VOL. 12, NO. 23, DECEMBER 2017 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

ROTATIONAL OSCILLATION OF A CYLINDER IN AIR FLOW

A. N. Ryabinin and N. A. Kiselev
Department of Hydro Aeromechanics, Faculty of Mathematics and Mechanics, Saint-Petersburg State University, St. Petersburg, Russia
E-Mail: a.ryabinin@spbu.ru

ABSTRACT

The paper describes the experimental study of shielding effects of the disk placed coaxially upstream of a cylinder. It not only reduces the drag of the cylinder. The disk changes the dynamic characteristics of the cylinder. Without a disk, an elastically fixed cylinder in the airflow performs rotational oscillations with constant amplitude. A disk of small diameter, located near the cylinder, reduces the amplitude of rotational oscillation. Increasing the distance between the disk and the cylinder causes the damped rotational oscillations. The influence of the aerodynamic force on the damping of the oscillations depends on the disk diameter and the gap between disk and cylinder. A mathematical model is proposed for describing the rotational steady and damped oscillation of a cylinder with a disk.

Keywords: cylinder, rotational oscillation, coaxial disk, mathematical model.

INTRODUCTION

It was obtained that the drag force of the cylinder of circular cross-section can be reduced with a disk attached to cylinder coaxially upstream [1, 2]. But dynamical characteristics of the cylinder with coaxial disk were not studied. A rotational oscillation of the configuration presented in the Figure-1 is the main subject of the present investigation.

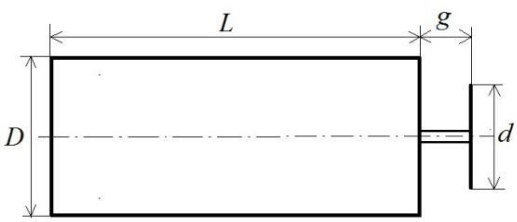

Figure-1. The scheme of the cylinder with coaxial disk.

If the aerodynamic forces acting on the body depend only on the instant angles of attack and sideslip, it has been proven that the quasi-steady approximation can be used to describe the oscillations of elastically fixed bodies [3, 4]. However, the quasi-steady approximation is inapplicable to rotational oscillations of bluff bodies, because the aerodynamic forces depend not only on the instant angles of attack and sideslip, but also on the derivatives of these angles with respect to time [5 - 8].

Mathematical model for rotational oscillation of the cylinder without coaxial disk was suggested in the paper [8]. Experimental study of cylinder rotational oscillations was described in [6, 7].

EXPERIMENTAL METHOD

The experiments were carried out in the subsonic wind tunnel AT-12 of Saint-Petersburg State University. The wind tunnel has open test section. The diameter of the outlet circular cross section of the nozzle is 1.5 m. The flow velocity in the test section varies from 0 to 40 m/s. Cylinder is fixed with the wire suspension. It could rotate around the horizontal axis that is perpendicular to the mean velocity vector of the oncoming stream. A steel tail holder is fixed to the downstream end of the cylinder. Two steel springs are attached to the holder (Figure-2). A semiconductor strain gauge registers the tension of one of the springs.

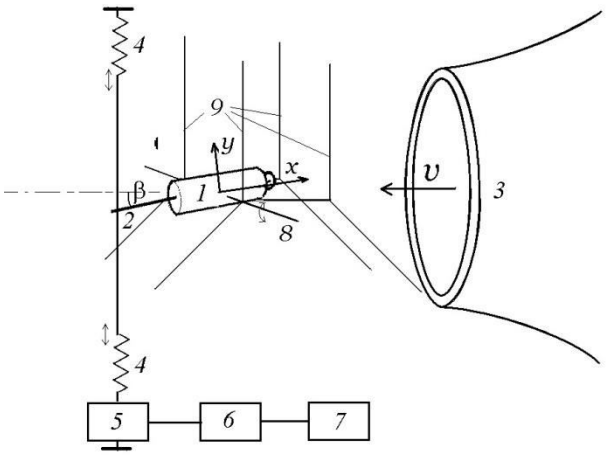

Figure-2. The scheme of experimental setup. 1- cylinder, 2- steel tail holder, 3 - nozzle, 4 - springs, 5 - semiconductor strain gauge, 6 - PC-oscilloscope, 7 - computer, 8 - rotational axis, 9 - wires.

PC-oscilloscope Velleman-PCS500A transferred the signal from strain gauge to the computer. The frequency of the records was equal to 100 Hz.

A typical sample of signal record of steady oscillation of the cylinder is shown in Figure-3.

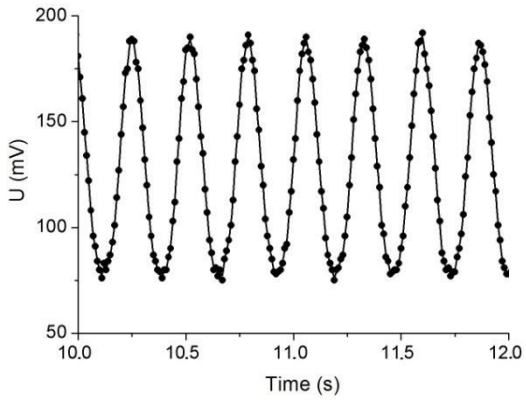

Figure-3. Record of the signal at steady oscillations.

A typical sample of signal record of damped oscillation of the cylinder is shown in Figure-4.

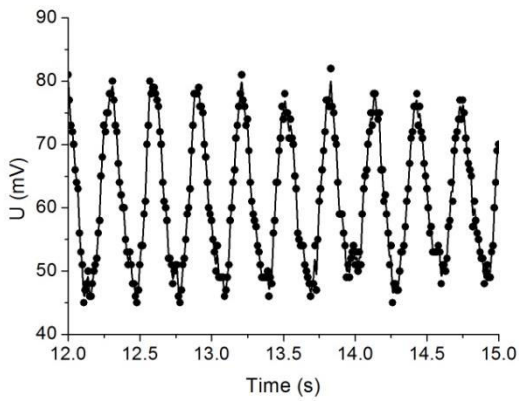

Figure-4. Record of the signal at damped oscillations.

The signal is proportional to the tension of the lower spring. It turned out that the oscillation frequency does not depend on the velocity of the air. In the absence of flow in the test section, the oscillations of the cylinders on the elastic suspension were damped, and the oscillation frequency remained the same as in the oscillations due to the airflow. This fact confirms the smallness of the aerodynamic forces in comparison with the elastic forces.
We suggested that the tension of the springs in the extremes of the dependence of the signal on time is equal to the tension of the springs under the action of a constant load, causing a deviation equal to the amplitude of the oscillations. This assumption made it possible to relate the amplitude of the oscillations to the maximum or minimum tension force of the lower spring detected by the instrument. Two calibration experiments were carried out. In one experiment, during the recording of the readings of the strain gauge a load of a known mass was hung to the point of attachment of the tail holder to the wire. Based on the measurement results, a change in the readings of the device, caused by a known force, was determined. In another calibration experiment, the displacement of the end of the tail holder was determined under the influence of suspending a load of known mass.

Based on the results of two calibration experiments, the coefficient relating the amplitude of the tension oscillations of the lower spring with the amplitude of the cylinder's rotational oscillations was determined.

PROCESSING OF EXPERIMENTAL RESULTS

As the basis of signal processing, it was suggested that the measured rotational angle of the cylinder β_i at time t_i is the sum of the harmonic function $B \cos \omega t_i + C \sin \omega t_i$, constant E and a random variable ξ_i with zero expectation, which is an experimental error:

$$\beta_i = B\cos\omega t_i + C\sin\omega t_i + E + \xi_i \tag{1}$$

The period of oscillations T and the angular frequency of oscillations $\omega = 2\pi/T$ are determined from the results of calculating the number of oscillations over a known time interval containing dozens of oscillation periods. Let n be the number of readings in one period, $i = 1, 2, 3, \ldots, n$. Then, multiplying formula (1) sequentially by $\cos \omega t_i$ and $\sin \omega t_i$ and calculating the arithmetic average over all readings in the period, we get

$$\frac{1}{n}\sum_{i=1}^{n}\beta_i \cos\omega t_i = B\frac{1}{n}\sum_{i=1}^{n}\cos^2\omega t_i + C\frac{1}{n}\sum_{i=1}^{n}\cos\omega t_i \sin\omega t_i + E\frac{1}{n}\sum_{i=1}^{n}\cos\omega t_i + \frac{1}{n}\sum_{i=1}^{n}\xi_i \cos\omega t_i, \tag{2}$$

$$\frac{1}{n}\sum_{i=1}^{n}\beta_i \sin\omega t_i = B\frac{1}{n}\sum_{i=1}^{n}\cos\omega t_i \sin\omega t_i + C\frac{1}{n}\sum_{i=1}^{n}\sin^2\omega t_i + E\frac{1}{n}\sum_{i=1}^{n}\sin\omega t_i + \frac{1}{n}\sum_{i=1}^{n}\xi_i \sin\omega t_i. \tag{3}$$

The sums in equations (2) and (3), which contain only trigonometric functions, can be calculated. The parameter E is determined in the experiment in the absence of oscillations. If n is larger then

$$\frac{1}{n}\sum_{i=1}^{n}\cos^2\omega t_i \approx 0.5, \quad \frac{1}{n}\sum_{i=1}^{n}\sin^2\omega t_i \approx 0.5,$$
$$\frac{1}{n}\sum_{i=1}^{n}\cos\omega t_i \sin\omega t_i \approx 0, \quad \frac{1}{n}\sum_{i=1}^{n}\cos\omega t_i \approx 0,$$
$$\frac{1}{n}\sum_{i=1}^{n}\sin\omega t_i \approx 0, \quad \frac{1}{n}\sum_{i=1}^{n}\xi_i \cos\omega t_i \approx 0.$$

Thus, from the equations (2) and (3) we can find the parameters B, C and calculate the amplitude of rotational oscillation $A = \sqrt{B^2 + C^2}$.

MATHEMATICAL MODEL OF ROTATIONAL OSCILLATIONS

The equation of motion of a cylinder elastically fixed in a flow has the form:

$$I\ddot{\beta} = L_a + L_s, \tag{4}$$

where I is the moment of inertia, L_a - moment of aerodynamic force, L_s - moment of the elastic force and frictional force. The dot above the symbol denotes differentiation with respect to time.

The components of the force moment can be represented in the form

$$L_a = \frac{\rho v^2}{2} LS\left(m + \frac{L\dot{\beta}}{v} m^{\dot{\beta}}\right), \quad L_s = -kl^2\beta - k_1\dot{\beta}. \tag{5}$$

In formula (5), ρ is the air density; v is the velocity of the incoming flow; L is the length of the cylinder;
S is the characteristic area that is equal to the area of the base of the cylinder; m is the coefficient of the aerodynamic force moment; $m^{\dot{\beta}}$ is the coefficient of the aerodynamic derivative; k is the spring rate; l is the distance from the axis of rotation to the end of the tail holder; k_1 is the coefficient corresponding to the viscous friction.

The coefficient of the moment is proportional toβ, and the rotational derivative coefficient is described by the formula:

$$m = -C_\beta \beta, \quad m^{\dot{\beta}} = C_{\dot{\beta}}\left(1 - \delta\beta^2\right). \tag{6}$$

Expressions (6) include the dimensionless parameters C_β, $C_{\dot{\beta}}$ and δ. After substituting expressions (5, 6) into the equation of motion (4), we obtain

$$\ddot{\beta} + \omega^2\beta + \Omega^2\beta = \mu\frac{v}{L}\left(1 - \delta\beta^2\right)\dot{\beta} - \mu k_2\dot{\beta}, \tag{7}$$

where

$$\omega^2 = kl^2/I, \Omega^2 = \frac{\rho v^2}{2I} LSC_\beta, \mu = \frac{\rho L^3 S}{2I} C_{\dot{\beta}}, k_2 = \frac{k_1}{\mu I}.$$

If ω^2 is much greater than Ω^2 then

$$\ddot{\beta} + \omega^2\beta = \mu\left[\frac{v}{L}\left(1 - \delta\beta^2\right) - k_2\right]\dot{\beta}, \tag{8}$$

Assuming μ to be a small parameter, the equation (8) can be solved by the Krylov-Bogolyubov method [9]. As a result, we obtain equations for the slowly varying amplitudes A and the phase φ of oscillation:

$$\dot{A} = \frac{A\mu}{2}\left[\left(\frac{v}{L} - k_2\right) - \frac{v}{L}A^2\frac{\delta}{4}\right], \quad \dot{\varphi} = 0. \tag{9}$$

In the case of steady oscillation with constant amplitude the derivative of the amplitude $\dot{A}$ is equal to zero and

$$A^2 = \frac{4}{\delta} - \frac{4k_2 L}{\delta}\frac{1}{v}. \tag{10}$$

Therefore, A^2 is linear function of $1/v$.

The first equation (9) can be transformed to the form

$$\eta = \frac{d\ln A}{dt} = \frac{\mu}{2}\left[\left(\frac{v}{L} - k_2\right) - \frac{v}{L}A^2\frac{\delta}{4}\right]. \tag{11}$$

EXPERIMENTAL RESULTS

Cylinder has the length L = 0.28 m, diameter D = 0.14 m (see Figure-1). Aspect ratio $\lambda = L/D = 2$. Experiments were performed without the coaxial disk and with the disk. The ratio of the disk diameter and cylinder diameter d/D was 0.536, 0.714 and 0.893. The ratio of the gap between disk and cylinder and cylinder diameter g/D was 0.5, 0.607 and 0.714.

Figure-5 shows the dependence of the square of oscillation amplitude of the cylinder without coaxial disk on $1/v$. Marks in the Figure-5 are near the straight line.

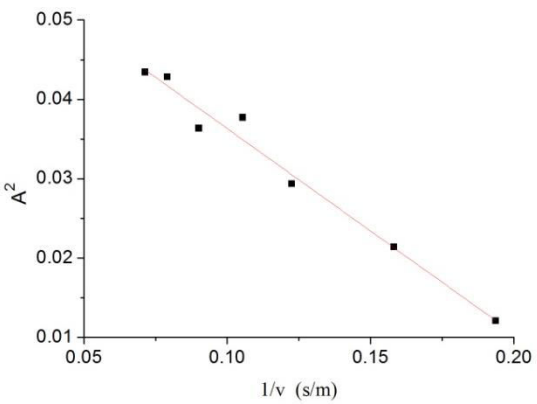

Figure-5. Dependence of square of amplitude A^2 on $1/v$. Steady oscillation of the cylinder without coaxial disk.

This fact is in accordance with the predictions of the mathematical model. It is possible to determine the parameters δ and k_2 in the equation (10) using the method of least squares.

The steady oscillation exists in the case of the cylinder with the small disk and the small gap between disk and cylinder. The dependence of A^2 on $1/v$ is presented in the Figure-6. However, the amplitude of oscillation in less than the amplitude in the case of the cylinder without disk.

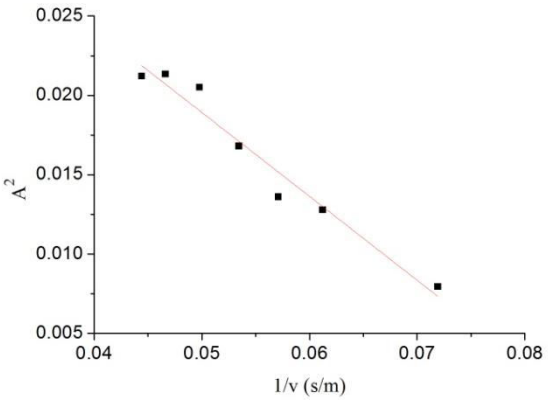

Figure-6. Dependence of square of amplitude A^2 on $1/v$. Steady oscillation of the cylinder with disk. $d/D = 0.536$, $g/D = 0.5$.

Two cases mentioned above confirm the predictions of the mathematical model (10).

Others configurations of the geometry demonstrate the damped oscillations at different velocity of inlet flow. In the Figure-7 there is the dependence of the logarithm of amplitude of oscillation on time.

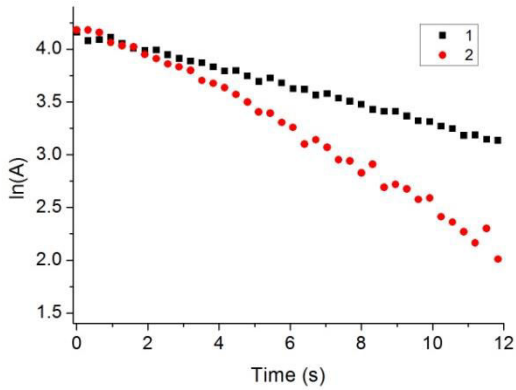

Figure-7. Dependence of the logarithm of amplitude of oscillation on time: 1 - flow velocity v is equal to zero, 2 - $v = 12.5$ m/s, $d/D = 0.893$, $g/D = 0.607$.

Dependence of ln (A) on time at $v = 0$ is a linear function. Parameter η does not depend on amplitude of oscillations. Another dependence in the Figure-7 at $v = 12.5$ m/s is not a linear function. If amplitude decreases then parameter η reduces. This type of damped oscillation exists if $d/D = 0.714$ or $d/D = 0.893$. If $d/D = 0.536$ and $g/D > 0.5$ then another type of damped oscillation is realized. Dependence of ln (A) on time for this type of oscillations is presented in the Figure-8. If amplitude of oscillation A decreases then parameter η increases.

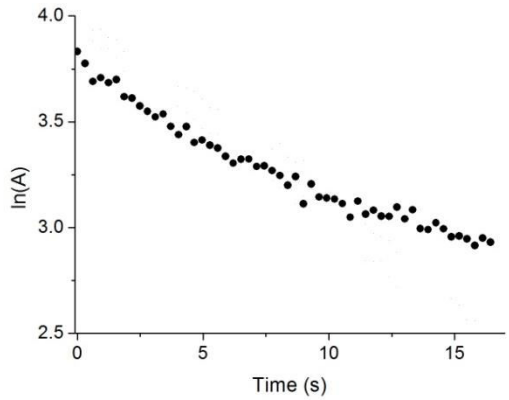

Figure-8. Dependence of the logarithm of amplitude of oscillation on time: $v = 19.1$ m/s, $d/D = 0.536$, $g/D = 0.714$.

Equation (11) can be presented in the form:

$$\eta_i = -\frac{\mu}{2}k_2 + \frac{\mu}{2L}v_i - \frac{\mu}{2}\frac{\delta}{4L}v_i A_i^{\,2}. \qquad (12)$$

The time range was divided into sections, each of which contained 7 periods of oscillation. For each section, a straight segment approximated the dependence of logarithm of the amplitude on time at velocity v_i. The slope of segment η_i and average values of the amplitudes A_i were determined. The method of least squares can be used to calculate the parameters of equation (12) $\mu k_2/2$, $\mu/(2L)$, $\mu\delta/(8L)$.

Data with a small amplitude of oscillations were excluded from the consideration due to the large error in determining the logarithm of the amplitude. Results of the data processing are presented in the Figure-9, Figure-10, Figure-11 and Figures-12.

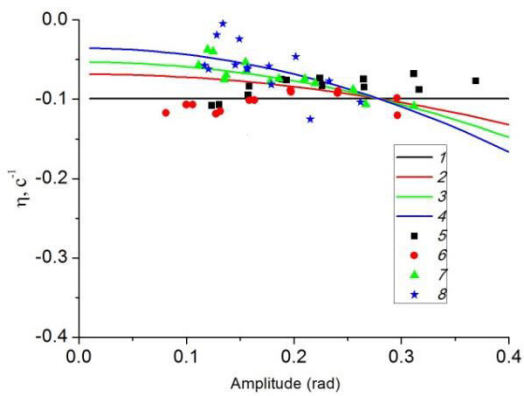

Figure-9. Dependence of η on amplitude of oscillation: $d/D = 0.536$, $g/D = 0.607$; 1, 5 - $v = 0$, 2; 6- $v = 9.8$ m/s; 3, 7- $v = 15.2$ m/s; 4, 8- $v = 20.9$ m/s; 1, 2, 3, 4- experiment; 5, 6, 7, 8 - approximation.

Solid line in the figures corresponds to the approximation (12).

For d/D = 0.536 and g/D = 0.607 (Figure-9) the damping of oscillations is provided by viscous friction. If the amplitude of oscillations is small then aerodynamic force reduces damping. Parameter η is growing. Increasing of g/D up to 0.714 (Figure-10) leads to the elimination of the influence of aerodynamic forces on the damping of oscillations.

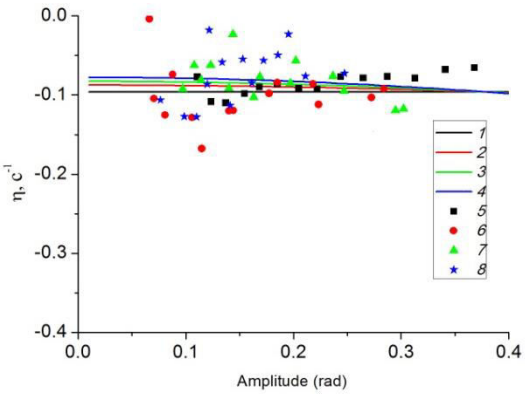

Figure-10. Dependence of η on amplitude of oscillation: d/D = 0.536, g/D = 0.714; 1, 5 - v = 0, 2; 6 - v = 10.2 m/s; 3, 7- v = 15.2 m/s; 4, 8- v = 21.0 m/s; 1, 2, 3, 4 - experiment; 5, 6, 7, 8 – approximation.

If d/D = 0.893 (Figure-11) then there is reducing of parameter η due to aerodynamic force at small amplitude of oscillation.

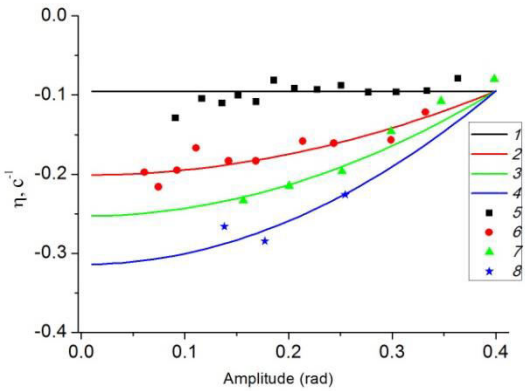

Figure-11. Dependence of η on amplitude of oscillation: d/D = 0.893, g/D = 0.607; 1, 5 - v = 0, 2; 6 - v = 10.2 m/s; 3, 7-v = 15.2 m/s; 4, 8-v = 21.1 m/s; 1, 2, 3, 4- experiment; 5, 6, 7, 8- approximation.

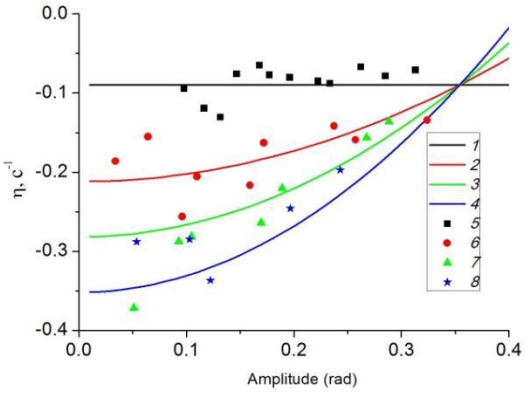

Figure-12. Dependence of η on amplitude of oscillation: d/D = 0.893, g/D = 0.714; 1, 5 - v = 0, 2; 6- v = 9.8 m/s; 3, 7- v = 15.4 m/s; 4, 8- v = 21.0 m/s; 1, 2, 3, 4- experiment; 5, 6, 7, 8- approximation.

Figure-12 shows that at g/D = 0.714 the influence of the aerodynamic force on oscillation damping increases.

CONCLUSIONS

A coaxial disk in front of the cylinder not only reduces the drag of the cylinder, but also changes the dynamic characteristics of the cylinder. Without a disk, an elastically fixed cylinder in the flow performs rotational oscillations with constant amplitude. A disk of small diameter, located close to the cylinder, reduces the amplitude of rotational oscillation. Increasing of the distance between the disk and the cylinder causes the damped rotational oscillations. The influence of the aerodynamic force on the damping of the oscillations depends on the disk diameter and the gap between disk and cylinder. The damping increases with the increasing of disk diameter and the distance between the disk and the cylinder. A mathematical model is proposed for describing the rotational oscillation of a cylinder with a disk.

REFERENCES

[1] Saunders W.S. 1966. Apparatus for reducing linear and lateral wind resistance in a tractor-trailer combination vehicle. US Patent No. 3241876.

[2] Koenig K, Roshko A. 1985. An experimental study of geometrical effects on the drag and flow field of two bodies separated by a gap. J. Fluid Mech. 156: 167-204.

[3] Parkinson G. V., Smith J. D. 1964. The square prism as an aeroelastic non-linear oscillator. Quarterly Journal of Mechanics and Applied Mathematics. 17: 225-239.

[4] Ryabinin A.N., Lyusin V.D. 2015. Galloping of small aspect ratio square cylinder. ARPN Journal of Engineering and Applied Sciences. 10(1): 134-138.

[5] Bratt J.B. 1963. Wind tunnel techniques for the measurements of oscillatory derivatives. Aeronautical research council report and memoranda. No. 3310: 53.

[6] Luschenko I.V., Ryabinin A.N. 2007. Experimental studies of the cylinder oscillations in the air stream. Vestnik Sankt-Peterburgskogo universiteta. Ser. 1. Issue 2: 120-123 [In Russian].

[7] Ryabinin A.N, Kiselev N.A. 2016. Effect of rotational axis position of the cylinder on its rotational oscillations in the air stream. Vestnik Sankt-Peterburgskogo universiteta. Ser. 1. 3(61): 2: 315-323 [In Russian].

[8] Ryabinin A.N., Tyurin B.F. 1993. The behavior of the load suspended under a helicopter. Vestnik Sankt-Peterburgskogo universiteta. Ser. 1. Issue 1: 87-91 [In Russian].

[9] Bogoliubov N.N., Mitrropolski Y.A. 1961. Asymptotic method in the theory of non-lenear oscillations. Gordon and Breach, New York, USA.