EZLAKI TRANSFORM HOMOTOPY PERTURBATION METHOD FOR TEMPERATURE FIELD OF A FLUID OVER A STRETCHING SHEET WITH UNIFORM HEAT FLUX

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ABSTRACT

Objectives: In this paper, the temperature distribution in the flow of a viscous incompressible fluid caused by the stretching sheet with uniform heat flux has been discussed. Methods/Analysis: The combination of Elzaki Transform and Homotopy perturbation method is applied for finding the solutions. Findings: The series solutions for velocity and temperature distribution are achieved by this method. Novelty/Improvement: The series solutions are obtained for the non linear equations caused by temperature field over a stretching sheet and the results are compared with the exact solutions, this method is seen as a better alternative method to some existing techniques for such realistic problems.

Keywords: elzaki transform method, prandtl number, homotopy perturbation method, series solution, kinematic viscosity.

1. INTRODUCTION

The flow and heat transfer of a viscous incompressible fluid in a channel has applications in technological fields, heat exchanger, reactor cooling etc. All these investigations are restricted to hydrodynamic flow and heat transfer problems, recently these problems have become more important to industry. Due to its wide range of applications, the stretching sheet problems have been studied by a number of researchers. Most solutions available are based on numerical techniques such as Keller box method, Runge Kutta method and finite element method. Datta *et al* [1] discussed the incomplete gamma function to study the behavior of temperature distribution over a stretching sheet. Pradhan *et al* [2] studied the MHD boundary layer flow over a nonlinear stretching sheet using implicit finite difference scheme. Noor *et al* [3] solved the higher dimensional initial boundary value problems by Variational Homotopy Perturbation method.

In this method the solution is given in an infinite series usually converging to an accurate solution. There are few investigators who have tried to study the flow of fluid over a stretching sheet and their behaviour under different conditions. Promise Mebine [4] introduced the boundary layer flow over an exponentially stretching surface by VIM Pade's method. Sarif *et al* [5] used Keller Box method to find the numerical solution of flow and heat transfer over a stretching sheet. Pallavi and Bhuma Devi *et al* [6] discussed the applications of Elzaki transform method to ordinary differential equations and partial differential equations. Sushila *et al* [7] studied an efficient analytical approach for MHD viscous flow over a stretching sheet via homotopy perturbation sumudu transform method. Elzaki Tarig *et al* [8] considered Elzaki and Sumudu transform for solving some differential equations. Elzaki Tarig *et al* [9] studied the connection between Laplace and Elzaki transforms. Prem Kiran *et al* [9] analyzed the porous medium equation using Elzaki transform homotopy perturbation method. A broad range of analytical and numerical methods have been used in the analysis of these stretching sheet models. An effective method is required to develop the analytical models to study the behaviour of stretching sheet which provide solutions conforming to physical reality.

The aim of this paper is to investigate the velocity and temperature distribution in the flow of a viscous incompressible fluid caused by stretching sheet behaviour using ETHPM and comparing with the exact solutions.

2.ELZAKI TRANSFORM HOMOTOPY PERTURBATION METHOD (ETHPM)

Consider a general nonlinear non-homogenous partial differential equation with initial conditions

$$Du(x,t)+Ru(x,t)+Nu(x,t)=g(x,t)$$
$$u(x,0)=h(x), u_t(x,o)=f(x) \qquad (1)$$

where D is the linear operator of order two, R is linear differential operator of less order than D, N is the general nonlinear differential operator and $g(x,t)$ is the source term.

Applying the Elzaki transform on both sides of equation (1) and inverse Elzaki transform, to find

$$u(x,t)=G(x,t)-E^{-1}\left(v^2 E[Ru(x,t)+Nu(x,t)]\right) \qquad (2)$$

where $G(x,t)$ represents the term arising from the source term and the given boundary conditions.
Now the nonlinear term can be decomposed as

$$N[u(x,t)]=\sum_{n=0}^{\infty} p^n H_n(u) \qquad (3)$$

Where $H_n(u)$ are given by

$$H_n(u_0, u_1, \ldots\ldots\ldots\ldots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{n=0}^{\infty} p^i u_i\right)\right]_{p=0}, n = 0,1,2,\ldots\ldots.. \quad (4)$$

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = G(x,t) - p\left[E^{-1}\left[v^2 E\left(\sum_{n=0}^{\infty} p^n u_n(x,t) + \sum_{n=0}^{\infty} p^n H_n(u)\right)\right]\right] \quad (5)$$

Comparing the coefficient of like powers of p, the following approximations are obtained.

$$p^0 : u_0(x,t) = G(x,t),$$
$$p^1 : u_1(x,t) = -E^{-1}\left[v^2 E\left[Ru_0(x,t) + H_0(u)\right]\right]$$
$$p^2 : u_2(x,t) = -E^{-1}\left[v^2 E\left[Ru_1(x,t) + H_1(u)\right]\right]$$

Then the solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots\ldots\ldots\ldots.. \quad (6)$$

3. MATHEMATICAL FORMULATION OF THE PROBLEM

Assuming boundary layer approximations, the equations of continuity, momentum and heat transfer in the usual notation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad (8)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\sigma}\frac{\partial^2 T}{\partial y^2} \quad (9)$$

where u and v are the velocity components in the x and y directions respectively, σ is the Prandtl number and ν is the kinematic viscosity.
Subject to the boundary conditions

$$u = \alpha x,\ v = 0,\ -\lambda\frac{\partial T}{\partial y} = A' \text{ at y = 0}$$
$$u \to 0,\ T \to T_\infty \text{ as } y \to \infty \quad (10)$$

Define a stream function

$$\psi = -(\alpha\nu)^{1/2} x f(\eta),\ \eta = (\alpha/\nu)^{1/2} y \quad (11)$$

$$u = \alpha x f'(\eta),\ v = -(\alpha\nu)^{1/2} f(\eta) \quad (12)$$

4. SOLUTION OF THE PROBLEM

Substitution of equation (10), (11) & (12) in equation (8) & (9) gives

$$f'^2(\eta) - f(\eta)f''(\eta) = f'''(\eta) \quad (13)$$

$$g''(\eta) + \sigma f(\eta)g'(\eta) = 0 \quad (14)$$

Subject to the boundary conditions

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0 \quad (15)$$

$$g'(0) = -1, g(\infty) = 0$$
(16)

In this section, we mainly solve our equation (13) and (14) with boundary conditions given in (15) and (16), by Elzaki transform Homotopy perturbation method.
Taking Elzaki transform of equation (13) & (14) on both sides, we get

$$E(f(\eta)) = u^3 + \alpha u^4 + u^3 E\left((f')^2 - ff''\right) \quad (17)$$

$$E(g(\eta)) = u^2 - u^3 - \sigma u^2 f(\eta)g'(\eta) \quad (18)$$

$$f(\eta) = \eta + \frac{\alpha\eta^2}{2} + p\left[E^{-1}\left[u^3 E\left[\sum_{m=0}^{\infty} p^m H_m(\eta) - \sum_{m=0}^{\infty} H'_m(\eta)\right]\right]\right] \quad (19)$$

$$g(\eta) = 1 - \eta - p\sigma E^{-1}\left(u^2 E\left(\sum_{m=0}^{\infty} p^m H_m(\eta)\right)\right) \quad (20)$$

Where $H_m(\eta)$ and $H'_m(\eta)$ are He's polynomial that represents the non linear terms

$$\sum_{m=0}^{\infty} p^m H_m(\eta) = (f'(\eta))^2$$
$$H_0(\eta) = (f_0')^2(\eta)$$
$$H_1(\eta) = 2(f_0')(\eta)f_1'(\eta)$$
$$H_2(\eta) = (f_1')^2(\eta) + 2f_0'(\eta)f_2'(\eta)$$

And for $H_m(\eta)$, we find that

$$\sum_{m=0}^{\infty} p^m H_m'(\eta) = f(\eta)f''(\eta)$$
$$H'_0(\eta) = f_0(\eta)f''_0$$

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$$H'_1(\eta) = f_0(\eta) f''_1 + f_1(\eta) f''_0(\eta)$$

$$H'_2(\eta) = f_0(\eta) f''_2 + f_1(\eta) f''_1(\eta) + f_2(\eta) f''_0(\eta)$$

Comparing the coefficient of like powers of p, we get

$$p^{(0)} : f_0(\eta) = \eta + \frac{\alpha\eta^2}{2}$$

$$p^{(1)} : f_1(\eta) = E^{-1}\left[v^3 E\left[H_0(\eta) - H'_0(\eta)\right]\right]$$

$$p^{(2)} : f_2(\eta) = E^{-1}\left[v^3 E\left[H_1(\eta) - H'_1(\eta)\right]\right]$$

$$p^{(0)} : f_0(\eta) = \eta + \frac{\alpha\eta^2}{2}$$

$$p^{(1)} : f_1(\eta) = \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120}$$

$$p^{(2)} : f_2(\eta) = \frac{\eta^5}{120} + \frac{\alpha\eta^6}{720} + \frac{\alpha^2\eta^7}{5040} + \frac{\alpha^3\eta^8}{40320}$$

The series solution is given by

$$f(\eta) = \lim_{n\to\infty} f_n$$

$$f(\eta) = \eta + \frac{\alpha\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120} + \frac{\alpha\eta^6}{720} + \frac{\alpha^2\eta^7}{5040} + \frac{\alpha^3\eta^8}{40320} + \frac{\alpha^2\eta^9}{362880} + \frac{\alpha^3\eta^{10}}{3628800} + \frac{\alpha^4\eta^{11}}{39916800} + \ldots\ldots \quad (21)$$

$$p^{(0)} : g_0 = 1 - \eta$$

$$p^{(1)} = g_1 = \frac{\sigma\eta^3}{6} + \frac{\sigma\alpha\eta^4}{24}$$

The series solution is given by

$$g(\eta) = \lim_{n\to\infty} g_n$$

$$g(\eta) = 1 - \eta + \frac{\sigma\eta^3}{6} + \frac{\sigma\alpha\eta^4}{24} + \frac{\sigma\eta^5}{120} - \frac{\sigma^2\eta^6}{240} + \frac{\sigma\alpha\eta^6}{720} + \frac{\sigma^2\alpha\eta^6}{72} - \frac{\sigma\alpha^2\eta^7}{504} + \frac{\sigma\alpha^2\eta^7}{5040} + \ldots\ldots\ldots \quad (22)$$

RESULTS AND DISCUSSIONS

Table-1. The comparison of the values obtained for $f(\eta)$ by the ETHPM with the exact solution

Exact solution		ETHPM	
η	$f(\eta)$	η	$f(\eta)$
0.1	0.0952	0.1	0.0997
0.2	0.1813	0.2	0.1993
0.3	0.2592	0.3	0.3000
0.4	0.3297	0.4	0.4026
0.5	0.3935	0.5	0.5081
0.6	0.4512	0.6	0.6175

From the Table-1 it is clear that the numerical values are presented for ETHPM and exact method. Therefore by observing the results obtained by ETHPM and exact solution method we found that the series solution obtained by ETHPM converges faster than the exact solution in the studied case.

Table-2. The comparison values obtained for heat flux $g'(\eta)$ by the ETHPM with the exact solution when Prandtl number $\sigma = 1$.

Exact solution		ETHPM	
η	$g'(\eta)$	η	$g'(\eta)$
0.1	-0.9952	0.1	-0.9952
0.2	-0.9815	0.2	-0.9814
0.3	-0.9600	0.3	-0.9601
0.4	-0.9321	0.4	-0.9330
0.5	-0.8990	0.5	-0.9023
0.6	-0.8617	0.6	-0.8711

Table-2 shows that the numerical computation of these values is used to study the effect of heat flux with assuming that Prandtl number one. ETHPM results coincide with exact solution results for temperature distribution. Numerical results show that the ETHPM solution are good approximations for the exact solution and reduces the computational work. We find that agreement is perfectly good.

6. CONCLUSIONS

The main aim of the present work is to demonstrate the combination of new integral transform Elzaki Transform and Homotopy perturbation method for temperature flow over a stretching sheet with heat flux and provides series solution. Numerical values are presented in the table. The obtained results of velocity and temperature distribution showed good agreement with the exact solution .The results suggest that the solutions are very close to exact solution and more efficient by Elzaki Transform Homotopy Perturbation method (ETHPM).

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