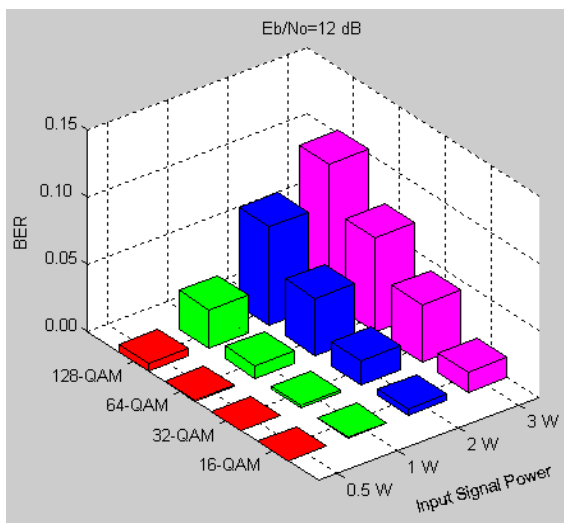
BER performance of M-QAM modulation schemes depends also on the input signal power. The impact of changing the input signal power on the BER of M-QAM variants where  $M=16, 32, 64$  and  $128$  for two values of  $E_b/N_0$ , namely,  $E_b/N_0 = 6$  dB and  $E_b/N_0 = 12$  dB is illustrated in Figure-7 and Figure-8, respectively.

To represent the results in the manner as shown in Figure-7 and Figure-8, the data sets obtained in BERTool, are exported to the Matlab workspace and are processed using `bar3` Matlab command. According to these results, it can be concluded that when the power of the input signal is increased, the BER of M-QAM modulation schemes for  $M=16, 32, 64$  and  $128$  is also increased. This means that the BER performance becomes worse.





**Figure-7.** BER for different QAM modulation orders as a function of input signal power at  $E_b/N_0 = 6$  dB.



**Figure-8.** BER for different QAM modulation orders as a function of input signal power at  $E_b/N_0 = 12$  dB.

From Figure-7 and Figure-8, it can be concluded that when the input signal power is increased from 0.5W to 3W, the value of BER for  $E_b/N_0 = 6$  dB is increased at about 28 times for 16-QAM, 9 times for 32-QAM, 4 times for 64-QAM and 3 times for 128-QAM. At the same time, for  $E_b/N_0 = 12$  dB, the increase of BER is approximately 2250 times for 32-QAM, 108 times for 64-QAM and 18.5 times for 128-QAM. The reason of BER performance reduction, when the input signal power is increased, is due to the proportional relation between the signal power and the noise variance (see Equation (12)).

## 5. CONCLUSIONS

In this paper, four M-QAM modulation schemes (16-QAM, 32-QAM, 64-QAM and 128-QAM) are studied in order to evaluate their BER performances in AWGN channel.

A simple simulation model of M-ary QAM is designed in Simulink environment. Moreover, the model is used with BERTool to illustrate its utilization to implement a Monte Carlo simulation approach in evaluating and comparing the performance of the different QAM modulation schemes. The theoretical approach of BERTool is also used to obtain BER performance curves of M-QAMs. The theoretical results are compared with the simulation results and it is clearly observed that the BER for all the studied modulation schemes decreases monotonically when the values of  $E_b/N_0$  are increased. From the curves for BER vs  $E_b/N_0$  it can be seen that as the order of QAM modulation increases, the BER is also increases. The simulation results illustrate that the BER performance of the modulation schemes becomes worse when the input signal power is increased. This is due to the proportional relation between the input signal power and the noise variance. From the results, it can also be concluded that for QAM orders with even bits (16-QAM and 64-QAM), the simulated BER curve coincides with the theoretical BER curve, while for QAM orders with odd bits (32-QAM and 128-QAM) the curves coincide only partially at low values of  $E_b/N_0$ . Based on the BER results for the case of odd bits per symbol, it can be recommended the use of cross-QAMs instead of rectangular QAMs. The cross-QAM is a better choice compared to rectangular QAM in terms of power efficiency.

Finally, it can be stated that Matlab/Simulink can be successfully used along with Matlab/BERTool in evaluating the performance not only of QAM modulation, but and of other digital modulation techniques, such as M-PSK, DQPSK, OQPSK, etc. The use of BERTool in conjunction with Simulink models to generate and evaluate BER data, can be helpful for many researchers, in the field of digital modulation techniques, in simplifying the process of passing from simulation to implementation without the necessity of being specialized hardware engineers.

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