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VARIATIONAL ITERATION TRANSFORM METHOD FOR SOLVING BURGER AND COUPLED BURGER'S EQUATIONS

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ABSTRACT

In this paper, Variational iteration transform method is employed to determine the exact solution of the Burger equation which is one-dimensional and coupled Burger's equations nonlinear partial differential equation. This method is combined form of the Laplace transform and Variational iteration method. The explicit solutions obtained were compared to the exact solutions. The method finds the solution without any restrictive assumptions and free from round-off errors and therefore reduce the numerical computation to a great extent. The method is tested on two examples and coupled Burger's equations. The results show that the new method is more effective and convenient to use and high accuracy of it is evident.

Keywords: Laplace transform, variational iteration transform method, burger's equation, and nonlinear partial differential equation.

INTRODUCTION

Burgers' equation is a fundamental partial differential equation in fluid mechanics. It occurs in various areas of applied mathematics, such as modelling of dynamics, heat conduction, and acoustic waves, it is named for Johannes Martin us Burgers (1895-1981). Localized in a small part of space, play a major role in several different fields such as hydrodynamic, plasma physics, nonlinear optic, etc. The investigation of exact solutions of these nonlinear equations is interesting and important. In the past several decades, many authors had paid attention to study solutions of nonlinear equations by using various methods, such as Backland transformation [1,7], Darboux transformation [34], inverse scattering method [13], Hirota's bilinear method [22], the tanh method [27], the sine-cosine method [40,41], the homogeneous balance method [35,42], the Riccati expansion method with constant coefficients [43,44]. "Recently, an extended tanh-function method and symbolic computation are suggested in [11] for solving the new coupled modified Burger's equations to obtain four kinds of soliton solutions." This method has some merits in contrast with the tanh-function method. It is not only using a simpler algorithm to produce an algebraic system, but also can pick up singular solutions with no extra effort [12,23,28,32,39].

Most of the developed schemes have their limitations like limited convergence, divergent results, linearization, discretization, unrealistic assumptions and no compatibility with the versatility of physical problems [8] in the Burger's model of turbulence [4]. It is solved analytically for arbitrary initial conditions [24]. Finite element methods have been applied to fluid problems, Galerkin and Petrov-Galerkin finite element methods involving a time-dependent grid [6,21]. Numerical solution using cubic spline global trial functions were developed in [31] to obtain two systems or diagonally dominant equations which are solved to determine the evolution of the system. A collocation solution with cubic spline interpolation functions used to produce three coupled sets of equations for the dependent variable and its two first derivatives [5].

Since exact solutions of most of the differential equations do not exist, approximation and numerical methods are used for the solutions of the FDEs Ali *et al.*[3] applied B-spline finite element methods to the solution of Burger's equation. The B-spline finite element approach applied with collocation method over a constant grid of cubic B-spline elements. Cubic B-spline had a resulting matrix system which is tri-diagonal and so solved by the Thomas algorithm. Soliman [33] used the similarity reductions to the partial differential equations from develop a scheme for solving the Burger's equation. The coupled system is derived by Esipov [10]. It is simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids, under the effect of gravity [30].

The Variational iteration method was first proposed by He [14–17] and was successfully applied to autonomous ODE in [18], to nonlinear polycrystalline solids [29], and other fields. The combination of a Laplace transform method, Variational iteration transform method, method of variation of constants and averaging method to establish an approximate solution of one degree of freedom weakly nonlinear system in [9]. The Variational iteration transform method has many merits and has much advantage over the Adomian method [37]. The aim of this paper is to extend the Variational iterations transform method proposed by He [14-17,19,20] to solve two different types such as the one-dimensional Burger's equation and coupled Burger's equations and made a comparison with the results obtained by the Adomian decomposition method [2, 25, 26, 36, 38].

Variational Iteration Transform Method (VITM)

To illustrate the basic idea of this method, we consider a general nonlinear non-homogeneous partial differential equation with initial conditions of the form

$$\mathfrak{D}u(x,t) + \mathcal{R}u(x,t) + \aleph u(x,t) = g(x,t) \tag{1}$$
$$u(x,0) = h(x), u_t(x,0) = f(x)$$

Where $\mathfrak{D}$ is the second order linear differential operator $\mathfrak{D} = \frac{\partial^2}{\partial t^2}$, $\mathcal{R}$is linear differential operator of less order than $\mathfrak{D}$, $\aleph$ represent the general nonlinear differential operator and g(x,t) is the source term. Taking Laplace Transform on both sides of Eq.(1)

$$L[\mathfrak{D}u(x,t)] + L[\mathcal{R}u(x,t)] + L[\aleph u(x,t)] = L[g(x,t)] \tag{2}$$

$$s^2L[u(x,t)] - su(x,0) - u_t(x,0) + L[\mathcal{R}u(x,t)] + L[\aleph u(x,t)] = L[g(x,t)] \tag{3}$$

Taking Inverse Laplace transform

$$u(x,t) = f(x) + th(x) + \frac{1}{s^2}L^{-1}[L(g(x,t))] - \frac{1}{s^2}L^{-1}[L(\mathcal{R}(x,t))] - \frac{1}{s^2}L^{-1}[L\aleph u(x,t)] \tag{4}$$

Derivative by $\frac{\partial}{\partial t}$ both sides of Eq.(4)

$$u_t(x,t) = h(x) + \frac{\partial}{\partial t}L^{-1}\left(\frac{1}{s^2}L\,\{g(x,t)\}\right) - \frac{\partial}{\partial t}L^{-1}\left(\frac{1}{s^2}L\{\mathcal{R}u(x,t)\}\right) - \frac{\partial}{\partial t}L^{-1}\left(\frac{1}{s^2}L\{\aleph u(x,t)\}\right) \tag{5}$$

By the correction function of irrational method

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t ((u_n)_\xi(x,\xi) + \frac{\partial}{\partial \xi}(L^{-1}\left(\frac{1}{s^2}L\{\mathcal{R}u_n(x,\xi)\}\right. + L^{-1}\left(\frac{1}{s^2}L\{\aleph u_n(x,\xi)\}\right) - L^{-1}\left(\frac{1}{s^2}L\{g(x,\xi)\}\right) - h(x)\Big))d\xi \tag{6}$$

Finally, the solution u(x,t) is given by

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

An illustrative paradigm

In this section, the (VITM) is implemented for tacking Burger's equations with initial conditions. We demonstrate the effectiveness of this method with two examples. Numerical results obtained by the proposed method are compared with Known results.

Example 1:

Consider one- dimensional Burger's equation of the form

$$u_t = u_{xx} - uu_x \tag{7}$$

The initial condition is

$$u(x,0) = 1 - \frac{2}{x} \tag{8}$$

Taking Laplace transform on both sides

$$L[u_t] = L[u_{xx}] - L[uu_x] \tag{9}$$

This can be written as

$$sLu(x,t) - u(x,0) = L\left[\frac{\partial^2 u}{\partial x^2}\right] - L\left[u\frac{\partial u}{\partial x}\right] \tag{10}$$

On applying the above specified initial conditions we get

$$sL\big(u(x,t)\big) - \left(1 - \frac{2}{x}\right) = L\left[\frac{\partial^2 u}{\partial x^2}\right] - L\left[u\frac{\partial u}{\partial x}\right] \tag{11}$$

$$L[u(x,t)] = \frac{1}{s} - \frac{2}{sx} + \frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right] - \frac{1}{s}L\left[u\,\frac{\partial u}{\partial x}\right] \tag{12}$$

Applying the inverse Laplace transform on both sides of Eq.(12), We get

$$u(x,t) = L^{-1}\left[\frac{1}{s}\right] + L^{-1}\left[\frac{1}{s}L(\frac{\partial^2 u}{\partial x^2})\right] - L^{-1}\left[\frac{2}{sx}\right] - L^{-1}\left[\frac{1}{s}L(u\frac{\partial u}{\partial x})\right]$$

$$u(x,t) = 1 - \frac{2}{x} + L^{-1}\left[\frac{L}{s}\left(\frac{\partial^2 u}{\partial x^2}\right)\right] - L^{-1}\left[\frac{1}{s}L\left(u\frac{\partial u}{\partial x}\right)\right] \tag{13}$$

Derivative by $\frac{\partial}{\partial t}$ both sides (13)

$$u_t(x,t) + \frac{\partial}{\partial t}\left[L^{-1}\left(\frac{1}{s}L\left(u\,\frac{\partial u}{\partial x}\right)\right)\right] - \frac{\partial}{\partial t}\left[L^{-1}\left(\frac{1}{s}L\left(\frac{\partial^2 u}{\partial x^2}\right)\right)\right] = 0 \tag{14}$$

Making the correction function is given

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t ((u_n)_\xi(x,\xi) + \frac{\partial}{\partial \xi}(L^{-1}(\frac{1}{s}L[u_n(x,\xi)(u_n(x,\xi)]) - L^{-1}(\frac{1}{s}L[(u_n)_{xx}(x,\xi)]))\,d\xi \tag{15}$$

We can use the initial condition to select

$$u_0(x,t) = u(x,0) = 1 - \frac{2}{x}$$

Using this selection into the correction functional gives the following successive approximations

$$u_0(x,t) = 1 - \frac{2}{x} \tag{16}$$

$$u_1(x,t) = u_0(x,t) - \int_0^t ((u_0)_\xi(x,\xi) + \frac{\partial}{\partial \xi}(L^{-1}\left(\frac{1}{s}L[u_0(x,\xi)(u_0)_x(x,\xi)]\right) + L^{-1}(\frac{1}{s}L[(u_0)_{xx}(x,\xi)])))d\xi$$

$$u_1(x,t) = 1 - \frac{2}{x} - \int_0^t \Bigg(0 + \frac{\partial}{\partial \xi}\left(L^{-1}\left(\frac{1}{s}L\left[\left(1-\frac{2}{x}\right)\left(\frac{2}{x^2}\right)\right]\right) + L^{-1}\left(\frac{1}{s}L\left[\frac{4}{x^3}\right]\right)\right)\Bigg)d\xi$$

$$u_1(x,t) = 1 - \frac{2}{x} - \frac{2t}{x^2} \quad (17)$$

$$u_2(x,t) = u_1(x,t) - \int_0^t \Bigg(((u_1)_\xi(x,\xi) + \frac{\partial}{\partial \xi}\left(L^{-1}\left(\frac{1}{s}L[u_1(x,\xi)(u_1)_x(x,\xi)]\right) + L^{-1}\left(\frac{1}{s}L[(u_1)_{xx}(x,\xi)]\right)\right)\Bigg)d\xi \quad (18)$$

$$u_2(x,t) = \left(1-\frac{2}{x}\right) - \frac{2t}{x^2} - \int_0^t \Bigg(\frac{-2}{x^2} + \frac{\partial}{\partial \xi}(L^{-1}(\frac{1}{s}L\left[\left(1-\frac{2}{x}-\frac{2\xi}{x^2}\right)\left(\frac{2}{x^2}+\frac{4\xi}{x^3}\right)\right]) - L^{-1}\left(\frac{1}{s}L\left[\frac{4}{x^3}-\frac{12\xi}{x^4}\right]\right)))$$

$$u_2(x,t) = \left(1-\frac{2}{x}\right) - \frac{2t}{x^2} - \frac{2t^2}{x^3} \quad (19)$$

$$\vdots$$

$$u_n(x,t) = 1 - \sum_{k=0}^{n} \frac{2t^k}{x^{k+1}} \quad (20)$$

Finally, the solution is

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = 1 - \sum_{k=0}^{\infty} \frac{2t^k}{x^{k+1}} = 1 - \frac{2}{x-t} \quad (21)$$

Example2:

Consider one dimensional Burger's equation of the form

$$u_t = u_{xx} - uu_x \quad (22)$$

Subject to the initial condition

$$u(x,0) = x \quad (23)$$

Taking Laplace transform on both sides

$$L[u_t] = L[u_{xx}] - L[uu_x] \quad (24)$$

This can be written as

$$sL[u(x,t)] - u(x,0) = L\left[\frac{\partial^2 u}{\partial x^2}\right] - L\left[u\frac{\partial u}{\partial x}\right] \quad (25)$$

On applying the above specified initial condition we get

$$sL[u(x,t)] - x = L\left[\frac{\partial^2 u}{\partial x^2}\right] - L\left[u\,\frac{\partial u}{\partial x}\right] \quad (26)$$

$$L[u(x,t)] = \frac{x}{s} + \frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right] - \frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right] \quad (27)$$

Applying the inverse Laplace transform to both sides of Eq.(27), we get

$$u(x,t) = L^{-1}\left[\frac{x}{s}\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]\right] - L^{-1}[\frac{L}{s}[u\frac{\partial u}{\partial x}]]$$

$$u(x,t) = x + L^{-1}\left[\frac{1}{s}L\left[\frac{\partial^2 u}{\partial x^2}\right]\right] - L^{-1}\left[\frac{1}{s}L\left[u\frac{\partial u}{\partial x}\right]\right] \quad (28)$$

Derivative by $\frac{\partial}{\partial t}$ both sides (28)

$$u_t(x,t) = \frac{\partial}{\partial t}(L^{-1}\left[\frac{1}{s}L\left(\frac{\partial^2 u}{\partial x^2}\right)\right] - L^{-1}[\frac{1}{s}L(u\frac{\partial u}{\partial x})])$$

Or

$$u_t(x,t) - \frac{\partial}{\partial t}\left(L^{-1}\left[\frac{1}{s}L\left(\frac{\partial^2 y}{\partial x^2}\right)\right] - L^{-1}\left[\frac{1}{s}L\left(u\frac{\partial u}{\partial x}\right)\right]\right) = 0 \quad (29)$$

Making the correction function is given

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \Bigg((u_u)_\xi(x,\xi) - \frac{\partial}{\partial \xi}\left(L^{-1}\left[\frac{1}{s}L\left(\frac{\partial^2 u_n}{\partial x^2}\right)\right] - L^{-1}\left[\frac{1}{s}L\left(u_n\frac{\partial u_n}{\partial x}\right)\right]\right)\Bigg)d\xi \quad (30)$$

We can use the initial condition to selectu$_0(x,t) = u(x,0) = x$. Using this selection into the correction functional gives the following successive approximations.

$$u_0(x,t) = x \quad (31)$$

$$u_1(x,t) = u_0(x,t) - \int_0^t \left((u_0)_\xi(x,\xi) - \frac{\partial}{\partial \xi}\left(L^{-1}\frac{1}{s}L\left[\frac{\partial^2 u_0}{\partial x^2}\right] - L^{-1}\left[\frac{1}{s}L\left(u_0\frac{\partial u_0}{\partial x}\right)\right]\right)\right)d\xi$$

$$= x - \int_0^t (0 - \frac{\partial}{\partial \xi}(L^{-1}\left[\frac{1}{s}L(0)\right] - L^{-1}[\frac{1}{s}L(x)]))d\xi \quad (32)$$

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$$u_2(x,t) = u_1(x,t) - \int_0^t \left(\left(u_1(x,\xi) - \frac{\partial}{\partial\xi}(L^{-1}\left[\frac{1}{s}L\left(\frac{\partial^2 u_1}{\partial x^2}\right)\right] - L^{-1}[\frac{1}{s}L(u_1\frac{\partial u_1}{\partial x})]\right)\right) d\xi$$

$$= x - tx - \int_0^t (-x - \frac{\partial}{\partial\xi}(L^{-1}\left[\frac{1}{s}L(0)\right] - L^{-1}[\frac{1}{s}L[(x-tx)(1-t)]]))d\xi$$

$$= x - xt + xt^2 \quad (33)$$

$$u_3(x,t) = u_2(x,t) - \int_0^t \left((u_2)_\xi(x,\xi) - \frac{\partial}{\partial\xi}L^{-1}\left[\frac{1}{s}L\left(\frac{\partial^2 u_2}{\partial x^2}\right)\right] - L^{-1}\left[\frac{1}{s}L\left(u_2 - \frac{\partial u_2}{\partial x}\right)\right])\right) d\xi$$

$$= x - xt + xt^2 - \int_0^t (-x + 2x\xi - \frac{\partial}{\partial\xi}\left(L^{-1}\left[\frac{1}{s}L(0)\right] - L^{-1}\left[\frac{1}{s}L\big((x - \xi x + x\xi^2)(1 - \xi + \xi^2)\big)\right]\right)\right) d\xi$$

$$u_3(x,t) = x - xt + xt^2 - xt^3 \quad (34)$$

$$\vdots$$

$$u_n(x,t) = \sum_{k=0}^{n} (-1)^k x t^k \quad (35)$$

Take $n \to \infty$

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

$$u(x,t) = \sum_{k=0}^{\infty} (-1)^k x t^k$$

$$u(x,t) = \frac{x}{1+t} \quad (36)$$

Example 3: Coupled Burger's equation

For the purpose of illustration of the Variational iteration transform method for solving the homogeneous form of coupled Burger's equations, we will consider the system of equations.

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0, \quad (37)$$

$$v_t - v_{xx} - 2vv_x + (uv)_x = 0 \quad (38)$$

The solutions of which are to be obtained subject to initial conditions.

$$u(x,0) = \sin(x), v(x,0) = \sin(x) \quad (39)$$

Applying the algorithm of Laplace transform on equations (37), (38) we have

$$L[u_t - u_{xx} - 2uu_x + (uv)_x] = 0 \quad (40)$$

$$L[v_t - v_{xx} - 2vv_x + (uv)_x] = 0 \quad (41)$$

$$sL[u(x,t)] - u(x,0) - L[u_{xx}] - 2L[uu_x] + L[(uv)_x] = 0 \quad (42)$$

$$sL[v(x,t)] - v(x,0) - L[v_{xx}] - 2L[vv_x] + L[(uv)_x] = 0 \quad (43)$$

Using the given initial conditions on equations (42), (43), we have

$$sL[u(x,t)] - \sin(x) - L[u_{xx}] - 2L[uu_x] + L[(uv)_x] = 0 \quad (44)$$

$$sL[v(x,t)] - \sin(x) - L[v_{xx}] - 2L[vv_x] + L[(uv)_x] = 0 \quad (45)$$

Then applying the inverse Laplace transform to equations (44), (45)

$$u(x,t) = L^{-1}\left[\frac{1}{s}\sin(x)\right] + L^{-1}\left(\frac{L}{s}[u_{xx}]\right) + 2L^{-1}\left(\frac{L}{s}[uu_x]\right) - L^{-1}(L[(uv)_x]) \quad (46)$$

$$v(x,t) = L^{-1}\left[\frac{1}{s}\sin(x)\right] + L^{-1}\left(\frac{L}{s}[v_{xx}]\right) + 2L^{-1}\left(\frac{L}{s}[vv_x]\right) - L^{-1}(L[(uv)_x]) \quad (47)$$

Derivative by $\frac{\partial}{\partial t}$ both sides to equations (46),(47)

$$u_t(x,t) = \frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[u_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[uu_x]\right) - L^{-1}\left(\frac{1}{s}L[(uv)_x]\right)\right) \quad (48)$$

$$v_t(x,t) = \frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[v_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[vv_x]\right) - L^{-1}\left(\frac{1}{s}L[(uv)_x]\right)\right) \quad (49)$$

Making the correction function are given

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[(u_n)_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[u_n(u_n)_x]\right) - L^{-1}\left(\frac{1}{s}L[(u_n v_n)_x]\right)\right)\right) d\xi \quad (50)$$

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[(v_n)_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[v_n(v_n)_x]\right) - L^{-1}\left(\frac{1}{s}L[(u_n v_n)_x]\right)\right)\right)d\xi \quad (51)$$

We can use the initial condition to select

$$u_0(x,t) = v_0(x,t) = u(x,0) = v(x,0) = \sin(x) \quad (52)$$

$$u_1(x,t) = u_0(x,t) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[(u_0)_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[u_0(u_0)_x]\right) - L^{-1}\left(\frac{1}{s}L[(u_0 v_0)_x]\right)\right)\right)d\xi \quad (53)$$

$$u_1(x,t) = \sin(x) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[-\sin(x)]\right) + 2L^{-1}\left(\frac{1}{s}L[\sin(x)\cos(x)]\right) - L^{-1}\left(\frac{1}{s}L[2\sin(x)\cos(x)]\right)\right)\right)d\xi \quad (54)$$

$$u_1(x,t) = \sin(x) - t\sin(x) \quad (55)$$

$$v_1(x,t) = v_0(x,t) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[(v_0)_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[v_n(v_0)_x]\right) - L^{-1}\left(\frac{1}{s}L[(u_0 v_0)_x]\right)\right)\right)d\xi \quad (56)$$

$$v_1(x,t) = \sin(x) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[-\sin(x)]\right) + 2L^{-1}\left(\frac{1}{s}L[\sin(x)\cos(x)]\right) - L^{-1}\left(\frac{1}{s}L[2\sin(x)\cos(x)]\right)\right)\right)d\xi \quad (57)$$

$$v_1(x,t) = \sin(x) - t\sin(x) \quad (58)$$

$$u_2(x,t) = u_1(x,t) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[(u_1)_{xx}]\right) + 2L^{-1}\left(\frac{1}{s}L[u_1(u_1)_x]\right) - L^{-1}\left(\frac{1}{s}L[(u_1 v_1)_x]\right)\right)\right)d\xi \quad (59)$$

$$u_2(x,t) = \sin(x) - t\sin(x) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[-\sin(x) + \xi\sin(x)]\right) + 2L^{-1}\left(\frac{1}{s}L[\sin(x)\cos(x) - 2\xi\sin(x)\cos(x) + \xi^2\sin(x)\cos(x)]\right) - L^{-1}\left(\frac{1}{s}L[2(\sin(x)\cos(x) - 2\xi\sin(x)\cos(x) + \xi^2\sin(x)\cos(x))]\right)\right)\right)d\xi \quad (60)$$

$$u_2(x,t) = \sin(x) - t\sin(x) + \frac{t^2}{2!}\sin(x) \quad (61)$$

$$v_2(x,t) = \sin(x) - t\sin(x) + \int_0^t \left(\frac{\partial}{\partial t}\left(L^{-1}\left(\frac{1}{s}L[-\sin(x) + \xi\sin(x)]\right) + 2L^{-1}\left(\frac{1}{s}L[\sin(x)\cos(x) - 2\xi\sin(x)\cos(x) + \xi^2\sin(x)\cos(x)]\right) - L^{-1}\left(\frac{1}{s}L[2(\sin(x)\cos(x) - 2\xi\sin(x)\cos(x) + \xi^2\sin(x)\cos(x))]\right)\right)\right)d\xi \quad (62)$$

$$v_2(x,t) = \sin(x) - t\sin(x) + \frac{t^2}{2!}\sin(x) \quad (63)$$

⋮

$$u_n(x,t) = \sin(x) - t\sin(x) + \frac{t^2}{2!}\sin(x) - \frac{t^3}{3!}\sin(x) + \cdots + \frac{t^n}{n!}\sin(x) \quad (64)$$

$$v_n(x,t) = \sin(x) - t\sin(x) + \frac{t^2}{2!}\sin(x) - \frac{t^3}{3!}\sin(x) + \cdots + \frac{t^n}{n!}\sin(x) \quad (65)$$

$$u_n(x,t) = \sin(x)\sum_{k=0}^{n}(-1)^k\frac{t^k}{k!} \quad (66)$$

$$v_n(x,t) = \sin(x) \sum_{k=0}^{n} (-1)^k \frac{t^k}{k!} \tag{67}$$

Finally, the solution is

$$u(x,t) = \lim_{n\to\infty} u_n(x,t)$$

$$= \sin(x) \sum_{k=0}^{\infty} (-1)^k \frac{t^k}{k!}$$

$$u_n(x,t) = \sin(x)\, e^{-t} \tag{68}$$

$$v(x,t) = \lim_{n\to\infty} v_n(x,t)$$

$$= \sin(x) \sum_{k=0}^{\infty} (-1)^k \frac{t^k}{k!}$$

$$v_n(x,t) = \sin(x)\, e^{-t} \tag{69}$$

CONCLUSIONS

In this paper, the Variational iteration transform method has been successfully applied to finding the solution of a Burger's and coupled Burger's equations. The solution obtained by the Variational iteration transform method is an infinite power series for appropriate initial condition, which can, in turn, be expressed in a closed form, the exact solution. The results show that the Variational iteration method is a powerful mathematical tool to solving Burger's and coupled Burger's equations; it is also a promising method to solve other nonlinear equations.

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