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EVALUATION THE IMAGE QUALITY AND THE OPTIMUM ABERRATIONS BALANCE FOR AN OPTICAL SYSTEM WITHDIFFERENT APERTURES

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ABSTRACT

The image of incoherently illuminated point object (Point Spread Function) will be analysed theoretically for imaging system with different apertures. Both free and defocused systems were considered. A new formula has been derived for determination of the PSF by integration over different square size pupil apertures. Our studies show that the optimum balance for square aperture with half diagonal =1 gives a best Strehl ratio, while the resolution of square aperture with area $=\pi$ have a higher resolution, and the tolerance of the different aperture solved analytically.

Keywords: image quality, PSF, aberration, strehl ratio.

1. INTRODUCTION

It is difficult to establish an exact measure of optical image quality since quality is an entity which usually finds itself subject to human judgment. When evaluating the performance of an optical imaging system, it is essential to understand the response of the system to the object being imaged. The Point Spread Function (PSF), which describes the imaging system response to a point object, is a measure of the intensity distribution of light emitted by the point source[1,2,3] and is analogous to the impulse response[4]. The point spread function is defined as the distribution of absorbed energy per unit area in the image plane when a point object [5] irradiates the imaging system. In the absence of aberration, the image quality of an optical system is limited by diffraction, which is the spreading of light at the edge on the aperture. Diffraction results a slight diffusion of the focus, when the pupil is circular, diffraction produced a circular pattern known as Airy disc [6]. Which is characterized by a central disc contains maximum image intensity surrounded by rings of lower intensity. Optical aberration and diffraction degraded the image of an object point. The Strehl ratio [7],which is an important Image quality-assessment parameter, especially when the optical system has some aberrations, indicates the ratio of the peak intensity of the aberrated PSF to the peak intensity of a diffraction limited PSF[8]:

$$SR = \left|\frac{A(0)}{A_0(0)}\right|^2 = |A_N(0)|^2$$

Where A(0) and $A_0(0)$ is the central amplitude for the optical system.

2. RELATED WORK

The intensity distribution of light for an optical system is an important image quality assessment parameter for any optical systems. Several works has been attempted for enhancement, optical system quality using various measurements. A. Miks *et al* [3] provides a theoretical analysis of the influence of primary aberrated on the accuracy of the optical system with respect to object system. Ali H.[8] investigated the influence of the distribution of the points on the accuracy of the results obtained for PSF. He used two kind of distribution: symmetric and modified distribution A. Srisailan *et al*[9] evaluated the parameters of an optical system using apodized Bartlett window. While Talib, A. [10] studied the effect of central elliptical obscuration on the PSF for the diffraction limited system. Tapashreer, R. [11]*et al*analysed the imaging capabilities by studied its PSF for the super oscillatory lens. Jeorgmin, K. *et al* [12] formulated the vectorial PSf of the direct oblique imaging system, and Kuo H. *et al* [13] studied the modulation transfer function as a quality criterion for an optical system, they calculate the MTF by using the spot diagram on the image plane. Al-Hamdani Ali studied the effect of aperture shape on the PSF [14,15,16,17] and on the aberration polynomial function [18,19,20,21].

3. MATHEMATICAL FORMULA FOR PSF

Considering wave propagation of light and the finite size of optical systems, the image of the point in the object plane is a diffraction pattern in the image plane. A point source in the focal plane will produce an expanding spherical wave, part of which enters the lens[22].The refractive action of the lens delays axial rays more near the center of the lens than at the edges, converting the expanding spherical wave into another spherical wave converging toward the image point [23]. If the illumination is incoherent, and varies randomly from one point to another , then the system is linear in intensity, and the response of the optical system to the point signal PSF can be obtained by taking the squared modulus of the complex amplitude function [24]:

$$p(u,v)= |A(u,v)|^2$$

Where $A(x,y)$is the complex amplitude function for arrays of circular apertures. The transfer of complex

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amplitude from the entrance pupil to exit pupil depends on numerous factors like diaphragm shape, reflection losses at the intermediate optical surfaces, light absorption in the lens materials, etc.[22,23]

It is useful to define the pupil function as the function that takes on a value of (one) inside the aperture, and (zero) outside. We can express the complex amplitude in the point (u, v) in the image plane by using Fourier transform [24,25] to pupil function $f(x,y)$.

$$A(u,v) = \int_y \int_x f(x,y) \cdot e^{i2\pi(ux+vy)} dxdy \quad (1)$$

u, v is the dimensionless coordinates

$$f(x,y) = \tau(x,y) \cdot e^{ikW(x,y)} \quad (2)$$

$\tau(x,y)$ represents the real amplitude distribution in exit pupil which called pupil transparency and often chooses equal to one.

$e^{ikW(x,y)}$: represents wavefront of aberration function[6].

$w(x,y)$: represents aberration function

(x,y): represent exit pupil coordinates

Wave aberrations can be expressed. Using Zernike polynomials [26,27] or Seidel polynomials [28,29].

In our work we will use Seidel aberration polynomials for further analysis

$$W(x,y) = w_{20}(x^2+y^2) + w_{40}(x^2+y^2)^2 + \ldots\ldots \quad (3)$$

W_{20}represents the defocus coefficient,W_{40}represents the coefficient of primary aberration

$$p(u,v) = N \left| \int_y \int_x \tau(x,y) e^{ikw(x,y)} e^{i2\pi(ux+vy)} dxdy \right|^2 \quad (4)$$

Where N is the normalizing factor and will be computed using MATHCAD. Let $z = 2\pi u$ and $m = 2\pi v$, which represent the dimensional coordinates. Then equation (4) can be written as

$$p(z,m) = N \left| \int_y \int_x e^{ikw(x,y)} e^{i(zx+my)} dxdy \right|^2 \quad (5)$$

The intensity distribution on the two axes (z, m) is symmetric, therefore one coordinates is sufficient so we can reduce it to one axis only, let (m = 0), then the equation (4)can be written as

$$p(z) = N \left| \int_y \int_x e^{ikw(x,y)} e^{i(zx)} dxdy \right|^2 \quad (6)$$

In our work we use different aperture as follows;

a) Considering a system with circular aperture of unit radius (r=1), The limit of integration is taken over a circular exit pupil of normalized area equal to N, so the integration of equation (6) was carried out over a circular exit pupil of the form $x^2 + y^2 = 1$.The limit of integration is taken from -1 to+1 and $-\sqrt{1-y^2}$ to $\sqrt{1-y^2}$ for y and x respectively.

$$p(z) = N \left| \int_{-1}^{1} \int_{-\sqrt{1-Y^2}}^{\sqrt{1-Y^2}} e^{ikw} e^{izx} dxdy \right|^2 \quad (7)$$

b) Considering a system with square aperture with half diagonal =1, The limit of integration is taken over a squarer exit pupil of normalized area equal to N, so the integration was carried out over the square exit pupil for area equal to 2.The limit of equation (6)is taken from -1/$\sqrt{2}$to 1/$\sqrt{2}$.

$$p(z) = N \left| \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \int_{-1/\sqrt{2}}^{1/\sqrt{2}} e^{ikw} e^{izx} dxdy \right|^2 \quad (8)$$

c) Considering a system with square aperture with area=π the integration was carried out over the square exit pupil for area equal =π. The limit of equation (6) is taken from -1/$\sqrt{2\pi}$ to 1/$\sqrt{2\pi}$.

$$p(z) = N \left| \int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} \int_{-\sqrt{\pi}/2}^{\sqrt{\pi}/2} e^{ikw} e^{izx} dxdy \right|^2 \quad (9)$$

The decrease in SR [31,8] of the imaging system given by the variance of the phase aberration of the system, the approach to minimize the variance of the wave front for the apertures used in this paper as follows:

a. Circular aperture with aberration function W is,

$$V = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} W_{20}{}^2\,(r^{\wedge}4) - \frac{1}{\pi^2} \left[\int_0^1 \int_0^{2\pi} W_{20}(r^2) dr d\varphi \right]^2 \quad (10)$$

b. Square aperture (half diagonal =1)is,

$$V = \frac{1}{\pi} \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} (W_{20}(x^2,y^2))^2 dxdy - \frac{1}{\pi^2} \left[\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} W_{20}(x^2,y^2) dxdy \right]^2 \quad (11)$$

c. Square aperture (area = π) is,

$$V = \frac{1}{\pi} \int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} \int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} (W_{20}(x^2,y^2))^2 dxdy - \frac{1}{\pi^2} \left[\int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} \int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} W_{20}(x^2,y^2) dxdy \right]^2 \quad (12)$$

By solving formulas (10, 11and 12) analytically for an optical system suffers from only defocus,the optimum focus error corresponds to the Strehl Ratio SR= 0.8 is:$W_{20} \leq 0.25\,\lambda$ for circular aperture; $W_{20} \leq 0.21\,\lambda$ for square aperture with area =π;$W_{20} \leq 0.33\,\lambda$for square aperture with half diagonal =1.

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4. NUMERICAL RESULTS

The PSF for aberration free systems with different apertures have been presented graphically in Figure-1.

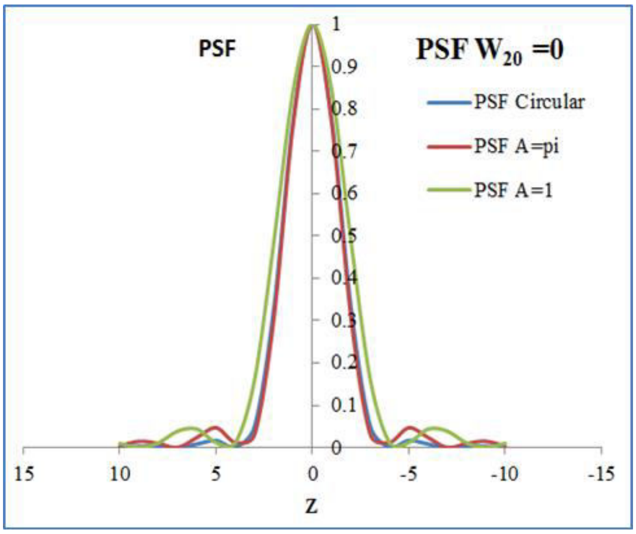

Figure-1. Normalized PSF profile for diffraction limited optical system (W20=0) with different apertures.

This Figure indicates that the resolution of the optical system with square aperture of area Pi is more than the systems with the other apertures. Figure-2, shows the effect of the focus errors (W20=0.25,0.5,0.75 and 1λ) als0 the effect of optimum balance for circular apertures (W20=-W40=1λ) on the PSF. The results indicate that the optimum balance for W20=0.25λand the balance for the spherical aberration value (W40=W20=1λ gave the expected value for PSF=0.8 which represent the Strehl ratio for circular apertures. Figure-3, shows the effect of defocus W20=0.25λ on the PSF. It is clear that the peak intensity system with square aperture of half diagonal equal one is larger than the other apertures. This result ensures that the optimum balance for this aperture for W_{20} (W20≤0.33 λ) is larger than the values evaluated above other apertures. So the optical system depth of focus and the depth of field are improved by using the new suggested square aperture of half diagonal equal one. The same phenomenon is shown in Figure-3, for optimum balance for primary spherical aberration W_{040}=0.5λ.

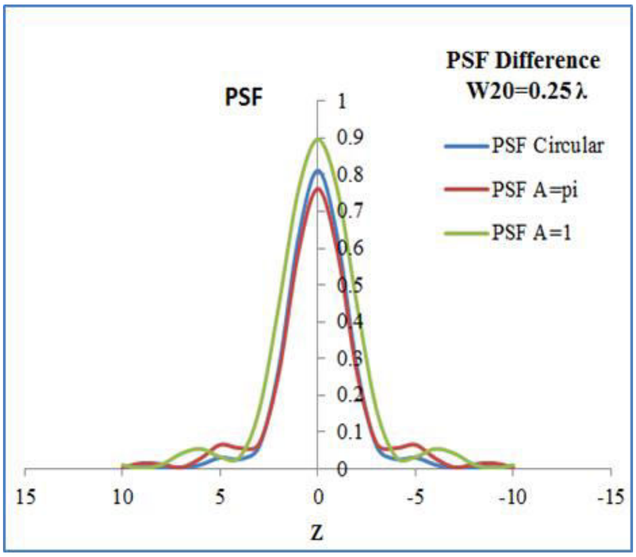

Figure-2.PSf profile for an optical system with different apertures suffer from defocus W_{20}=0.25λ.

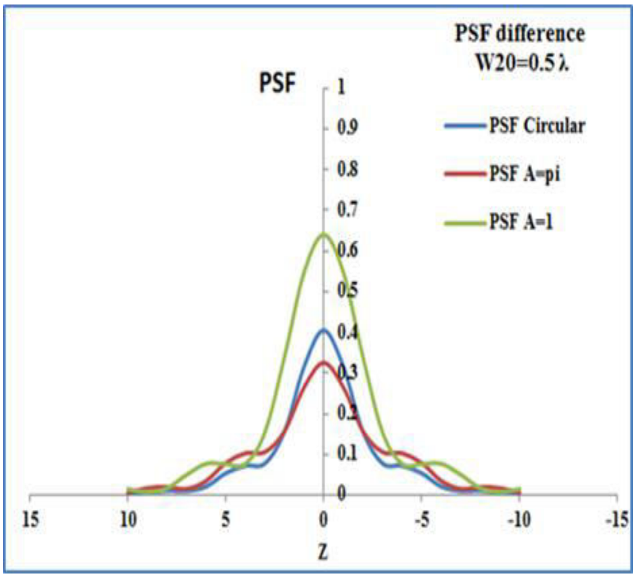

Figure-3. PSF profile for the diffraction limited optical system with different apertures suffer from focus error (W20=0. 5λ)

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Table-1.PSF Vis focus error W_{20} for optical system with circular aperture.

Z	PSF					
	W=0	*W=0.25*	*W=0.5*	*W=0.75*	*W=1*	*W=OP*
-10	7.56E-05	3.50E-04	1.68E-03	5.44E-03	0.012	2.36E-03
-9	2.97E-03	3.69E-03	6.32E-03	0.011	0.016	1.66E-03
-8	3.44E-03	5.08E-03	0.011	0.022	0.029	3.23E-03
-7	1.79E-06	1.83E-03	0.01	0.025	0.035	9.49E-03
-6	8.51E-03	0.012	0.021	0.031	0.032	0.024
-5	0.017	0.031	0.054	0.058	0.038	0.028
-4	1.09E-03	0.027	0.072	0.082	0.049	5.66E-03
-3	5.10E-02	0.062	0.078	0.07	0.04	0.04
-2	3.33E-01	0.277	0.156	0.054	0.015	0.265
-1	7.75E-01	0.629	0.316	0.072	1.40E-03	0.62
0	1.00E+00	0.811	0.405	0.09	1.82E-14	0.8
1	7.75E-01	0.629	0.316	0.072	1.40E-03	0.62
2	3.33E-01	0.277	0.156	0.054	0.015	0.265
3	5.10E-02	0.062	0.078	0.07	0.04	0.04
4	1.09E-03	0.027	0.072	0.082	0.049	5.66E-03
5	1.70E-02	0.031	0.054	0.058	0.038	0.028
6	8.51E-03	0.012	0.021	0.031	0.032	0.024
7	1.79E-06	1.83E-03	0.01	0.025	0.035	9.49E-03
8	3.44E-03	5.08E-03	0.011	0.022	0.029	3.23E-03
9	2.97E-03	3.69E-03	6.32E-03	0.011	0.016	1.66E-03
10	7.56E-05	3.50E-04	1.68E-03	5.44E-03	0.012	2.36E-03

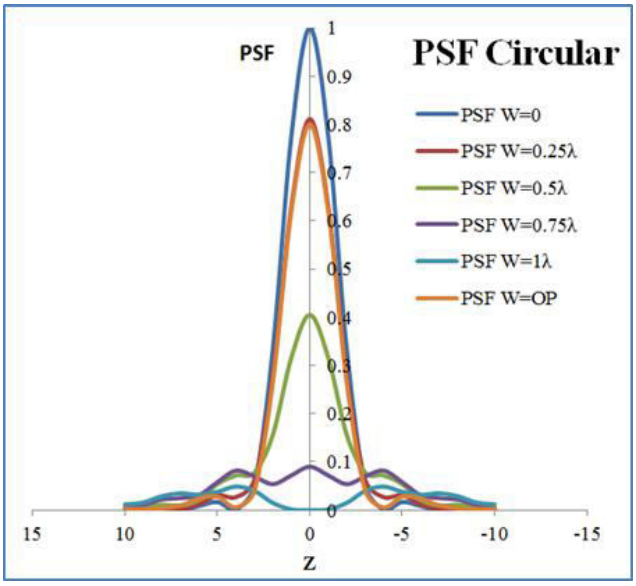

Figure-4. PSF profile for the diffraction limited optical system with circular aperture with different focus error W_{20}.

Figures (4, 5 and 6) represent the effect of different values of focus errors (W20=0.0,0.25,0.5,0.75,1.0λ). From these figures and tables 1, 2 and 3, it is shown that the new square aperture with half diagonal equal one is less affected by all values of focus error. Also the effect on the optimum balance between focus error and primary spherical aberration for circular apertures with area Pi (i.e. W20= -W40) is used in all the three apertures.

Table-2. PSF Vis focus error W_{20} for square aperture with area equal Pi.

Z	PSF					
	W=0	*W=0.25*	*W=0.5*	*W=0.75*	*W=1*	*W=OP*
-10	3.62E-03	4.16E-03	5.42E-03	5.83E-03	4.80E-03	1.14E-03
-9	1.50E-02	1.60E-02	1.70E-02	0.014	8.59E-03	2.06E-03
-8	0.01	0.013	0.02	0.02	0.013	3.60E-03
-7	1.64E-04	4.29E-03	0.015	0.02	0.014	0.013
-6	2.40E-02	0.029	0.035	0.028	0.015	0.027
-5	0.047	0.066	0.079	0.05	0.02	0.021
-4	1.20E-02	0.057	0.103	0.07	0.026	1.09E-03
-3	3.10E-02	0.07	0.106	0.07	0.026	0.048
-2	3.06E-01	0.261	0.157	0.064	0.019	0.242
-1	7.64E-01	0.589	0.265	0.068	0.013	0.507
0	1.00E+00	0.761	0.325	0.073	0.011	0.634
1	7.64E-01	0.589	0.265	0.068	0.013	0.507
2	3.06E-01	0.261	0.157	0.064	0.019	0.242
3	3.10E-02	0.07	0.106	0.07	0.026	0.048
4	1.20E-02	0.057	0.103	0.07	0.026	1.09E-03
5	4.70E-02	0.066	0.079	0.05	0.02	0.021
6	2.40E-02	0.029	0.035	0.028	0.015	0.027
7	1.64E-04	4.29E-03	0.015	0.02	0.014	0.013
8	1.00E-02	0.013	0.02	0.02	0.013	3.60E-03
9	1.50E-02	1.60E-02	1.70E-02	0.014	8.59E-03	2.06E-03
10	3.62E-03	4.16E-03	5.42E-03	5.83E-03	4.80E-03	1.14E-03

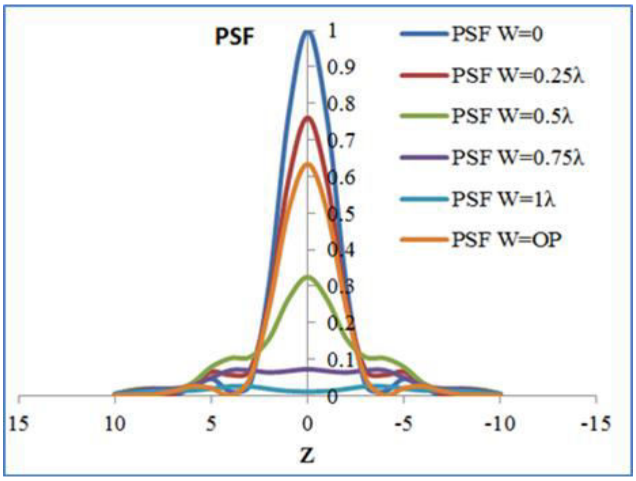

Figure-5. Intensity profile for different amount of W_{20} (square aperture with area equal Pi).

Table-3. PSF Vis focus error W_{20} for square aperture with half diagonal equal one.

Z	PSF					
	W=0	W=0.25	W=0.5	W=0.75	W=1	W=OP
-10	0.01	0.011	0.015	0.019	0.021	6.72E-03
-9	1.61E-04	1.79E-03	6.88E-03	0.014	0.02	3.53E-04
-8	0.011	0.012	0.017	0.022	0.024	0.02
-7	0.039	0.042	0.05	0.053	0.045	0.062
-6	4.40E-02	0.055	0.077	0.087	0.074	0.079
-5	0.012	0.033	0.076	0.102	0.092	0.047
-4	1.20E-02	0.033	0.077	0.103	0.094	0.026
-3	1.61E-01	0.161	0.155	0.133	0.095	0.127
-2	4.88E-01	0.446	0.34	0.215	0.113	0.386
-1	8.44E-01	0.758	0.548	0.314	0.141	0.681
0	1.00E+00	0.896	0.64	0.36	0.156	0.812
1	8.44E-01	0.758	0.548	0.314	0.141	0.681
2	4.88E-01	0.446	0.34	0.215	0.113	0.386
3	1.61E-01	0.161	0.155	0.133	0.095	0.127
4	1.20E-02	0.033	0.077	0.103	0.094	0.026
5	1.20E-02	0.033	0.076	0.102	0.092	0.047
6	4.40E-02	0.055	0.077	0.087	0.074	0.079
7	3.90E-02	0.042	0.05	0.053	0.045	0.062
8	1.10E-02	0.012	0.017	0.022	0.024	0.02
9	1.61E-04	1.79E-03	6.88E-03	0.014	0.02	3.53E-04
10	1.00E-02	0.011	0.015	0.019	0.021	6.72E-03

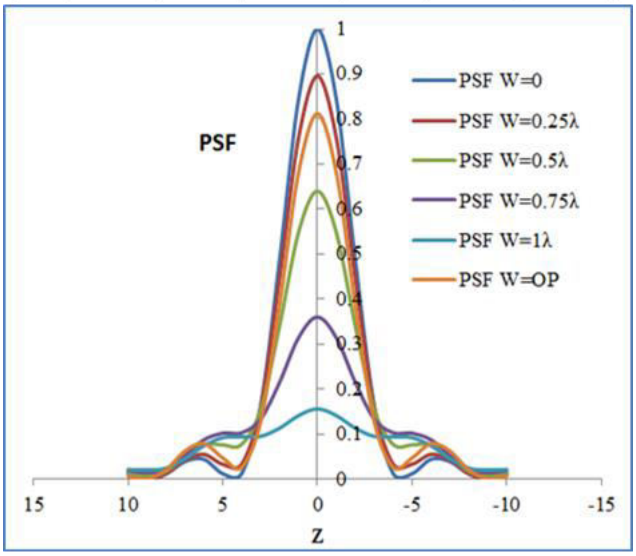

Figure-6. Intensity profile for different amount ofW_{20}(square aperture with half diagonal equal one).

The results from the Figures 4,5,6 and the tables 1,2,3 are SR=0.811, 0.761 and 0.821 for circular, square of area Pi and the square of the half diagonal equal one apertures respectively. This indicates that each aperture shape must have its own variance, its aberration optimum balance values and its Strehl ratio value, so one must derive a new optimum balance aberration values for each aperture shape individually.

CONCLUSIONS

Our work provides a theoretical analysis of the intensity PSF of the optical system with variable aperture in both of defocus and free aberration system. The influence for different pupil aperture are tested. The mathematical expression i particularly useful in testing the quality of the optical system, we indicate from our studies that the optical system with square area =π have a higher resolution than the other systems. While for small values of W_{20} the system of square aperture with half diagonal =1 is less sensitive to the defocus so it has a large depth of focus and this type of aperture is useful in in tracing a guidance system. The results indicate that one must derive a special formula for aberration optimally balanced for each aperture shape.

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