ABSTRACT

The article deals with the problem of finding the place and time of switching on section heaters, with the stabilization of the temperature field based on the Green's function. The question of replacing conventional heating elements with impulse ones with the relay control principle is studied. The control system and the stabilization of the temperature field are considered. Mathematical models are presented and the temperature field is studied for a three-dimensional control object. Based on the results of this work, a patent for a utility model has been obtained.

Keywords: green's function, thermal field, sample interval, object of the management, analysis, synthesis.

INTRODUCTION

Under the conditions of economic growth, the issue of the rational use of energy resources is becoming more and more important. The inefficient use of these resources leads to an increase in the cost of these goods. Therefore, the task of finding energy-saving technologies is an urgent one [2-9].

One of the key energy resources is electricity. When we are carrying out various technological processes, products require processing with temperature. At present, it is done in electric furnaces, while stream processing is done in electric furnaces of the pipeline type. One of the major drawbacks of electric furnaces of this type is the high cost of maintaining energy, as the incandescing of one heating element consumes energy equal to 0.12% of the value of the product. In a tunnel furnace, hexagonal silicon carbide channel lines are installed along the entire length of the firing chamber. Such long heating elements require a long heating time and a lot of energy. That, in turn, has a significant impact on the final cost of the product. Let us consider the possibility of cost reduction with the help of pulsed heating elements. They, in turn, will heat the silicon carbide channel lines to maintain the predetermined temperature in the section of the furnace. Short-term powering will save energy and as a result, the cost of the product will decrease. This way we will also get the method of calculating the optimum number of heating sources, depending on the values in the temperature field.

Statement of a problem. We pose the problem of calculating the optimum amount of the heating elements arrangement in the isotropic channel line. Let us consider the algorithm of calculating the optimal arrangement of the pulsed heating elements:

a) Introduce initial values of the system into the system: \( n \) - the number of members in the Fourier series; \( l \) - the length of the channel line; \( t \) - time; \( x \) - the point (the coordinate on the axis \( X \)) of the temperature sensor location; \( \zeta \) - the point (the coordinate on the axis \( X \)) of the heating element location; \( \tau \) - the time of switching on the point source, \( a^2 \) - the given coefficient of the thermal diffusivity of the object material management, the given temperature value \( T_{\text{giv}} = \text{const} \).

b) Set and solve the problem of the object temperature field stabilization at a different number of heating elements.

c) At the time when control force is created, we need to determine the place and the time of the heating elements turn on.

Let us consider the algorithm in more detail. We will use the Green's function in order to solve the problem of calculating the temperature field. It will create control force on the system as a whole.

\[
G(x, t; x', \tau) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos \left( \frac{n \pi x}{l} \right) \sin \left( \frac{n \pi x'}{l} \right) \exp \left( -\frac{n \pi \tau}{l} \right)
\]

However, we consider a dynamic system, so we will consider the Green's function taking into account the function of the initial heating. Such an approach would enable us to define the unambiguous temperature value in the isotropic channel line.

\[
T(x, t) = \sum_{l=1}^{\infty} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos \left( \frac{n \pi x}{l} \right) \sin \left( \frac{n \pi x'}{l} \right) \exp \left( -\frac{n \pi \tau}{l} \right) \sin \left( \frac{n \pi x'}{l} \right)
\]

We stabilize the temperature at a certain point \( T_{\text{giv}} = \text{const} \) located on the length of the channel line.

\[
T(x, t) = \sum_{l=1}^{\infty} \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos \left( \frac{n \pi x}{l} \right) \sin \left( \frac{n \pi x'}{l} \right) \exp \left( -\frac{n \pi \tau}{l} \right) \sin \left( \frac{n \pi x'}{l} \right)
\]

The problem is reduced to the solution of a finite number of the systems of inequalities with respect to the parameters of input impact or object parameters:

\[
T \leq T(x, t) = T(x, t_{\text{giv}}) \leq T^*
\]
Over time, the temperature at the point \( x_i \) decreases and reaches the value \( T_{giv} = \text{const} \) at some point of time \( t = \tau_i \). Then the source \( \xi_i \) is turned on, which corresponds to the sensor \( x_i \) and creates the thermal impact on all points of the channel rod. Currently, it is necessary to calculate the location of the thermal heating element; in order to do that, let us express the value of the variable \( \xi_i \) at time \( t = \tau_i \).

\[
\xi_i = \arcsin\left(\frac{2}{\pi d} \sum_{n=1}^{\infty} \frac{e^{-\left(\frac{\pi n d}{l}\right)^2 \left(t - \tau_i\right)}}{G(x, t, \xi_i, \tau)} \sin \left(\frac{\pi n d}{l}\right)\right) * \frac{l}{n\pi};
\]

In order to make the decision valid for all values of the temperature curve, it is necessary to enter an excessive amount of heating elements into the system at the initial time. Since the system will generate control force only at points where the temperature is below a predetermined value, this action will not cause excessive calculation. Then, in order to solve the problem, it is necessary to calculate the moments of the heating elements activation. Since the number of heating elements is excessive, the accuracy of the point calculation and the heating element location detection will be sufficient. However, it is worth noting, that if you take the multiple locations of the heating elements, then at the mathematical calculation we will find the coordinate with the greatest possible accuracy of the calculation. That, in turn, leads to the calculation of the point location of the heating element.

Therefore, it can be concluded that the precision of finding the heating point location depends on the number of location points of the heating elements. We will analyze the behavior of the temperature field for the object that possesses the following characteristics: \( l = 10, a'^2 = 0.01, k = 10, d = 9, \xi_i = x_i = 1, T_{giv} = 0.2, \xi_1 \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \).

Table 1. Table of results calculation by the number of sensors.

<table>
<thead>
<tr>
<th>Number of Sensors</th>
<th>d = 9</th>
<th>d = 8</th>
<th>d = 7</th>
<th>d = 6</th>
<th>d = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>tmas[1,690]</td>
<td>2.03</td>
<td>2.01</td>
<td>1.99</td>
<td>1.95</td>
<td>1.89</td>
</tr>
<tr>
<td>tmas[2,690]</td>
<td>3.82</td>
<td>3.73</td>
<td>3.59</td>
<td>3.39</td>
<td>3.07</td>
</tr>
<tr>
<td>tmas[3,690]</td>
<td>5.15</td>
<td>4.87</td>
<td>4.48</td>
<td>3.91</td>
<td>3.07</td>
</tr>
<tr>
<td>tmas[4,690]</td>
<td>5.86</td>
<td>5.27</td>
<td>4.48</td>
<td>3.39</td>
<td>1.89</td>
</tr>
<tr>
<td>tmas[5,690]</td>
<td>5.86</td>
<td>4.87</td>
<td>3.59</td>
<td>1.95</td>
<td>1.75</td>
</tr>
<tr>
<td>tmas[6,690]</td>
<td>5.15</td>
<td>3.73</td>
<td>1.99</td>
<td></td>
<td>2.12</td>
</tr>
<tr>
<td>tmas[7,690]</td>
<td>3.82</td>
<td>2.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tmas[8,690]</td>
<td>2.03</td>
<td>2.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tmas[9,690]</td>
<td>3.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 shows that, for example, when installing 9 control actions on a channel line 0.5 meters long only 2, 3, 5 heating elements will be used. When stabilization temperature and the power of the heating elements are changed, other heating elements will be used. So we can conclude that this method of calculating the optimal location of the heating elements is applicable to all thermal processes for which there is a solution in the form of the Green's function.

Let us estimate the error in the location of the heating elements depending on the number of located heating elements. We assume that at the extremities of the control object the value of the temperature field will take the value \( T_{giv} \). Then, this event will happen at the points \( x_1 \) and \( x_9 \) and at the time \( \tau_1 = (\frac{l}{a'^2}) \ln \left(\frac{2 \sin \pi x_1 y_{giv} \sum \sin \frac{\pi n}{l} \xi_i}{\pi \rho c} \right) \). At this point, the turn-on of the heating elements will take place at the points \( \xi_1 \) and \( \xi_9 \). These heating elements will create temperature impulse that will be expressed as follows:

\[
T(x, t - \tau_1) = \frac{2 \xi_1}{l} \sum_{n=1}^{\infty} \frac{\sin \left(\frac{\pi n d}{l}\right)}{G(x, t, \xi_1, \tau)} \sin \left(\frac{\pi n d}{l}\right) \sin \left(\frac{\pi n d}{l} \xi_1\right);
\]

This signal is added to an already existing temperature field of the object (the function of the initial heating) and will begin to have an impact on adjacent points of the spatial distribution object. Then the moment for the time \( t > \tau_1 \) for the middle of the segment will look as follows:

\[
T(x, t, \tau_1) = T(x, t) + 2T(x, t - \tau_1)
\]

Or:

\[
T(x, t, \tau_1) = \frac{2 \xi_1}{l} \sum_{n=1}^{\infty} \left( -\left(\frac{\pi n}{l}\right)^2 \sin \left(\frac{\pi n d}{l}\right) \sin \left(\frac{\pi n d}{l} \xi_1\right) \right) + \frac{4 \xi_1}{l} \sum_{n=1}^{\infty} \left( -\left(\frac{\pi n}{l}\right)^2 \sin \left(\frac{\pi n d}{l} \xi_1\right) \right) \sin \left(\frac{\pi n d}{l} \xi_1\right).
\]
If the point of observation (temperature sensor) will be placed at some other place than in the middle of the interval, then the momentum will look as follows:

\[ T(x,t,\tau_1) = T(x,t) + T(x,t - \tau_1) + T(x,t - \tau_1,\xi_0) \]

We divide this formula into two summands, due to the fact that the heating points are at different distances from the observation point. The function \( T(x,t - \tau_1) \), variable in time, cannot be described by one harmonic component of the Fourier series, as they influence the control object for a relatively small period of time. Thus, the temperature pulse source has an impact on all the points of the object, but the time of temperature distribution in the time channel depends on several factors. Consequently, it is necessary to find the value of the delay distribution in the time channel.

\[ \text{If } 0 < x < L, 0 < y < L, 0 < z < L; \]

It should be also noted that each zone of the tunnel furnace has its own temperature mode. Therefore, when considering the movement of a body in a tunnel furnace, the values of the thermal field should be calculated independently from other zones, changing the input parameters of the system.

The principle of the operation and technical implementation of tunnel furnaces are identical to each other; however, it is necessary to consider furnaces in more detail, especially those ones that are designed for baking bread and calcinating brick. This is due to the fact that, depending on the temperature and technical implementation, boundary conditions and behavioral functions of the temperature field will differ. Let us consider the principles of operation of various types of tunnel furnaces.

Tunnel furnaces used in bakery and confectionery belong to the furnaces of the pipeline type. The pipeline type furnaces are arranged in such a way that the cooking process is split into sections with different temperature regimes. Electric tunnel belt furnaces of the pipeline type are produced with a standard width of 0.6 to 4 m (with 0.1 m intervals). They may be mono-, bi- and multi-storey depending on the production volume. A furnace can also contain a steam moisturizing zone. The axonometric section of the sample of an electric furnace with a mash zone, duo-thermal zone, hydraulically pulled netted-wiry belt and tape drive with two fixed movable cone-frontal gearboxes see in Figure-1.

This furnace is used for baking bread. The specifications of the furnaces used for baking bread are shown in Table-2.

Table-2. Specifications of small and medium-sized baking furnaces.

<table>
<thead>
<tr>
<th>Type PPP</th>
<th>width $X$</th>
<th>width $S$</th>
<th>width $Y$</th>
<th>length $L$</th>
<th>height $P$</th>
<th>power (kg/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bread 1.5 kg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPP 0.6 7.5.xx1-2E</td>
<td>600</td>
<td>1,600</td>
<td>2,250</td>
<td>8,220</td>
<td>2,850</td>
<td>135</td>
</tr>
<tr>
<td>PPP 0.6 8.3.xx1-2E</td>
<td>600</td>
<td>1,600</td>
<td>2,250</td>
<td>8,870</td>
<td>2,850</td>
<td>150</td>
</tr>
<tr>
<td>PPP 0.6 9.0.xx1-2E</td>
<td>600</td>
<td>1,600</td>
<td>2,250</td>
<td>9,520</td>
<td>2,850</td>
<td>162</td>
</tr>
<tr>
<td>PPP 0.6 9.8.xx1-2E</td>
<td>600</td>
<td>1,600</td>
<td>2,250</td>
<td>10,170</td>
<td>2,850</td>
<td>176</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type PPP</th>
<th>width $X$</th>
<th>width $S$</th>
<th>width $Y$</th>
<th>length $L$</th>
<th>height $P$</th>
<th>power (kg/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bread 1.5 kg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPP 1.0 12.5.xx1-2E</td>
<td>1,000</td>
<td>2,000</td>
<td>2,650</td>
<td>8,220</td>
<td>2,850</td>
<td>225</td>
</tr>
<tr>
<td>PPP 1.0 13.8.xx1-2E</td>
<td>1,000</td>
<td>2,000</td>
<td>2,650</td>
<td>8,870</td>
<td>2,850</td>
<td>248</td>
</tr>
<tr>
<td>PPP 1.0 15.1.xx1-2E</td>
<td>1,000</td>
<td>2,000</td>
<td>2,650</td>
<td>9,520</td>
<td>2,850</td>
<td>272</td>
</tr>
<tr>
<td>PPP 1.0 16.4.xx1-2E</td>
<td>1,000</td>
<td>2,000</td>
<td>2,650</td>
<td>10,170</td>
<td>2,850</td>
<td>295</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type PPP</th>
<th>width $X$</th>
<th>width $S$</th>
<th>width $Y$</th>
<th>length $L$</th>
<th>height $P$</th>
<th>Power (kg/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bread 1.5 kg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPP 1.4 17.5.xx1-2E</td>
<td>1,400</td>
<td>2,400</td>
<td>3,050</td>
<td>8,220</td>
<td>2,850</td>
<td>315</td>
</tr>
<tr>
<td>PPP 1.4 19.3.xx1-2E</td>
<td>1,400</td>
<td>2,400</td>
<td>3,050</td>
<td>8,870</td>
<td>2,850</td>
<td>447</td>
</tr>
<tr>
<td>PPP 1.4 21.1.xx1-2E</td>
<td>1,400</td>
<td>2,400</td>
<td>3,050</td>
<td>9,520</td>
<td>2,850</td>
<td>380</td>
</tr>
<tr>
<td>PPP 1.4 23.0.xx1-2E</td>
<td>1,400</td>
<td>2,400</td>
<td>3,050</td>
<td>10,170</td>
<td>2,850</td>
<td>414</td>
</tr>
</tbody>
</table>

A product which moves on the pipeline through the furnace falls into various sections, where it is either warmed or cooled depending on the chosen process temperature curve. The heating is carried out by continuous heating elements, located over the entire width of the furnace. In view of the presence of continuous heating elements in the temperature field, it can be considered a homogeneous area, as it is situated in the
range of permissible values. The general form of the
temperature curve depends on the product that is being
processed.

To heat electric furnaces continuous metallic and
nonmetallic heating elements are used – they are arranged
over the entire length of the heating zone. The most
common metal elements at present contain:
a) Nickel-chromium alloys to the temperatures of 1,000-
1,100 °C; the elements consisting of chromium-
aluminum-titanium-steel to the temperature of 1,200
°C;
b) Heating elements "Kanthal" (Sweden) consisting of
chromium-aluminum-cobalt-steel to temperatures up
to 1,300 °C;
c) For higher temperatures a number of the heating
elements is used, namely "Silit", "Quartz-silit" (Sweden), "Globar" (USA), "KЭН", "ВП", "KЭНБ", and others.

Depending on the task, we select the composition
of the atmosphere of the heating elements work. The
atmosphere of work affects the technical characteristics of
alloys and their life dramatically. For example, when
creating a tunnel furnace, alkaline atmosphere is used for
glass molding, which reduces the life of the heating
element to 1,500 hours out of 36,000 hours in the air
atmosphere. Table-3 lists some characteristics of silicon
carbide channel lines used for heat treatment of various
products in given atmospheres. The heating element of
silicon carbide is a special type of a linear heater. The task
of the heater is to convert electrical energy into heat. As
seen in Figure-3, a silicon carbide heating element consists
of two cold ends and the heating zone. The heating zone of
the element is self-adhering SiC. The crystallized net
recrystallizes at high temperatures and forms a heating
zone. The main element in production is green SiC which
is a great type of a semiconductor. However, the use of the
elements of this type puts some constraints on the
installation and maintenance of the heating element. For
example, the elements cannot be placed closer than two
diameters of such elements. Or, they cannot be placed
closer than 1.5 diameters to a wall or other reflecting
body. In the first place, this is due to the bending of heat
from the heating element. Thus, when these requirements
are followed, the possibility of heat dissipation in space is
damaged, which leads to the local overheating of the
channel line and accordingly its failure (see Table-3).

Table-3. Lifetime of SiC heating elements.

<table>
<thead>
<tr>
<th>Application</th>
<th>Type of the furnace</th>
<th>Working mode</th>
<th>Working temperature, °C</th>
<th>Atmosphere</th>
<th>Life duration, hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcination of ceramics</td>
<td>tunnel furnace</td>
<td>permanently</td>
<td>1,350-1,400</td>
<td>air</td>
<td>7,000-9,000</td>
</tr>
<tr>
<td>Agglomerate of tech. ceramics</td>
<td>chamber furnace</td>
<td>with breaks</td>
<td>1,250-1,380</td>
<td>air or nitrogen</td>
<td>8,000-12,000</td>
</tr>
<tr>
<td>Agglomerate of tech. ceramics</td>
<td>tunnel furnace</td>
<td>permanently</td>
<td>1,100-1,250</td>
<td>Content of lead in the air</td>
<td>4,000-6,000</td>
</tr>
<tr>
<td>Hot metal processing</td>
<td>chamber furnace</td>
<td>with breaks</td>
<td>850-1,150</td>
<td>air</td>
<td>16,000-20,000</td>
</tr>
<tr>
<td>Hot metal processing</td>
<td>tunnel furnace</td>
<td>permanently</td>
<td>1,000-1,200</td>
<td>exogas, 20% H2 max</td>
<td>8,000-12,000</td>
</tr>
<tr>
<td>Production of glass</td>
<td>special chamber furnace</td>
<td>permanently</td>
<td>1,350</td>
<td>alkali vapors</td>
<td>1,000 – 1,500</td>
</tr>
<tr>
<td>Production of float glass</td>
<td>tunnel furnace</td>
<td>permanently</td>
<td>800-1,200</td>
<td>N2/H2 A</td>
<td>35,000-70,000</td>
</tr>
<tr>
<td>Heat treatment of aluminum</td>
<td>mixers, melting boilers</td>
<td>permanently</td>
<td>800</td>
<td>air with alkali</td>
<td>7,000-14,000</td>
</tr>
</tbody>
</table>

These heating elements differ in some features,
namely:

a) Higher maximum operating temperature of the
element up to 1,425 °C (1,250 °C for Nichrome);
b) Longer lifespan (2-4 times longer);
c) Higher specific load;
d) Higher maximum operating temperature of the
element up to 1,425 °C;
e) Higher resistivity;
f) Smaller specific weight;
g) Higher adhesion of the oxide layer, which prevents
contamination of the furnace and the processed
products.

Silit rods consisting of silicon carbide with minor
additions are the most widely used in electric furnaces for
the calcination of ceramic products? The appearance and
technical configuration of a Silit channel line is shown in
Figure-2 where B - hot zone, L - its length, A - diameter.
The area of application of this channel line is stipulated by
its greater resistance to the chamber atmosphere. And the
use of silicon carbide structure increases its energy value significantly.

Figure-2. Silicon carbide heating element.

Many of them have been successfully produced by domestic industry. Domestic analogues are marked the following way: КЭН, ВП, КЭНБ. Structurally, they have the same build and shape. Higher temperature is obtained by the use of silicon carbide alloy.

Numerical experiment. A tunnel furnace of the pipeline type has a number of advantages, as well as some drawbacks. One of the major disadvantages of electric furnaces is their high cost when it comes to maintaining energy – the calcination of one heating element requires the energy which costs 0.12% of the cost of one brick. The best thing to do, in our opinion, is to reduce the cost due to the use of pulsed heating elements. The short-term turning on will save energy and, as a result, will save the cost of bricks.

Due to the fact that in all cases the mathematical model of the thermal process is equal to:

\[
\frac{\partial T}{\partial t} = \alpha \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right],
\]

where:

- \( T \) - temperature, \( \alpha \) - thermal diffusivity;
- \( x, y, z \) - coordinates;
- \( \partial \) - partial derivative;
- \( t \) - time.

It is possible to find such positioning of the heating elements, which would allow to recreate such a temperature curve under pulsed heating. Let us try to do this with the help of the Green's function. We stabilize the temperature in the set of the segments constituting the temperature curve. To do this, we write a program in Delphi to calculate such a situation. We calculate the values in the temperature field in each of the forty-two sections. To do this, we write a program in Delphi environment which can be used to calculate the location and time of each turn of the heating element for each of the 42 positions. For this purpose, the program requires the input of 129 variables which characterize the physical properties of each of the furnace sections (section length, required temperature value in the section, the number of the heating elements).

By setting different parameters in the developed program, you can receive the calculation of locations of the heating elements depending on the set temperature curve of the process. The accuracy of the location setup of the heating element depends on the number of the arranged heating elements so that the program will include only those heating elements which are necessary for leading out of the control object to the desired temperature range.

This program makes it possible to calculate how much and where it is necessary to place dot heating elements in order to obtain a predetermined curve in the temperature field.

We evaluate the effectiveness of replacing the heating method of continuous heating elements, from a permanent item to the pulse point elements. For this purpose, we carry out a study of energy costs to heat silicon carbide channel line, which in turn will transmit heat throughout the chamber. The calculation of the power of silicon carbide heating is conducted according to the formula:

\[
N = D \cdot L \cdot II \cdot W
\]

Where: \( N \) - the power of the heater, W; \( D \) - diameter of the working zone of the heater, cm; \( L \) - length of the working zone of the heater, cm; \( II \) - average specific power rating (W/cm²).

The specific power rating of the silicon carbide heating element is determined by the atmosphere according to the following schedule.

Thus, knowing that a silicon carbide channel line has the diameter of 25 x 400 x 1200 mm, \( R = 0.87 \) ohms \pm 10% at the temperature of 1,070 °C, one can calculate that this channel line will have the power of:

\[
N = 2.5 \cdot 40 \cdot 3.14 \cdot 6 = 1,884 \text{ W}
\]

It is also necessary to know the operating voltage applied to the channel lines. It will be calculated from the Ohm's law based on the formula

\[
U = \sqrt{N \cdot R}
\]

Where: \( U \) - voltage; \( N \) - heater power; \( R \) - resistance. It should be noted that when calculating the resistance of the power we chose the resistance corresponding to the temperature mode. However, for other cases with a higher temperature the resistance changes. Thus, for example, at 1,400 °C the resistance is higher by 20% and comprises 4.1 ohms.

Together with the internal resistance, the Watt load on the element also grows. The working load in the furnace is achieved when the furnace is at 1,400 °C, but it can be reduced by the use of other types of atmospheres. There is no lower limit for the heating element; however, the minimum given load for the entire areal surface of the channel line is achieved at a temperature of 900° C. The following formula can be used for other cases of calculation:

\[
W = D \cdot L \cdot II
\]

To evaluate the effectiveness we take the silicon carbide heating element with the following features:

Dimensions: 25 x 400 x 1,200 mm.
Power at the temperature 1,070 °C (0.87 ohms): 1,884 W
Power at the temperature 1,400 °C (1.1 ohms): 1,800 W
Working mode: permanent.

Working mode: pulse (calculated).

The result of the calculation of the heater power at the output to specified temperature mode is shown in Table-4.

Table-4. The result of the calculation of the heater power.

<table>
<thead>
<tr>
<th>Current temperature, °C</th>
<th>Calculated power according to technical documentation, W/cm²</th>
<th>Power of the heater in the application of methods of pulsed heating, W/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>200</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>300</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>400</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>500</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>600</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>700</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>800</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>900</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>1,000</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>1,100</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>1,200</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>1,300</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>1,400</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>1,400</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

There is no gain in power, but it should be noted that by the pulse heating the time of the operation of the heating element is saved. We calculate the time of turning on the heating elements. The start-up time will be calculated according to the formula:

\[
\tau = \frac{1}{\sigma \rho \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right)} \ln \left( \frac{8 \sin \frac{\pi}{\alpha_1} \sin \frac{\pi}{\alpha_2} \sin \frac{\pi}{\alpha_3} \sin \frac{\pi}{\alpha_4} \sin \frac{\pi}{\alpha_5} \sin \frac{\pi}{\alpha_6} \sin \frac{\pi}{\alpha_7} \sin \frac{\pi}{\alpha_8}}{4 \mu_0 C_\text{eff}} \right)
\]

When temperature fields were stabilized according to the data listed above, we obtained the results presented in Table-5.

Table-5. The time of the heating elements turning on, depending on their number.

<table>
<thead>
<tr>
<th>d = 8</th>
<th>d = 7</th>
<th>d = 6</th>
<th>d = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turned on = 4.43</td>
<td>Turned on = 4.33</td>
<td>Turned on = 4.27</td>
<td>Turned on = 4.23</td>
</tr>
<tr>
<td>Turned on = 2.01</td>
<td>Turned on = 2.03</td>
<td>Turned on = 2.01</td>
<td>Turned on = 2.01</td>
</tr>
<tr>
<td>Turned on = 1.8</td>
<td>Turned on = 1.8</td>
<td>Turned on = 1.8</td>
<td>Turned on = 1.8</td>
</tr>
<tr>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
</tr>
<tr>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
</tr>
<tr>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
</tr>
<tr>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
<td>Turned on = 1.4</td>
</tr>
<tr>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
<td>Turned on = 1.3</td>
</tr>
</tbody>
</table>

Based on these results, it is possible to observe the dependence of the number of the heating elements and on the time of turning on; for example, when there are 7 heating elements, silicon carbide channel line is warmed to the temperature of 1,400 °C, when the heating elements...
are turned on every 10 seconds. Consequently, by the average time of work of the heating element which comprises 3,000 hours in continuous mode, with the help of this method of the heating, the pulse time of the heater will reduce to 1,500 hours. As a result, energy will be saved. However, to heat the channel line to the temperature of 1,400 °C the pulsed heater requires a lot of power in a pulsed mode, more than during continuous heating.

The total average power at a continuous heating amounts to $3,000 \cdot 5 = 15,000$ watts. And at the pulsed heating, it amounts to $1,200 \cdot 8 = 12,000$ watts. It comprises 80% of the used resources.

$$\frac{1,200 \cdot 100}{15,000} = 80\%$$

Then the benefit from the use of this technique comprises 20 percent for a perfect furnace.

Let us verify the stability of the developed system according to Popov criterion. We consider a silicon carbide channel line, performing the heating of the product in the tunnel furnace of the pipeline type. The mathematical model of the heating in the furnace of this type would look as follows:

$$\frac{\partial T}{\partial t} = a^2 \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right];$$

$$0 < x < L_x, 0 < y < L_y, 0 < z < L_z;$$

For this model, the transfer function for the n-mode will look as follows:

$$W_{n}(s) = \frac{T_n(x, y, z = z_n)}{C_n(s) \sin(\phi_n - \gamma) \sin(\phi_n - \gamma)} = \frac{\exp[\beta_{n,z} z]}{\exp[-\beta_{n,z} z]} \cdot \frac{\exp[\beta_{n,y} y]}{\exp[-\beta_{n,y} y]} \cdot \frac{\exp[\beta_{n,x} x]}{\exp[-\beta_{n,x} x]}$$

<table>
<thead>
<tr>
<th>$Y(z)$</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>5</th>
<th>6</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Graphically, Hurwitz angle looks the same way as shown in Figure-3.

**Figure-3.** Gurwitz angle.

We construct a hodograph according to the given equations and transfer function. Figures 4-5 show the hodographs for three spatial modes of the system.

**Figure-4.** Relative position of the hodograph and Popov straight line $n = 1$. 

Where $s$ - Laplace operator, $L_x, L_y, L_z$ - spatial coordinates; $t$ - time; $x, y, z$ - the point of the heating element location; $\tau$ - time of the point source turning on, $a^2$ - the given coefficient of thermometric conductivity, $\phi(x)$ - initial temperature distribution, $z$ - the given number ($0 < z < L_z$).
Having analyzed the system, we can come to the conclusion about the stability of the control process when the proposed method is used. On the basis of stability, we make the corresponding conclusions about the stability of the processes in distributed non-linear systems of automatic control on the basis of the Green's function. Also, it can be concluded that the presented systems are adequate due to the correct usage of mathematical apparatus and the theory of automatic control systems.

**Technical implementation.** The existing heating elements, including the heating elements produced by the company "Kanthal", have a short life due to the aggressive impact of the atmosphere of the tunnel furnace. The main factor that reduces the life of a product is the run-out of continuous heaters located across the outer surface of the heater. In this work, we consider the case of locating the heating elements on the inside of the channel line rather than on the outer surface of the heater. Technically, it is realized due to the creation of a hollow heater, followed by the introduction of the pulse sources (see Figures 6-7).

The installation of the heating elements in the furnace does not require large structural changes, as the dimensions of the heating element will remain the same. The heating method will change, namely the end supply of electricity. Thus, in existing furnaces, the electricity is supplied to the outer side of the heating element, and in our case, the supply of electric power will be performed on the inner side of the heater. However, this improvement does not require any structural changes.

**CONCLUSIONS**

The above-given technique considers the possibility of replacing continuous heating elements on pulse elements. The novelty and the technical peculiarity of this article are explained by the following:

a) The usage of an innovative approach to the heating of hexagonal silicon carbide structures is an important task, since channel lines made of this alloy are used for the calcination of ceramics, bricks and other products.

b) This method is not designed solely for hexagonal silicon carbide structures, and has a general form which can be easily adapted to other alloys as well.

c) The proposed methodology would reduce the final cost of the product by saving energy resources of the enterprise.

d) This method, along with the software and hardware complex for the stabilization of the temperature field of tunnel furnaces of the pipeline type, will allow us to solve a wide range of tasks required in the modern industry [1, 18-20].

Thus, the developed method can be generalized for the whole class of systems for which there is a fundamental solution (Green's function). At that, the loss of simplicity of the expression of the Green function naturally causes an increase in the cost of the calculation process. However, if you compare current costs spent on low-efficiency heating elements, the use of mathematical modeling for the calculation of the heating elements location is justified [6-20].
It should be noted that it would be useful to consider the choice of sampling parameters of the control actions for a system, the boundary problem of which contains non-zero boundary conditions. The fundamental solution (Green's function) of the boundary problem of such systems has the form that is different from the type considered in the work. It is necessary to explore the possibility of expanding the working area of the object, that is, the area that enables us to achieve the desired output function with a given accuracy. But this is the subject of further research.

REFERENCES


[16] Rapoport E., Pleshivtseva Y. Combined Optimization of Metal Hot Forming Line with Induction Pre-
Heating. In IEEE 26th Convention of Electrical and Electronics Engineers, Israel, Eilat.


