



ANALYSIS OF SURFACE CONTACT STRESS FOR A SPUR GEAR OF MATERIAL STEEL 15Ni2Cr1Mo28

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ABSTRACT

The main factors that cause the failure of gears are the bending stress and contact stress of the gear tooth. Stress analysis has been a key area of research to minimize failure and optimize design. This paper gives a finite element model for investigation of the stresses in the tooth during the meshing of gears. The model involves the involute profile of a spur gear for material Steel 15Ni2Cr1Mo28. The geometrical parameters, such as the face width and module, are considered important for the variation of stresses in the design of gears. Using modeling software, 3-D models for different modules in spur gears were generated, and the simulation was performed using ANSYS to estimate the bending and contact stresses. The Hertzian equation is used to calculate the contact stress. The results of the theoretical stress values are compared with the stress values from the finite element analysis.

Keywords: gearing, transmission system, root bending stress, surface contact stress, finite element analysis.

INTRODUCTION

Spur gears have straight teeth, are mounted on parallel shafts and are mainly used to create very large gear reductions. The pressure angle is an important factor for spur gears to prevent undercutting when the number of teeth is small and to adjust the center distance. Quality spur gears can be easily manufactured since the axial force is not produced [1].

The calculation of the tooth bending strength and surface durability of normal and high contact ratios may be sufficient for preliminary designs or standardized purposes, but the stresses calculated using those simple equations derived from the linear theory of elasticity and the Hertzian contact model are not in good agreement with experimented results [2]. The load distribution along the line of contact for a model, obtained from the minimum elastic potential criterion is considered to calculate the stresses of spur gear drives with transverse contact ratios. The load conditions are calculated, and the contact stress and the nominal tooth-root stress are computed [3]. The contact stress between two gear teeth is analyzed for different contact positions, which represents a pair of mating gears during rotation. Each case is represented by a sequence position of contacts between these two teeth. The spur and helical gear for different modules and face width are designed and the stresses are computed through AGMA, Lewis and Hertz equation [4]. The results are validated through von mises stresses in finite element model.

Models in the CAD software have been used to explain the stress and displacement field to determine the maximum equivalent stress and maximum displacement [5]. The quasi-static characteristic of finite element analysis allows the model to accurately simulate the distribution of equivalent stress and displacement change in the process of teeth meshing. The results agree well with the actual meshing. The stress calculated for a pair of gears using the Lewis formula, Hertz equation, and AGMA standards is comparable with FEA, and the PRO-E

software and finite element software are good tools to define a safe design [7, 10].

A complex problem is divided into smaller and simpler problems that can be solved using the existing knowledge of the mechanics of materials and mathematical tools.

Contact stress (Hertz equation)

The stresses on the surfaces of gear teeth are usually determined using formula derived from the work of H. Hertz; frequently, these stresses are called Hertz stress. Hertz determined the width of the contact band and the stress pattern when various geometric shapes were loaded against each other.

The Hertz formula can be applied to spur gears quite easily by considering that the contact condition of gears are equivalent to those of cylinders having the same radius of curvature at the point of contact [6, 9].

$$R_1 = r_{p1} + \sin\alpha \quad [1]$$

$$R_2 = r_{p2} + \sin\alpha \quad [2]$$

The Hertz equation for contact stresses in the teeth then takes the following form:

$$\sigma_c = -C_p \left[\frac{K_v F_t}{b \cos \alpha} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{\frac{1}{2}} \quad [3]$$

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)} \right]^{\frac{1}{2}} \quad [4]$$

$$\sigma_c = C_E \sqrt{F_t k_o k_v k_s \frac{k_m C_f}{d_p b Y_j}} \quad [5]$$

(Negative sign because σ_c is a compressive stress)
Input Parameters of Spur Gear

**Table-1.** Geometric input parameters for spur gear.

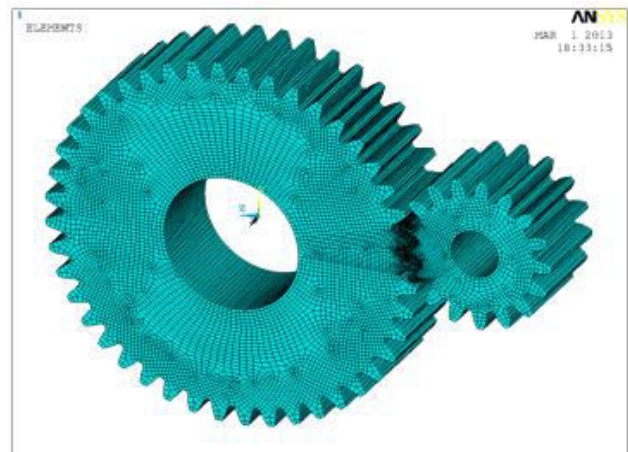
Description	Gear	Pinion
Material	Steel 15Ni2Cr1Mo28	Steel 15Ni2Cr1Mo28
Number of teeth(Z)	63	18
Young's Modulus(E)	$2.08 \times 10^5 \text{ N/mm}^2$	$2.08 \times 10^5 \text{ N/mm}^2$
Speed (N)	228 rpm	800 rpm
Power (P), kW	45	45
Poisson Ratio	0.3	0.3
Normal Module (m), mm	2,3,4,5,6,7	2,3,4,5,6,7
Normal Pressure Angle	20°	20°

Table-2. Results obtained for different modules of spur gear.

Module(m)							
Description	Formula used	2	3	4	5	6	7
Pitch Diameter (d) mm	$m \cdot Z_1$	36	54	72	90	108	126
Circular Pitch (P_c) mm	$\pi d_1 / Z_1$	6.28	9.42	12.56	15.7	18.84	21.98
Diameter Pitch (P_d)	Z_1 / d_1	36	54	72	90	108	126
Centre Distance (a) mm	$m(Z_1 + Z_2) / 2$	6.28	9.42	12.56	15.7	18.84	21.98
Velocity	$(3.14 \cdot d_1 N) / 60$	0.50	0.33	0.25	0.20	0.17	0.14
Velocity factor K_v	$(6 + v) / 6$	81	121.5	162	202.5	243	283.5

Table-3. Comparison of maximum contact stress values by hertz approach and AGMA for different modules in spur gear.

S. No	Module, m (mm)	Contact stress, σ_c (N/mm ²)	
		Hertz equation	AGMA
	2	2556.16	2544.96
	3	1485.17	1316.82
	4	963.26	937.85
	5	663.4	676.32
	6	467.52	471.76
	7	306.25	320.52

**Figure-1.** Spur gear mesh for module 6.

The comparison of theoretical contact stress values obtained from Hertz approach and AGMA for different modules is shown in Table-3. The contact stress values obtained by using Hertz equation and AGMA are relatively similar.

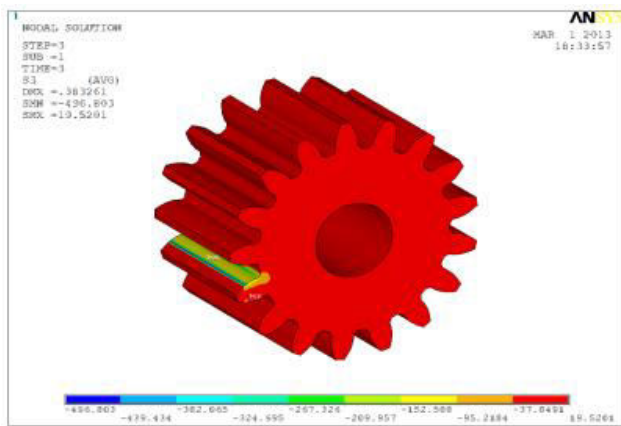


Figure-2. Contact stress distribution plot of spur gear for module 6.

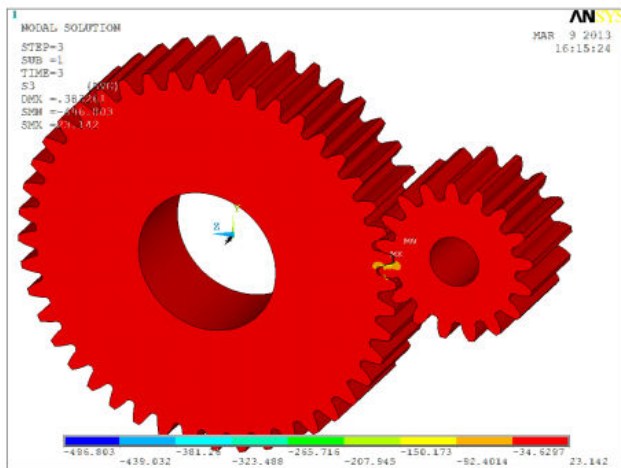


Figure-3. FEA contact stress value of spur gear for module 6.

The meshing of the gear with pinion of module 6 is shown in Figure-1 using ANSYS. The material of the gear and pinion is Steel 15Ni2Cr1Mo28. This analysis is used to obtain the contact stress values for the above profile.

The contact stress distribution plots are shown in Figures 2 and 3 for the gear with module 6 in which the maximum contact stress is 326.26 N/mm². This maximum contact stress value is almost identical with the values obtained by AGMA and Lewis equations.

Table-4 compares the contact stress values obtained by using the Hertz approach, AGMA and ANSYS for different modules. The contact stress obtained

by using the three methods decreases as the module increases. The contact stress values obtained by using the three methods are relatively similar and are shown in Graph-1.

RESULT AND DISCUSSIONS

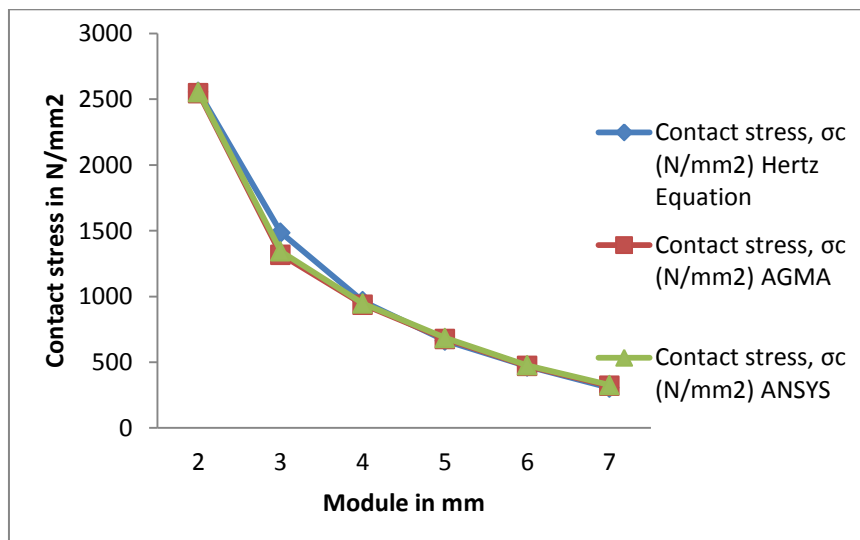
The findings in spur gear clearly show that the contact stress negatively correlates from 2552.5 N/mm² to 326.26 N/mm² with the module ranging from 2 mm to 7 mm respectively, with a maximum difference of 1.86% and minimum difference of 0.29% between AGMA and ANSYS results for material Steel 15Ni2Cr1Mo28 with 63 teeth in gear and 18 teeth in pinion. As a result, module with a larger face width is preferred in order to determine the material strength during the manufacture of gears for both materials.

CONCLUSIONS

In spur gear, the design of the teeth is purely based on bending and contact stresses. The contact stress using AGMA for different modules in spur gear were calculated for Steel 15Ni2Cr1Mo28 material. The contact stresses were also calculated for spur gear using the Hertz equation. The results obtained for the contact stress by AGMA, and Hertz equation is validated using the FEA approach. The spur gear tooth profile is geometrically modeled by applying constraints and suitable loads for Steel 15Ni2Cr1Mo28 material. Meshing was performed using the finite element method. The analysis results yielded by ANSYS were compared with the AGMA and Lewis equation. The results of spur gears for Steel 15Ni2Cr1Mo28 clearly show that the contact stresses decrease with an increase in the module. Hence, higher modules can be preferred for larger power transmission with minimum bending stress values.

**Table-4.** Comparison of maximum contact stress values by different modules in spur gear.

S. No	Module, m (mm)	Contact stress, σ_c (N/mm ²)			Difference [%]
		Hertz equation	AGMA	ANSYS	
1	2	2556.16	2544.96	2552.5	0.29
2	3	1485.17	1316.82	1342.15	1.86
3	4	963.26	937.85	945.26	0.7
4	5	663.4	676.32	684.25	1.1
5	6	467.52	471.76	476.51	0.9
6	7	306.25	320.52	326.26	1.7

**Graph-1.** Contact stress values comparison for different modules in spur gear.**NOMENCLATURE**

AGMA	-	American Gear Manufacturers Association
F_a	-	Axial force
a	-	Center distance between shafts
P_c	-	Circular Pitch
ρ	-	Density of the material
P_d	-	Diametric pitch
b	-	Face width in mm
n	-	Factor of safety
i	-	Gear (or) transmission ratio
Y_i	-	Geometry factor
σ_c	-	Induced contact stress
Y	-	Lewis Form factor
K_{bl}	-	Life factor for bending
k	-	Load concentration factor
K_m	-	Load distribution factor
d_p, d_g	-	Pitch circle diameter of pinion, gear
V_l	-	Pitch Line Velocity
v_1, v_2	-	Poisson's ratio of pinion, gear
P	-	Power transmitted in Kw
r_{g1}, r_{g2}	-	Radii of curvature of gear
r_{p1}, r_{p2}	-	Radii of curvature of pinion
K_s	-	Size factor

N_1, N_2	-	Speed of pinion, gear in rpm
K_σ	-	Stress concentration factor for the fillet
C_f	-	Surface condition factor
F_t	-	Tangential force
E_1, E_2	-	Young's modulus of pinion, gear
α	-	Normal pressure angle
Z_1, Z_2	-	Number of teeth in pinion, gear
K_o	-	Overload factor

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