PERIODIC ORTHOGONAL BINARY SIGNALS WITH ZERO CROSS CORRELATION

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ABSTRACT

This paper presents a method of constructing orthogonal binary sequences with any positive even integer lengths. We show that periodic binary signals, coded in accordance with these orthogonal binary sequences, contain groups of signals with zero cross correlation. It is possible to construct an ensemble of binary signals with zero cross correlation properties by using only one coded signal from each of these groups. When the ensemble of binary signals is transmitted through one communication channel, they do not interfere with each other and can be separated on the receiver side without tight synchronization. The results of this paper can be used in asynchronous CDMA communication, telemetric networks (e.g. Wireless Body Area Networks (WBAN), Wireless Sensor Networks (WSN)) and optical systems.

Keywords: binary signals cross correlation, CDMA, multiple access interference, walsh-hadamard codes.

INTRODUCTION

This paper presents construction methods of matrices of binary orthogonal sequences. These matrices have a N_nx N_2dimension, where $N_n = 2^n$ (n =1, 2, 3, …) is the number of rows (i.e., the number of binary sequences) and N_2(N_2 is an even positive integer: N_2 = 2, 4, 6, 8, 10, …) is the number of columns (i.e., the length of the sequences). These matrices are similar to Walsh-Hadamard (WH) matrices [1] in that they exhibit orthogonality properties and the presence of groups of sequences which correspond to periodic signals with zero cross correlation (CC), i.e. periodic signals with zero multiple access interference (MAI). These periodic signals do not have mutual frequency components [12] and they are orthogonal in the time domain with the presence of any time shift between them. WH sequences corresponding to periodic signals with zero CC, i.e. zero MAI, are also named Cyclically Orthogonal subsets (groups) of Walsh functions [6], or Cyclic Orthogonal WH Codes (COWHC) [7, 8], or signals with ideal cross correlation (ICC) [10]. Periodic binary orthogonal sequences with zero cross correlation for certain even values ofN_2 were considered in [2, 3], however the authors did so under conditions that the sequences have the same least period (Appendix 1).Periodic sequences, including binary sequences, with zero cross correlation were also considered in [11], but under conditions related with their autocorrelation properties (Appendix 2). This paper does not impose either of the two sets of conditions from [2, 3, 11] for periodic binary orthogonal sequences with zero CC.

FORMATION OF ORTHOGONAL BINARY MATRICES

Binary orthogonal functions were introduced by J. L. Walsh in 1923 [1]. These orthogonal functions are widely used in the theory of discrete signals. In practice, binary signals, coded in accordance with Walsh functions, are used in modern CDMA wireless communication systems [4] as channelization codes. As is well known, orthogonal WH matrices exist only for $N_n = 2^n$, where n = 1, 2, 3, ….

Walsh-Hadamard matrices can be represented as

$$H_{Nn} = \begin{bmatrix} H_{Nn/2} & H_{Nn/2} \\ H_{Nn/2} & -H_{Nn/2} \end{bmatrix},$$

where H_1 = [+] is the single or elementary Hadamard matrix and $N_n = 2^n$ is the size of the WH matrix

It is possible to represent the same matrices in another way, namely

$$H_{Nn} = \begin{bmatrix} H_{Nn/2} & H_{Nn/2} \\ H_{Nn/2} & -H_{Nn/2} \end{bmatrix} = H_2 \otimes H \otimes \ldots H_2 \otimes H_1 =$$
$$= (H_2\otimes)^n \otimes H_1 = (H_2\otimes)^n, \quad (1)$$

where matrix $H_2$is the Hadamard second order matrix

$$H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix} = \begin{bmatrix} + & + \\ + & - \end{bmatrix}.$$

The symbol "⊗" is called the symbol of Kronecker multiplication (Kronecker product) and is defined as [5]:

$$A \otimes B = \begin{bmatrix} a_{11} * B & \cdots & a_{1n} * B \\ \vdots & \ddots & \vdots \\ a_{m1} * B & \cdots & a_{mn} * B \end{bmatrix},$$

where A is a matrix of size m x n and B is also a matrix.

In general, Kronecker multiplication is a non-commutative procedure and thus $A \otimes B \neq B \otimes A$. In this context, the notation $(H_2\otimes)^n$ means applying the Kronecker multiplication procedure n times.

Thus, the representation of matrix H_{Nn} (1) is formed step by step in n steps of Kronecker multiplications of the elementary Hadamard matrix $H_1$by the second order matrix H_2 (1). During this (step by step)

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procedure, the length of sequences (length of rows) and the number of rows (i.e., number of sequences) of matrices H_{Nn} are increased by a factor of two after each Kronecker multiplication by matrix H_2.

For instance, for $N_n= 2^n =2$ (n=1) and $N_n= 2^n = 4$ (n=2), WH matrices have the following form (Figure-1)

$$H_2= H_2\otimes H_1 = H_2\otimes[+] = \begin{bmatrix} + & + \\ + & - \end{bmatrix},$$

$$H_4 = H_2\otimes H_2\otimes H_1=H_2\otimes H_2= \begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix}. \qquad (2)$$

We will show that by using the same Kronecker multiplication procedure (1), it is possible to create orthogonal binary systems which consist of orthogonal binary sequences with the length of any even number N_2, where N_2 = 2, 4, 6, 8, 10, 12, ….Note that orthogonal binary sequences can exist only for even integers $N = N_2$ = 2, 4, 6, 8, 10, 12, ….

Walsh-Hadamard Matrices

$\mathbf{N_1 = 1}$

$H_1 = [+]$ Elementary Hadamard matrix

$\mathbf{N_n = 2^n = 2 \quad (n = 1)}$

$$H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix} = \begin{array}{c|cc} & \downarrow k & \downarrow G \\ \begin{bmatrix} + & + \\ + & - \end{bmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} & \begin{matrix} G_1 \\ G_2 \end{matrix} \end{array}$$

$\mathbf{N_n = 2^n = 4 \quad (n = 2)}$

$$H_4 = (H_2\otimes)^2 = H_2\otimes H_2 =$$

$$= H_2\otimes\begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} =$$

$$= \begin{array}{ccc} & \downarrow k & \downarrow G \\ \left[\begin{array}{cc|cc} + & + & + & + \\ + & - & + & - \\ \hline + & + & - & - \\ + & - & - & + \end{array}\right] & \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{matrix} G_1 \\ G_2 \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} G_3 \end{matrix} \end{array}$$

$\mathbf{N_n = 2^n = 8 \quad (n = 3)}$

$$H_8 = (H_2\otimes)^3 = H_2\otimes H_4 =$$

$$= \begin{array}{ccc} & \downarrow k & \downarrow G \\ \left[\begin{array}{cccc|cccc} + & + & + & + & + & + & + & + \\ + & - & + & - & + & - & + & - \\ + & + & - & - & + & + & - & - \\ + & - & - & + & + & - & - & + \\ \hline + & + & + & + & - & - & - & - \\ + & - & + & - & - & + & - & + \\ + & + & - & - & - & - & + & + \\ + & - & - & + & - & + & + & - \end{array}\right] & \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{matrix} G_1 \\ G_2 \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} G_3 \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} G_4 \end{matrix} \end{array}$$

Figure-1. Walsh-hadamard matrices.

The natural sequence of positive integers N consists of even integers N_2 (N_2 =2, 4, 6, 8, 10, …) and odd integers N_1 (N_1= 1, 3, 5, 7, 9, 11, …). Any even integer N_2can be represented as the function of an odd integer N_1, i.e. $N_2 = f(N_1, n)$ and can be represented in multiplication form:

$N_2= f(N_1, n)= N_1*2^n$, where n = 1, 2, 3, 4, 5, ….

The following are examples of this representation for several values of N_1:

if N_1=1, $N_2 = f(N_1)$= 2, 4, 8, 16, 32, 64, …,
if N_1=3, $N_2 = f(N_1)$ = 6, 12, 24, 48, 96, …,
if N_1=5, $N_2 = f(N_1)$ = 10, 20, 40, 80, 160, …, (3)
if N_1=7, $N_2 = f(N_1)$ = 14, 28, 56, 112, 224, …,
and etc.

We introduce the matrix A_{N1} as a matrix-row consisting of a binary sequence with N_1elements, where N_1 are odd integers (N_1 = 1, 3, 5, 7, 9, …), as

$$A_{N1}=\underbrace{[+ \; + \; - \; + \; - \; - \cdots\cdots\cdots]}_{N_1}, \qquad (4)$$

where N_1 is the length of the sequence. Each of the elements of matrix-row A_{N1}can have only two values, namely, plus or minus.

We utilize the same approach to forming binary orthogonal matrices as shown in (1) using matrix-row A_{N1} (4) instead of elementary Hadamard matrix H_1 =[+]. With this change, matrix A_{N1} will be multiplied by matrix H_2(step by step) from the left side using the same multiplication symbol $\otimes$ called Kronecker multiplication or Kronecker product

$$A_{N2}(N_1, n)= H_2\otimes \ldots\ldots\otimes H_2\otimes H_2\otimes A_{N1} =$$

$$= (H_2\otimes)^n\otimes A_{N1} = H_{Nn}\otimes A_{N1} =$$

$$= \begin{array}{cc} & \downarrow k \\ \begin{bmatrix} A_{N1} & A_{N1} & \cdots & A_{N1} \\ A_{N1} & -A_{N1} & \cdots & -A_{N1} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1} & -A_{N1} & \cdots & A_{N1} \end{bmatrix} & \begin{matrix} 0 \\ 1 \\ \cdots \\ N_n - 1 \end{matrix} \end{array}. \qquad (5)$$

where $N_2 = N_1 * 2^n$, $N_n = 2^n$ (n = 1, 2, 3, …) , and H_{Nn} is a WH square matrix with size N_n,

$$H_{Nn}= (H_2\otimes)^n= \begin{array}{cc} & \downarrow k \\ \begin{bmatrix} + & + & + & + & \cdots & \cdots & + \\ + & - & + & - & \cdots & \cdots & - \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ + & - & - & + & \cdots & \cdots & + \end{bmatrix} & \begin{matrix} 0 \\ 1 \\ \cdots \\ N_n - 1 \end{matrix} \end{array} \qquad (6)$$

and k (k = 0, 1, 2, 3, …., N_2-1) is the matrix row number.

We observe from (5) that matrix $A_{N2}(N_1, n)$is the result of the Kronecker multiplication procedure of matrix A_{N1} by matrix H_2 n times, step by step. For instance, the first step is $H_2 \otimes A_{N1}$, the second step is $H_2 \otimes (H_2 \otimes A_{N1})$, the third step is $H_2 \otimes (H_2 \otimes (H_2 \otimes A_{N1}))$, etc., and $(H_2 \otimes)^n \otimes A_{N1}$ after n steps. During this procedure, the length of matrix rows (length of matrix sequences) and the number of matrix rows increases by a factor of two after each multiplication step. In the common case, matrices A_{N2} (N_1, n) are rectangular matrices with size2^nx ($N_1 * 2^n$), where $N_n=2^n$ is the number of rows and ($N_1 * 2^n$) is the number of columns (5), (6). The number of rows $N_n=2^n$ defines the number of binary sequences and the number of columns ($N_1 * 2^n$) = N_2 defines the length of orthogonal sequences of matrices A_{N2} (N_1, n).

Thus, for any even integer N_2(N_2 = 2, 4, 6, 8, 10, …),the Kronecker multiplication of matrix-row A_{N1}(4)by the WH square matrix H_{Nn} of size $N_n= 2^n$(5) represents matrix A_{N2} (N_1, n).And if the value of N_1 (length of matrix row A_{N1}(4)) and the value of n (number of multiplication steps) are known, it is possible to find the length of rows (length of sequences)and number of rows (number of sequences) of matrix A_{N2}(N_1, n)(5), i.e. $N_2 = N_1 * N_n = N_1 * 2^nandN_n=2^n$ respectively (5), (6). For example, for n =1 we have

$$A_{N2}(N_1, n=1) = (H_2\otimes)^n \otimes A_{N1} = \\ = H_2 \otimes A_{N1} = \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N1} = \begin{bmatrix} A_{N1} & A_{N1} \\ A_{N1} & -A_{N1} \end{bmatrix} \quad (7)$$

and for n =2 we have

$$A_{N2}(N_1, n=2) = (H_2\otimes)^n \otimes A_{N1} = \\ = H_2 \otimes H_2 \otimes A_{N1} = H_2 \otimes (H_2 \otimes A_{N1}) = \\ = H_2 \otimes \begin{bmatrix} A_{n1} & A_{n1} \\ A_{n1} & -A_{n1} \end{bmatrix} = \begin{bmatrix} A_{N1} & A_{N1} & A_{N1} & A_{N1} \\ A_{N1} & -A_{N1} & A_{N1} & -A_{N1} \\ A_{N1} & A_{N1} & -A_{N1} & -A_{N1} \\ A_{N1} & -A_{N1} & -A_{N1} & A_{N1} \end{bmatrix}. \quad (8)$$

EXAMPLES OF ORTHOGONAL BINARY SEQUENCES

Consider the following three examples:

N_1 = 1:

In the N_1 = 1 case, matrix row A_{N1} (4) consists of only one element, namely

$A_{N1=1}= [+]$,

Where N_1 is the number of elements in matrix-row A_{N1} (4) and $N_1$is also the length of sequence A_{N1}.

In this case, matrix A_{N2}may be represented (5), for any n = 1, 2, 3, 4, … , as

$$A_{N2}(N_1,n) = (H_2\otimes)^n \otimes A_{N1=1} = \\ = H_{Nn} \otimes A_{N1=1} = H_{Nn} \otimes [+] = H_{Nn}, \quad (9)$$

where H_{Nn} is the WH square matrix of size $N_n = 2^n$ (6). Thus, when N_1 = 1, matrices A_{N2}(N_1, n) are the same as WH matrices H_{Nn} because matrix $A_{N1=1}$= [+] is the same as the elementary Hadamard matrix H_1 = [+] which is used to create WH matrices (1).

This approach (9) can be utilized to create orthogonal binary systems with sequences of length $N_n= N_1 * 2^n$(N_1=1, n =1, 2, 3, 4, …), or with a length N_n = 2, 4, 8, 16, … (1), (3). WH matrices for $N_2=N_n$ = 2 (n=1), $N_2 = N_n$ = 4 (n=2) and $N_2 = N_n$ = 8 (n=3) are represented above in (2) and in Figure-1.

N_1 = 3:

In the N_1 =3 case, matrix-row $A_{N1}$is represented as

$$A_{N1=3} = [+ \ - \ +], \quad (10)$$

where N_1 = 3is the number of elements in matrix-row A_{N1} (4). The original matrix-row A_{N1}can be taken arbitrarily as a sequence from the all binary sequences with the length N_1 = 3. In this case, the matrix A_{N2}(N_1,n)can be represented in the form of (5):

$$A_{N2}(N_1,n) = (H_2\otimes)^n \otimes A_{N1=3} = \\ = H_{Nn} \otimes A_{N1=3} = H_{Nn} \otimes [+ \ - \ +]. \quad (11)$$

Using this approach (11), one can create orthogonal binary systems with the sequences of length $N_2 = N_1 * 2^n = 3 * 2^n$(n =1, 2, 3, 4, …) or of length N_2= 6, 12, 24, 48, 96, … (3). Examples of matrices A_{N2} (N_1,n)for N_2 = 6, N_2= 12 (n=1 and n=2 respectively) and N_1=3 (10) are represented in Figure-2.

N_1 = 5:

In the N_1=5 case, it is possible to create orthogonal binary systems with sequences of length $N_2 = N_1 * 2^n = 5 * 2^n$(n =1, 2, 3, 4, …) or with length N_2= 10, 20, 40, 80, 160, … (3).Examples of matrices A_{N2}(N_1,n) for N_2= 10, N_2= 20 (n =1 and n =2 respectively) and N_1=5 are represented in Figure-3. Matrix A_{N1} (Figure-3) is represented as $A_{N1=5}=[+ \ + \ + \ - \ +]$.

We observe that in the N_1=7 case, it is possible to create orthogonal binary matrices with sequences of length $N_2 = N_1 * 2^n = 7 * 2^n$(n =1, 2, 3, 4, …) or with sequences length of N_2= 14, 28, 56, 112, 224, … (3), and in the N_1=9 case, it is possible to create orthogonal binary matrices with length $N_2 = N_1 * 2^n = 9 * 2^n$(n =1, 2, 3, 4, …) or with length N_2= 18, 36, 72, 144, … (3), and for any even number N_2using the same approach.

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$$\mathbf{N_1 = 3 \quad A_{N1=3} = [+ - +]}$$

$$\mathbf{n = 1, \ N_2 = N_1 * 2^n = 6}$$

$$A_{N2=6}\,(N_1,n) = H_2 \otimes A_{N1=3} =$$

$$= \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N1=3} =$$

$$= \begin{bmatrix} \mathbf{A_{N1=3}} & \mathbf{A_{N1=3}} \\ \mathbf{A_{N1=3}} & \mathbf{-A_{N1=3}} \end{bmatrix} =$$

$$\begin{array}{rll} & \downarrow k & \downarrow G \\ = \left[\begin{array}{ccc|ccc} + & - & + & + & - & + \\ \hline + & - & + & - & + & - \end{array}\right] & \begin{array}{l} 0 \\ 1 \end{array} & \begin{array}{l} G_1 \\ G_2 \end{array} \end{array}$$

$$\mathbf{n = 2, \ N_2 = N_1 * 2^n = 12}$$

$$A_{N2=12}\,(N_1,n) = (H_2\otimes)^2 \otimes A_{N1=3} =$$

$$= H_2 \otimes A_{N2=6} =$$

$$= \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N2=6} =$$

$$\begin{array}{rll} & \downarrow k & \downarrow G \\ = \left[\begin{array}{cccccc|cccccc} + & - & + & + & - & + & + & - & + & + & - & + \\ + & - & + & - & + & - & + & - & + & - & + & - \\ \hline + & - & + & + & - & + & - & + & - & - & + & - \\ + & - & + & - & + & - & - & + & - & + & - & + \end{array}\right] & \begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{array}{l} G_1 \\ G_2 \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} G_3 \end{array} \end{array}$$

Figure-2. Matrices $A_{N2}(N_1,n)$, $N_1=3$.

$$\mathbf{N_1 = 5 \quad A_{N1=5} = [+ + + - +]}$$

$$\mathbf{n = 1, \ N_2 = N_1 * 2^n = 10}$$

$$A_{N2=10}\,(N_1,n) = H_2 \otimes A_{N1=5} = \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N1=5}$$

$$\begin{array}{rll} & \downarrow k & \downarrow G \\ A_{N2=10}\,(N_1,n) = \left[\begin{array}{ccccc|ccccc} + & + & + & - & + & + & + & + & - & + \\ \hline + & + & + & - & + & - & - & - & + & - \end{array}\right] & \begin{array}{l} 0 \\ 1 \end{array} & \begin{array}{l} G_1 \\ G_2 \end{array} \end{array}$$

$$\mathbf{n = 2, \ N_2 = N_1 * 2^n = 20}$$

$$A_{N2=20}\,(N_1,n) = (H_2\otimes)^2 \otimes A_{N1=5} =$$

$$= H_2 \otimes A_{N2=10} = \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N2=10} =$$

$$\begin{array}{rll} & \downarrow k & \downarrow G \\ = \left[\begin{array}{cccccccccc|cccccccccc} + & + & + & - & + & + & + & + & - & + & + & + & + & - & + & + & + & + & - & + \\ + & + & + & - & + & - & - & - & + & - & + & + & + & - & + & - & - & - & + & - \\ \hline + & + & + & - & + & + & + & + & - & + & - & - & - & + & - & - & - & - & + & - \\ + & + & + & - & + & - & - & - & + & - & - & - & - & + & - & + & + & + & - & + \end{array}\right] & \begin{array}{l} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{array}{l} G_1 \\ G_2 \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} G_3 \end{array} \end{array}$$

Figure-3. Matrices $A_{N2}(N_1,n)$, $N_1=5$

In all of examples (Figure-1, Figure-2, and Figure-3), the value k represents the number of matrix $A_{N2}(N_1,n)$rows (number of binary sequences), where k = 0, 1, 2, 3, 4, ……, 2^n-1. The total number of rows is $N_n= 2^n$ and the total number of columns (the length of sequences) is $N_2= N_1{*}2^n$ (n =1, 2, 3, 4, …).As previously described, in the common case matrices A_{N2} (N_1, n) are rectangular matrices with size N_nx ($N_1{*}2^n$), where $N = N_n = 2^n$ is the number of rows and ($N_1{*}2^n$) is the number of columns. In only one case, when N_1 =1, matrices A_{N2} (N_1, n)are square Walsh-Hadamard matrices with size $N_n= 2^n$sincematrix A_{N1}consists of a matrix-row with only one element: A_{N1} = [+] (9).

PROPERTIES OF ORTHOGONAL BINARY SEQUENCES

The primary properties of matrices A_{N2} (N_1, n) (N_2 = 2, 4, 6, 8, 10, …) are determined by the properties of WH matrices H_{Nn} (1). The first property of interest of WH matrices is the orthogonal property, whereby mutual orthogonal properties exist between of all rows (or between all binary sequences) of matrices H_{Nn}. In our case, matrices A_{N2} (N_1, n) are orthogonal. This is evident from the structure of A_{N2} matrices (5), (6), (7), and (8).

In [6, 9, 10] it was shown that Walsh-Hadamard matrices of size $N_n = 2^n$ have one even more important property; namely all WH matrices consist of groups of sequences with zero cross correlation (CC) (i.e., with zero multiple access interference (MAI) or with ideal cross correlation (ICC) properties) when applied to periodic signals. The number of groups G (G_1,G_2, G_3, …, G_{n+1}) of sequences with zero CC is defined by the size of the WH matrices and equals n+1. New groups of sequences appear after each step of the Kronecker multiplication procedure during formation of WH matrices (1). After the first multiplication step (n=1), group G_1 and group G_2 appear. After the second multiplication step (n=2), group G_3 appears, and etc. Figure-1. The first group, group $G_{1,}$ consists of a sequence which corresponds to the k=0 row (Figure-1). The second group, group G_2, corresponds to the k=1 row. The third group, group G_3, consists of two sequences which correspond to the k=2 and k=3 rows. The fourth group, group G_4, consists of four sequences which correspond the k=4, k=5, k=6, and k=7 rows, and etc. The last group, group G_{n+1}, consists of sequences which correspond to the k=N/2 through k=N-1 rows. By using the same frequency approach as in [10, 12], it is possible to show that matrices A_{N2} (N_1,n) (5) also exhibit the zero CC property of WH matrices. Groups G (G_1, G_2, G_3, …, G_{n+1}) are represented in Figure-2 and Figure-3 in comparison with the same groups of sequences of WH matrices (Figure-1). Applying the same approach to periodic signals coded by WH sequences, it was shown [10, 12] that each group of signals G_1, G_2, G_3, …, G_{n+1} has its own discrete frequencies which are different for each group G of signals. However all signals which belong to the same group G have mutual frequencies and therefore these periodic signals do not have zero cross correlation properties. By selecting sequences from different groups (but only one sequence from each group), it is possible to form an ensemble of signals which correspond to periodic

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coded signals with zero CC (i.e., with zero MAI or with ICC properties). There can be no more than n+1 total coded signals in this ensemble for any even integer $N_2 = f(N_1, n) = N_1 * 2^n$.

GENERAL APPROACH TO FORMATION OF ORTHOGONAL BINARY SEQUENCES WITH ZERO CROSS CORRELATION

As stated earlier, matrices rows A_{N1} can be taken arbitrarily as sequences of '+'s and '-'s with length N_1. The total number of different sequences A_{N1} depends on the length of sequence N_1. It easy to show that total number of different binary sequences with length N_1 is equal to 2^{N1}. Usually we use only half of these sequences, namely, $m = 2^{N1-1}$, because the second half of the sequences is the "mirror reflection" of the first half with exactly the reverse polarity.

All of the different binary sequences of length N_1 can be represented in matrix form as

$$A_{N1,m} = \underbrace{\begin{bmatrix} + & + & + & \cdots & \cdots & + & + \\ + & - & + & \cdots & \cdots & - & + \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ + & + & - & \cdots & \cdots & \cdots & + \\ + & - & - & \cdots & \cdots & \cdots & - \end{bmatrix}}_{N_1} = \begin{array}{c} \\ \begin{bmatrix} A^0{}_{N1} \\ A^1{}_{N1} \\ \cdots \\ A^{m-2}{}_{N1} \\ A^{m-1}{}_{N1} \end{bmatrix} \end{array} \begin{array}{c} \downarrow k \\ 0 \\ 1 \\ \cdots \\ m-2 \\ m-1 \end{array} = [A^k{}_{N1}]. \tag{12}$$

Matrix $A_{N1,m}$ consists of m rows and N_1 columns (length of sequences N_1), where m is the total number of different binary sequences $m = 2^{N1-1}$ with a length of N_1 and k is a row number (k = 0, 1, 2, 3, …, m-1). For example, $A^k{}_{N1}$ is the sequence which corresponds to row number k. None of the binary sequences are orthogonal because the sequence lengths N_1 are odd numbers.

All m binary sequences with length N_1 can be used to create matrices $A_{N2,m}(N_1,n)$ if we consider the application of the Kronecker multiplication procedure to matrix $A_{N1,m}$ (12) as in (5)

$$A_{N2,m}(N_1,n) = H_{Nn} \otimes A_{N1,m} = (H_2\otimes)^n \otimes A_{N1,m} =$$
$$= \begin{bmatrix} A_{N1,m} & A_{N1,m} & \cdots & A_{N1,m} \\ A_{N1,m} & -A_{N1,m} & \cdots & -A_{N1,m} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1,m} & -A_{N1,m} & \cdots & A_{N1,m} \end{bmatrix} \begin{array}{c} \downarrow k \\ 0 \\ 1 \\ \cdots \\ N_n - 1 \end{array}, \tag{13}$$

where H_{Nn} is the WH square matrix with size $N_n = 2^n$ (n = 1, 2, 3, …) and $N_2 = N_1 * 2^n$ (6).

In the $N_1 = 3$ case, matrix $A_{N1=3,m}$ can be represented as

$$A_{N1=3,m} = \begin{bmatrix} + & + & + \\ + & - & + \\ + & + & - \\ + & - & - \end{bmatrix} \begin{array}{c} \downarrow k \\ 0 \\ 1 \\ 2 \\ 3 \end{array}, \tag{14}$$

where $m = 2^{N1-1} = 4$ is the number of different binary sequences with length of N_1 which have only plus signs at the beginning of each sequence and k is the row number (k = 0, 1, …, m -1). After the first multiplication step (n=1) (13), we have:

$$A_{N2=6,m}(N_1=3, n=1) = H_2 \otimes A_{N1=3,m} =$$
$$= \begin{bmatrix} + & + \\ + & - \end{bmatrix} \otimes A_{N1=3,m} = \begin{bmatrix} A_{N1=3,m} & A_{N1=3,m} \\ A_{N1=3,m} & -A_{N1=3,m} \end{bmatrix} \tag{15}$$

After the second multiplication step (n=2) (13), we have:

$$A_{N2=12,m}(N_1=3, n=2) = (H_2\otimes)^n \otimes A_{N1=3,m} =$$
$$= H_2 \otimes (H_2 \otimes A_{N1=3,m}) =$$
$$= \begin{bmatrix} A_{N1=3,m} & A_{N1=3,m} & A_{N1=3,m} & A_{N1=3,m} \\ A_{N1=3,m} & -A_{N1=3,m} & A_{N1=3,m} & -A_{N1=3,m} \\ A_{N1=3,m} & A_{N1=3,m} & -A_{N1=3,m} & -A_{N1=3,m} \\ A_{N1=3,m} & -A_{N1=3,m} & -A_{N1=3,m} & A_{N1=3,m} \end{bmatrix}. \tag{16}$$

Matrices $A_{N2=6,m}$ (N_1=3, n=1) (15) and $A_{N2=12,m}$ (N_1=3, n=2) (16) are represented in Figure-4 and Figure-5 respectively. It is important to note that these matrices are not orthogonal matrices, because matrix $A_{N1=3,m}$ (14) is not orthogonal. In Figure- 4 and Figure-5, after the first step of Kronecker multiplication (n=1), Group 1 and Group 2 appear and in this case (m>1) these groups consist of m sequences (for N_1 =3, m = 4). However, when using WH matrices (Figure-1) and matrices A_{N2} (N_1=3, n=1) (Figure-2), after the first step of Kronecker multiplication (n=1), Group 1 and Group 2 consist of only one sequence. This difference appears in the case of WH matrices (Figure-1) and orthogonal matrices A_{N2} (N_1, n) (Figure-2) because only one sequence A_{N1} (4), (5) was used to create these matrices.

$$\mathbf{N_1 = 3, \quad m = 2^{N1-1} = 4}$$

$$A_{N1,m} = \begin{bmatrix} + & + & + \\ + & - & + \\ + & + & - \\ + & - & - \end{bmatrix} \begin{array}{c} \downarrow k \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

$$\mathbf{N_2 = N_1 * 2^n = 6 \quad (n=1)}$$

$$A_{N2=6,m}(N_1,n) = H_2 \otimes A_{N1=3,m} = \begin{bmatrix} + & + & + & + & + & + \\ + & - & + & + & - & + \\ + & + & - & + & + & - \\ + & - & - & + & - & - \\ + & + & + & - & - & - \\ + & - & + & - & + & - \\ + & + & - & - & - & + \\ + & - & - & - & + & + \end{bmatrix} \begin{array}{c} \downarrow k \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \downarrow G \\ \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} G_1 \\ \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} G_2 \end{array}$$

Figure-4. Matrix $A_{N2,m}(N_1,n)$, N_1=3, n=1, m=4.

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In all the matrices represented in the first three figures (Figure-1, Figure-2, and Figure-3), Group 1 (G_1) sequences and Group 2 (G_2) sequences appear after the first Kronecker multiplication (n=1). After the second Kronecker multiplication (n=2), Group 3 (G_3) sequences appear, after the third multiplication (n=3), Group 4 (G_4) sequences appear, etc. And after each step of Kronecker multiplication, the number of rows (number of sequences) and number of columns (length of sequences) of the matrices both increase by a factor of two.

When we consider matrix $A_{N2=6,m}(N_1=3,\ n=1)$ (Figure-4), we observe (16) that after the first Kronecker multiplication (n=1) (15) there are eight binary sequences of length $N_2 = N_1 * 2^n = 6$. The first four of them (k = 0, 1, 2, 3) belong to Group 1 (G_1) and none of them are orthogonal (15). The other four sequences (k = 4, 5, 6, 7) belong to Group 2 (G_2) and all of them are orthogonal to any Group 1 (G_1) sequences. Consequently periodic signals coded by any sequences of Group 2 (G_2) have zero

$$\mathbf{N_1 = 3,\quad m = 2^{N1\text{-}1} = 4}$$

$$\mathbf{N_2 = N_1 * 2^n = 12\ \ (n=2)}$$

$$A_{N2=12,m} = (H_2\otimes)^2\otimes A_{N1=3,m} = H_2\otimes A_{N2=6,m} = \begin{array}{ccc} & \downarrow k & \downarrow G \\ \left[\begin{array}{cccccc|cccccc}
+&+&+&+&+&+&+&+&+&+&+&+\\
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\end{array}\right] & \begin{array}{c} 0\\1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15 \end{array} & \begin{array}{l} \left.\begin{array}{c}\\ \\ \\ \\ \end{array}\right\} G_1 \\ \left.\begin{array}{c}\\ \\ \\ \\ \end{array}\right\} G_2 \\ \left.\begin{array}{c}\\ \\ \\ \\ \\ \\ \\ \\ \end{array}\right\} G_3 \end{array} \end{array}$$

Figure-5. Matrix $A_{N2,m}(N_1,n)$, N_1=3, n=2, m=4.

CC with signals coded by any sequences of Group 1 (G_1). However, periodic signals which correspond to sequences inside of the same group are not orthogonal and do not have zero cross correlation between of them. As the result of this property, using all eight of these sequences of matrix $A_{N2=6,m}$ (N_1=3, n=1) (Figure-4), it is only possible to create pairs of periodic signals with zero cross correlation. One signal of each pair corresponds to any sequence from Group 1(G_1) and the other signal of the pair corresponds to any sequence from Group 2(G_2). The total number of different pairs of periodic signals with zero cross correlation will be $m * m = m^2 = 2^{2(N1-1)}$, which in the $N_1 = 3$ case equals 16. In other words, from binary sequences with length $N_2 = 6$, using this approach it is theoretically possible to create 16 different pairs of periodic signals with ICC properties. We observe that some of these periodic signals are distinguished from other periodic signals only by time shifts or "phase" shifts. For instance, for $A_{N2=6,m}$ (N_1=3, n=1) (Figure-4), periodic signals corresponding to k=1 and k=2 and periodic signals corresponding to k=4, k=6, and k=7 exhibit a periodic shift. As a result of this property, there will be only 3 x 2 = 6, and not 16 as calculated above, different pairs of periodic signals with length $N_2 = 6$.

After the second Kronecker multiplication step (n=2) (16), there are 16 sequences of length N_2 =12 in matrix $A_{N2=12,m}$ (N_1=3, n=2)(Figure-5) and Group 3 (G_3) appears. The first four sequences (k =0, 1, 2, 3) belong to Group 1 (G_1), another four sequences (k =4, 5, 6, 7) belong to Group 2 (G_2), and the remaining eight sequences (k =8, 9, 10, …, 15) belong to Group 3 (G_3). All sequences of any groups G are orthogonal to any sequences which belong to other groups and their corresponding signals have zero cross correlation. But sequences within each groups G are not orthogonal and their corresponding signals do not have zero cross correlation between any combination of them. This result means that by using all of these sixteen sequences of matrix $A_{N2=12,m}$ (N_1=3, n=2) (Figure-5), it is possible to create no more than triplets (sets of three signals) of periodic signals with zero cross correlations, where all three of these signals correspond to sequences from three different groups G of sequences, namely, G_1, G_2 and G_3. The total number of triplets (three signals per set) of periodic signal with ideal cross correlation properties will be $m * m * 2m = 2m^3 = 4 * 4 * 8 = 2^7 = 128$. In other words, it is possible to create 128 different triplets (three signals per set) of periodic binary signals with zero cross correlation from binary sequences with length $N_2 = 12$. Of course, as mentioned above for the $A_{N2=6,m}$ (N_1=3, n=1) case, it is necessary to take into account that some of these periodic signals will be distinguished from the other periodic signals only by time shifts or "phase" shifts. For instance, for $A_{N2=12,m}$ (N_1=3, n=2) (Figure-5), periodic signals corresponding to k=1 and k=2 (G_1), periodic signals corresponding to k=4, k=6 and k=7 (G_2), and periodic signals corresponding to k=8 and k=12, k=9 and k=13, k=10 and k=14, k=11 and k=15 (G_3), exhibit a periodic phase shift. As the result of this property, there will be only 3 x 2 x 4 = 24, and not 128 as theoretically calculated, different triplets of periodic signals with length $N_2 = 12$.

For N_1=5, there are m binary sequences with length $N_1 = 5$, where $m = 2^{N1-1} = 16$, and none are orthogonal to each other. After the first step of Kronecker multiplication (n = 1), there are 32 different binary sequences of length N_2 =10. The first 16 sequences belong to Group 1 (G_1), the remaining 16 belongs to Group 2 (G_2), and the total number of different pairs of periodic signals with zero cross correlation is $m^2 = 2^{2(N1-1)}$, which in the $N_1 = 5$ case equals 256. In other words, from binary sequences with length $N_2 = 10$ it is possible to create 256 different pairs of periodic signals with zero cross correlation. After the second step of Kronecker multiplication for $A_{N1=5,\ m}$ (16), there are 64 binary sequences of length $N_2 = 20$. And the theoretical total number of different triplets (three signals per set) of periodic signals with zero cross correlation are m * m * 2m = $2m^3$ = 16 * 16 * 32 = 2^{13} = 8192. Because some of these periodic signals are only distinguished from other periodic signals by time shifts or "phase" shifts, the number of different pairs and triplets with zero CC will total much less than 8192 as previously described for $N_1 = 3$.

As a generalization, it is possible to calculate how many different sets of periodic signals will exist after the third, fourth, etc. steps of Kronecker multiplication of matrix $A_{N1,m}$ (13) for any even integer $N_2 = f(N_1, n) = N_1 * 2^n$ (3). However, the total number of signals with zero cross correlation per one set (pair, triplet, quadruplet, etc.) is no larger than n +1 (n =1, 2, 3, 4, …).

Binary phase coded radio signals corresponding to these band based signals share the same properties [12]. Some samples of filters for separation of these zero cross correlation signals were presented in [12].

CONCLUSIONS

A method to create orthogonal binary sequences for any positive even number N_2 = 2, 4, 6, 8, 10, … is presented. For even values of $N_n = 2^n$ (N_n = 2, 4, 8, 16, …), these orthogonal binary sequences correspond to binary sequences of Walsh-Hadamard matrices. For even values of N_2 = 6, 10, 12, 14, 18, …, the orthogonal binary sequences have the same primary properties as Walsh-Hadamard functions, namely the orthogonal property and the presence of some groups of signals with zero cross correlation (i.e. with zero MAI or with the ICC property) when applied to periodic signals. The method of creating ensembles of periodic binary signals with ideal cross correlation properties for any even values of N_2 = 6, 10, 12, 14, 18, …. is also considered. By applying these results to radio signals, it is possible to create ensembles of independent orthogonal radio signals.

APPENDIX 1

The generation of periodic orthogonal binary sequences of the same least period with zero cross correlation was discussed in [2, 3]. It was shown that pairs of these binary sequences of length and least period N_2 = r x 2^n can be synthesized for all positive integers n and any nonprime odd integer r. As an example, the reference presents the pair of periodic orthogonal binary sequences (sequences C and D) of the same least period with zero cross correlation for N_2 = 18 (r = 9, n=1).

N_2 =18

C = (+ - + + - + + - + - + -- + -- + -)
D = (- + + + --- + + + + --- + + + -)

Using the results of [2,3], it is possible to create pairs of periodic orthogonal binary sequences of length and least period for any even $N_2 = N_1$ x 2^n, where n = 1 and N_1 (N_1 = 3, 5, 9, 11, …) are odd integers. These pairs of sequences have zero cross correlation. The following are examples of these pairs of sequences (sequences C_1, D_1, and sequences C_1, D_2) of length and least period N_2 = 6, 10, and 14:

N_2 = 6, (N_1 = 3, n = 1)
C_1 = (+ - + - + -)
D_1 = (+ + -- + +)
C_1 = (+ - + - + -)
D_2 = (+ ---- +)

N_2 = 10, (N_1 = 5, n = 1)
C_1 = (+ - + - + - + - + -)
D_1 = (+ + -- + + -- + +)
C_1 = (+ - + - + - + - + -)
D_2 = (+ + + ---- + + +)

N_2 = 14, (N_1 = 7, n = 1)
C_1 = (+ - + - + - + - + - + - + -)
D_1 = (+ + + --- + + --- + + +)
C_1 = (+ - + - + - + - + - + - + -)
D_2 = (+ - + - + + + + + + - + - +)

APPENDIX 2

Some sets of periodic binary sequences, called periodic perfect cross correlation (PPCC) sets of binary sequences with zero cross correlation for N=6, N=8 and N=12, were synthesized in [11] under a condition related with their autocorrelation properties.

These two binary sequences for N=6 are S_1 = (+ + + + + +) andS_2 = (+ + + - - -). We observe that these sequences correspond to two sequences of matrix $A_{N2=6,m}$(N_1=3, n=1) for N=6 with k=0 and k=4 (Figure-4). These two sequences belong to two different Groups, G_1 and G_2, therefore periodic signals corresponding to these two sequences have zero cross correlation. As shown in this paper, using all eight sequences of matrix $A_{N2=6,m}$ (N_1=3, n=1) (Figure-4), for N=6 it is possible to create 6 different pairs of periodic signals with zero cross correlation including the pair of sequences which were synthesized in [11].

The four binary sequences for N=8 which form the set of sequences with zero cross correlation were synthesized in [11]. These four sequences are S_1 = (+ + + + + + + +), S_2=(+ - + - + - + -), S_3=(+ - - + + - - +), and S_4=(+ - - + - + + -) and all are WH sequences for N=8 with k=0, k=1, k=3, and k=7 (Figure-1). Ensembles of signals corresponding to this set of WH sequences have zero cross correlation because all of these sequences belong to different Groups, namely G_1, G_2, G_3 and G_4 (Figure-1). As shown in [6, 10, 12], for N=8 it is possible to create ensembles of four signals from WH sequences, i.e. set of four signals, with zero cross correlation properties. Each signal in these ensembles corresponds to WH sequences from different Groups, namely G_1, G_2, G_3 and G_4 (Figure-1). Only two different ensembles, i.e. different sets, can be synthesized from four signals corresponding to WH sequences for N=8. The first ensemble corresponds to sequences with k=0, k=1, k=3, and k=7, i.e. S_1=(+ + + + + + + +), S_2=(+ - + - + - + -),

S_3=(+ - - + + - - +), and S_4=(+ - - + - + + -) and the second ensemble corresponds to sequences with k=0, k=1, k=3, and k=6, i.e. S_1 = (+ + + + + + + +), S_2 = (+ - + - + - + -), S_3 = (+ - - + + - - +) and S_4 = (+ + - - - - + +). The first ensemble corresponds to sequences which were synthesized in [11]. In theory, there should be eight different ensembles constructed from the four signals corresponding to WH sequences for N=8, but some of the WH sequences for N=8 only differed by a time shift, i.e. phase shift, applied to the periodic signals. The WH sequences which only differed by a time shift for N=8 correspond to k=2 and k=3, k=4 and k=6, and k=5 and k=7 (Figure-1). Furthermore, in two ensembles for N=8 above, one can replace the k=2 sequence with the k=3 sequence, the k=4 sequence with the k=6 sequence, and the k=5 sequence with the k=7 sequence.

Three sequences for N=12 with zero cross correlation were synthesized as the set of sequences in [11]. These three sequences are S_1 = (+ + + + + + + + + + + +), S_2 = (+ + + - - - + + + + - - -) and S_3 = (+ + + - - - - - - + + +). These sequences correspond to three sequences of matrix $A_{N2=12,m}$ (N_1=3, n=2)for N=12 with k=0, k=4, and k=12 (Figure-5), and all three sequences belong to three different Groups, namely G_1, G_2, and G_3, and therefore periodic signals corresponding to these three sequences have zero cross correlation. As shown in this paper, using all 16 sequences of matrix $A_{N2=12,m}$ (N_1=3, n=2) (Figure-5), for N=12 it is possible to create 24 different triplets of periodic signals, i.e. ensembles of three signals, with zero CC including the triplet of sequences above synthesized in [11].

REFERENCES

[1] J. L. Walsh. 1923. A closed set of normal orthogonal functions. Amer. J. Math. 45: 5-24.

[2] J. MacWilliams. 1967. An example of two cyclically orthogonal sequences with maximum period. IEEE Transactions on Information Theory. IT-13: 338-339.

[3] Domenic Calabro, Joseph Paolillo. 1968. Synthesis of Cyclically Orthogonal Binary Sequences of the Same Least Period. IEEE Transactions on Information Theory. IT-14(5): 756-759.

[4] William Stallings. 2005. Wireless Communications and Networks. 2e. Pearson Prentice Hall.

[5] J. W. Brewer. 1978. Kronecker Products and Matrix Calculus in System Theory. IEEE Transactions on Circuits and Systems, 25, No. 9, pp. 772-781.

[6] Y. Xia, Pu. Li, X. Shan, Y. Ren. 2002. Cyclically orthogonal subsets of Walsh functions. The 8th International Conference on Communication Systems. 1: 107-111.

[7] Jamshid Abouei. 2003. A set of Cyclic Orthogonal Codes Acquired from Walsh-Hadamard Matrix. In Proc. 34th International Mathematics Conference, Shahrood, Iran. pp. 813-819.

[8] Ali Tawfiq, Jamshid Abouei, Konstantinos N. Plataniotis. 2012. Cyclic Orthogonal codes In CDMA-Based Asynchronous Wireless Body Area Networks. In Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'12), Kyoto, Japan, pp. 1593-1596.

[9] A. V. Titov. 1982. Spectral Characteristics of Walsh-Hadamard Functions. Izvestia Vuzov. Radioelectronica (Radio electronics and Communication Systems). XXV(6): 91 (abstract only, No. 5078-81).

[10] V. L. Koblov, A. V. Titov, L. I. Iankovskiy. 1983. Properties of Spectral Characteristics of Walsh Functions. Questions of Radioelectronics. Ser. General Questions of Radioelectronics (Voprosiy Radioelectroniki. Ser. OVR), Issue 9, pp. 24-29. Published by Russian Institute of Radionavigation and Time (http://www.rirt.ru).

[11] Lifang Feng, Pingzhi Fan, Li Hao. 2007. On the Spreading Sequences with Perfect Periodic Cross Correlation Properties. 3rd International Workshop on Signal Design and its Application in Communication (IWSDA 2007). Chegdu, China. pp. 53-55.

[12] A. V. Titov and G. J. Kazmierczak. 2017. Periodic binary signals with zero cross correlation based on Walsh sequences. ARPN Journal of Engineering and Applied Science. 12(3): 746-752.