A NEW EQUIVALENT CIRCUIT OF THE THREE-PHASE INDUCTION MOTOR (CASE STUDIES: CURRENT AND POWER FACTOR OF THE MOTOR)

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ABSTRACT

Characteristics of the three-phase induction motors can be analyzed by using a conventional equivalent circuit. The parameters of the circuit can be obtained through several experiments results in the laboratory such as dc test, no-load test, and blocked-rotor test. All data must be gotten accurately if they are used for predicting the characteristics of the three-phase induction motor. If one data is not gotten accurately, the characteristics of the motor can not be predicted accurately. This study is aimed to give a simple equivalent circuit for analyzing the characteristics of the 3-phase induction motors by using only the nameplate data and no-load test data of the motor. So, the blocked-rotor test and dc test of the motor are not required for the purpose of this circuit. This study is focused to discuss about the line current and power factor of the three-phase induction motor. The object used in this study was the 3-phase induction motor of 1.5 HP, 380/220V, Y/Δ, 2.7/4.7 A, 4 poles, 50 Hz, 1400 RPM. The results of this study show that the equivalent circuit proposed in this study can be used to predict the characteristics of the three-phase induction motor, especially the input current and power factor of the motor with an accurate rate above 90%.

Keywords: three-phase induction motor, new equivalent circuit, characteristics of the motor.

INTRODUCTION

Three-phase induction motors are widely used in many sections, especially in industrial sectors because these motors are simple and robust. These motors are usually produced in high power ratings. These motors are normally operated by using a three-phase power supply. On specific application conditions, these motors can be operated on single-phase power by installing the capacitor banks to the windings of the motor[1]-[6]. There are many kinds of modification of single-phase power supply can be used for operating the three-phase induction motor on single-phase supply such as single-phase inverter and PWM control technique [7], [8]. If the three-phase induction motor operates on power supply system, the motor can be analyzed by using characteristics of the motor that monitored during operation. In other condition, the motor can also be analyzed by using an equivalent circuit to predict the characteristics of the motor[9]. The equivalent circuit can be used to predict the characteristics of the motor on various load conditions, even in blocked rotor condition.

Several equations had been developed in conventional method for calculating the current and power factor of the 3-phase induction motors. The parameters used in conventional circuit must be obtained from three experimental results such as dc test, no-load test and blocked-rotor test[1], [3], [6], [10]-[15]. Also, some application like automatic rescue devices (ARD) [16] and the reference frame theory to the dynamic analysis of a three-phase induction motor fed from a single-phase supply have been developed [2]. But this method needs some parameters that are received from three experimental results such as dc test, no-load test and blocked-rotor test. If any of the given data is not accurate, the parameter are used for predicting the characteristics of the motors can not be done accurately. So, this study is proposed to give a new equivalent circuit for predicting the characteristics of the three-phase induction motor by using only the data of the no-load test and full-load test of the motor (nameplate data).

EQUIVALENT CIRCUIT AND PREVIOUS WORKS

Three-phase Induction motor is the three-phase alternating current motor that widely used in many applications. The motor has a strong construction and easy to operate. The naming is derived from the fact that this motor operates by induction the stator magnetic field to the rotor, so that the motor is called the induction motor. The current generated by the rotor (moving parts) is not obtained from a particular source, but is an induced current as a result of the relative difference between the rotation of the rotor and the rotating magnetic field generated by the stator current.

When the stator winding of the three-phase induction motor is connected to a three-phase power supply, the induced voltages will produce rotor currents and torque to the rotor. Then, the rotor will start rotating and reach a steady-state speed ‘N’ that is less than synchronous speed ‘Ns’, at which the stator rotating field rotates in the air gap of the motor. The difference between the rotor speed and the stator rotating magnetic field rotates in the air gap by referencing to rotating the magnetic field rotates in the air gap is called the slip ‘s’ that can be described as follows.

\[ s = \frac{N_s - N}{N_s} \]  
(1)
Where:
\( s \) = slip
\( N_s \) = stator speed rotating field rotates in the air gap (RPM)
\( N \) = rotor speed rotating (RPM)

**Conventional method**

The three-phase induction motors are usually supplied by a balance three-phase voltage source. Therefore, the three-phase induction motors can be analyzed by using a one phase supply equivalent circuit. Figure-1 show a conventional equivalent circuit for analyzing the three-phase induction characteristics when operated on three-phase systems [9]. All data impedances of the motor must be obtained by doing some testing on the motor such as dc-source test, no-load test, and blocked-rotor test. If one data is not true, the characteristics of the motor can not be predicted well [3], [6], [9].

![Figure-1. Conventional equivalent circuit of the three-phase induction motor per phase[9].](image)

By referring to Figure-1 can be explained that
\( V_1 \) = phase voltage source
\( R_1 \) = resistance of the stator windings per phase
\( X_1 \) = inductive reactance of the stator windings per phase
\( R'_2 \) = resistance of the rotor windings per phase referred to the stator
\( X'_2 \) = inductive reactance of the rotor windings per phase referred to the stator
\( X_m \) = magnetic reactance of the motor per phase
\( I_1 \) = current through the stator windings
\( I'_2 \) = current through the rotor windings referred to the stator
\( s \) = slip

\( P_{ag} = (I'_2)^2 \frac{R'_2}{s} = (I'_2)^2 \left( R'_2 + R'_2 (1 - s) \right) \) (2)

Then, the power losses in the rotor circuit (rotor copper loss) is
\( P_2 = (I'_2)^2 R'_2 \) (3)

The mechanical power developed \( (P_{mech}) \) by the induction motor can be calculated as follows.
\( P_{mech} = (I'_2)^2 \left( R'_2 \left( \frac{1 - s}{s} \right) \right) \) (4)

When the three powers are compared to the slip (s), it can be made as follows.
\( P_{ag} : P_2 : P_{mech} = 1 : s : (1 - s) \) (5)

The current \( (I_1) \) and power factor \( (pf) \) of the induction motor can be calculated from Figure-1 as follows.
\( I_1 = \frac{V}{Z_{tot}} \) (6)
\( pf = \left[ \frac{R_{tot}}{Z_{tot}} \right] \) (7)

Where:
\( Z_{tot} = R_{tot} + jX_{tot} \) = the total impedance of the motor circuit
\( R_{tot} \) = the total resistance of the motor circuit
\( X_{tot} \) = the total reactance of the motor circuit

All data of the impedance of the induction motor must be obtained accurately from three experimental result such as dc test, no-load test and blocked-rotor test.

**Automatic Rescue Devices (ARD)**

Automatic Rescue Device (ARD) is a method of sensorless low range speed and parameter identification estimation devoted to lift automatic rescue devices [16]. The equivalent circuit of this method is shown in Figure-2.
From the equivalent circuit shown in Figure-2, the equivalent impedance can be determined as follows.

\[ Z_{eq} = R_{eq} + jX_{eq} \]  \hspace{1cm} (8)

Where:
\[ R_{eq} = \text{equivalent motor resistance} \]
\[ X_{eq} = \text{equivalent motor reactance} \]
\[ Z_{eq} = \text{equivalent motor impedance} \]

From equation (8) can be explained that the equivalent resistance \( R_{eq} \) and the equivalent reactance \( X_{eq} \) are

\[ R_{eq} = R_s + \frac{\omega^2 s L_M T_r}{1 + (s \omega T_r)^2} \]  \hspace{1cm} (9)

\[ X_{eq} = \omega L_s' + \frac{\omega L_M}{1 + (s \omega T_r)^2} \]  \hspace{1cm} (10)

Where:
\[ R_s = \text{stator resistance} \]
\[ L_s = \text{stator transient inductance} \]
\[ L_M = \text{referred magnetizing inductance} \]
\[ T_r = \text{rotor time constant} \]
\[ \omega = \text{stator angular frequency} \]

The active (P) and reactive (Q) power of induction motor are

\[ P = (\overline{T_s})^2 R_{eq} \]  \hspace{1cm} (11)

\[ Q = (\overline{T_s})^2 X_{eq} \]  \hspace{1cm} (12)

Where:
\[ \overline{T_s} = \text{stator space phasor current} \]

Then, the active power delivered to the rotor \( P_{sr} \) and the reactive power delivered to the rotor \( Q_{sr} \) are respectively as follows:

\[ P_{sr} = P - (\overline{T_s})^2 R_{eq} \]  \hspace{1cm} (13)

\[ Q_{sr} = Q - (\overline{T_s})^2 \omega L_M \]  \hspace{1cm} (14)

The referred rotor resistance \( R_{ref} \) can be determine as follows:

\[ R_{ref} = \frac{L_M}{T_r} \]  \hspace{1cm} (15)

By substituting equation (15) in (9) and (10) then are obtained as follows:

\[ R_{eq} = \frac{P}{|\overline{T_s}|^2} = R_s + \frac{\omega^2 s L_M^2 / R_{ref}}{1 + (s \omega L_M / R_{ref})^2} \]  \hspace{1cm} (16)

\[ X_{eq} = \frac{Q}{|\overline{T_s}|^2} = \omega L_s' + \frac{\omega^2 L_M}{1 + (s \omega L_M / R_{ref})^2} \]  \hspace{1cm} (17)

By comparing equations (13) and (14) respectively with equations (16) and (17), then both the active and reactive power delivered to the rotor from the stator are

\[ P_{sr} = \frac{A_{sr}}{|\overline{T_s}|^2} \]  \hspace{1cm} (18)

\[ Q_{sr} = \frac{A_{sr} Q_{sr}}{|\overline{T_s}|^2} \]  \hspace{1cm} (19)

By dividing equation (18) by (19) then will be obtained as follows:

\[ \frac{P_{sr}}{Q_{sr}} = \frac{\omega A_{sr}}{R_{ref}} \]  \hspace{1cm} (20)

Then, the equation (20) can be substituted in the second term of the denominator in equation (19) giving the value of the referred magnetizing inductance as follows:

\[ L_M = Q_{sr} \frac{1 + \frac{P_{sr}^2}{Q_{sr}^2}}{\omega |\overline{T_s}|^2} = \frac{P_{sr}^2 + Q_{sr}^2}{\omega |\overline{T_s}|^2} Q_{sr} = \frac{A_{sr}^2}{|\overline{T_s}|^2} \]  \hspace{1cm} (21)

Where:
\[ A_{sr} = \text{apparent power delivered to the rotor} \]

By substituting equation (21) in (20), the expression of the motor slip (s) is obtained as follows:
The speed can be calculated as follows:

\[
\omega = \frac{P_{sr}}{\omega L_M Q_{sr}} = \frac{P_{sr}}{A_{sr} L_M Q_{sr}} \tag{22}
\]

The speed can be calculated as follows:

\[
\Omega_r = (1 - s) \frac{\omega}{p} \tag{23}
\]

Where:

\[\Omega_r = \text{rotor angular speed}\]

Previous works and proposed method

The 3-phase induction motors have multiple windings placed in the slots. The equivalent circuit model for one coil of the motors can be drawn as shown in Figure-3[17].

By referring to Figure-3 can be explained that

\[R_{OC} = \text{no load test resistance of the motor’s windings per phase}\]

\[X_{OC} = \text{no load test reactance of the motor’s windings per phase}\]

\[X_A = \text{additional reactance of the motor (usually capacitive reactance)}\]

\[I_1 = \text{phase current through to the motor at full load}\]

\[I_{OC} = \text{no load test current through the windings of the motor}\]

\[I_R = \text{the additional current through the resistance of the motor}\]

\[I_A = \text{the additional current through the additional reactance}\]

\[V_1 = \text{phase voltage of the motor}\]

Figure-4 is a simple equivalent circuit model that can be used for analyzing the characteristics of the 3-phase induction motors when operated on the 3-phase power system. By using Figure-4 we only need the data of no-load test and full-load test (nameplate data) of the motor. So, we need not DC test and blocked-rotor test for analyzing the characteristic of the motor. By referring to Figure-4, the magnitude of the no load test impedance \(Z_{OC}\) of the motor can be written as follows:

\[
Z_{OC} = R_{OC} + jX_{OC} \tag{24}
\]

There are two ways to determine the impedance of no-load test \(Z_{OC}\) of the motor depending on the motor windings connection system. When the motor’s windings are connected in ‘Delta’ connection standard, the magnitude of the ‘\(Z_{OC}\)’ for delta standard \(Z_{OC(\Delta)}\) becomes:

\[
Z_{OC(\Delta)} = \frac{\sqrt{3} V_{LL(\Delta)}}{I_{L(LC)}} \tag{25}
\]

Then, when the motor’s windings are connected in ‘Wye’ connection standard, the magnitude of the ‘\(Z_{OC}\)’ for wye standard \(Z_{OC(Y)}\) becomes:

\[
Z_{OC(Y)} = \frac{V_{LL(Y)}}{\sqrt{3} J_{L(LC)}} \tag{26}
\]

Where:

\[V_{LL} = \text{line to line voltage source on no load test}\]

\[I_{L(LC)} = \text{line current on no load test}\]

\[Z_{OC(Y)} = Z_{OC} \text{ for wye standard motor}\]

\[Z_{OC(\Delta)} = Z_{OC} \text{ for delta standard motor}\]

The full-load current of the motor (nominal current of the motor) can be obtained from full-load test data or from the data written on the nameplate of the motor. If the motor is operated on ‘Delta (\(\Delta\))’ connection standard, then \(I_1\) from Figure-4 is equal to the full load current divided by 1.73205. Then, if the motor is operated...
on ‘Wye (Y)’ connection standard, \(I_1\) from Figure-4 is
equal to the full load current (nominal current). So, the
line current magnitude of the 3-phase induction motors for
‘\(\Delta\)’ and ‘\(Y\)’ connection standard can be written as follows:

\[ I_{L(\Delta)} = \sqrt{3}I_1 \]  
(27)

\[ I_{L(Y)} = I_1 \]  
(28)

The currents ‘\(I_Z\)’, ‘\(I_R\)’ and ‘\(I_A\)’ from the Figure-4
then can be written as follows:

\[ I_Z \angle \theta = I_1 \angle \phi - I_{OC} \angle \beta \]  
(29)

\[ I_R = I_Z \cdot \cos(\theta) \]  
(30)

\[ I_A = I_Z \cdot \sin(\theta) \]  
(31)

Where:

\(\phi\) = the angle of the full load current

\(\beta\) = the angle of the no-load current

\(\theta\) = the angle of the current ‘\(I_Z\)’

The magnitude of \(R_1\), \(R_2\) and \(X_A\) from the
Figure-4 then can be calculated as follows:

\[ X_A = \frac{V_1}{I_A} \]  
(32)

\[ R_1 + \frac{R_2'}{s} = \frac{V_1}{I_R} \]  
(33)

By assuming \(R_1 = R_2'\), then will be obtained as
follows:

\[ R_1 = R_2' = \frac{V_1}{(1+1/s)I_R} \]  
(34)

For certain conditions, the data of power factor of
the motor is not given on the motor nameplate, so that the
motor should be operated directly under full load
condition to find the power factor of the motor. By
referring to Anthony’s research about the capacitance of
the run capacitor ‘\(C_{r_y}\)’ that used to operate the 3-phase
induction motor on single phase supply, the full load
power factor of the motor can be calculated accurately as
shown in formula of equation (35) to equation (38) [5].

\[ Cr_y = k \frac{I_L}{(12.5664)(f)(V_{LN})} \]  
(35)

Where:

\(I_L\) = line current of the 3-phase induction

\(V_{LN}\) = line to neutral of the single phase

\(Cr_y\) = capacitance of the run capacitor for

\(k\) = constant factor

When the ‘\(k\)’ is 1 (for the motor is operated with a
close to 3-phase rating), the run capacitor in equation (35)
should have ‘reactive power (Q_c)’ as follows.

\[ Q_c = \omega C_{r_y} (V_c)^2 = 2V_{LN} \cdot I_L \]  
(36)

Where:

\(V_C\) = voltage on capacitor

\(\omega\) = 2.\(\pi\).\(f\)

\(f\) = frequency

If the full-load reactive power of the motors ‘\(Q_M\)’
is compared against to the ‘\(Q_c\)’ from the equation (36), the
magnitude of ‘\(Q_M = 0.983 Q_c\)’ (referring to the rotor speed
standard of the motor) or ‘\(Q_M = 0.997 Q_c\)’ (referring to the
nominal current of the motor). Therefore, the magnitude of
‘\(Q_M\)’ would be defined as follows:

\[ Q_M = 0.99Q_c \]  
(37)

Then, the power factor of the motor can be
calculated as follows.

\[ \cos \phi = \cos(\tan^{-1}\phi) \]  
(38)

Where:

\[ \tan \phi = \frac{Q_M}{S_M} \]  
(39)

and,

\[ S_M = \sqrt{3}V_{LN} \cdot I_L \]  
(40)

\(\cos \phi\) = power factor of the motor

\(S_M\) = apparent power of the motor

**METHODOLOGY**

The motor used in this study was the 3-phase
induction motor of 1.5 HP, 380/220V, \(\Delta/\Delta\), 2.7/4.7 A, 4
poles, 50 Hz, 1400 RPM. The study is focused about for
calculating the line current and power factor of the motor
by using the equivalent circuit as shown in Figure-4. The
equivalent circuit and the formulas created in this study will be compared with the experiment results in the laboratory. The motor is operated by using ‘Wye’ connection standard winding. The circuit equipment and accessories used for operating the 3-phase induction motor is shown in Figure-5.

RESULT AND DISCUSSIONS

The results of this study are given in Table-1 and Table-2 as below.

Table-1. The input current during operation against the calculation results by using the formulas created.

<table>
<thead>
<tr>
<th>No.</th>
<th>$I_L$ (exp)</th>
<th>$I_L$ (c)</th>
<th>Error (%)</th>
<th>$N_r$ (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.829</td>
<td>1.902</td>
<td>-4.013</td>
<td>1481</td>
</tr>
<tr>
<td>2</td>
<td>1.872</td>
<td>1.874</td>
<td>-0.118</td>
<td>1476</td>
</tr>
<tr>
<td>3</td>
<td>1.949</td>
<td>1.924</td>
<td>1.303</td>
<td>1469</td>
</tr>
<tr>
<td>4</td>
<td>1.998</td>
<td>1.999</td>
<td>-0.025</td>
<td>1460</td>
</tr>
<tr>
<td>5</td>
<td>2.113</td>
<td>2.138</td>
<td>-1.193</td>
<td>1452</td>
</tr>
<tr>
<td>6</td>
<td>2.206</td>
<td>2.217</td>
<td>-0.490</td>
<td>1444</td>
</tr>
<tr>
<td>7</td>
<td>2.323</td>
<td>2.253</td>
<td>3.026</td>
<td>1434</td>
</tr>
<tr>
<td>8</td>
<td>2.436</td>
<td>2.413</td>
<td>0.944</td>
<td>1423</td>
</tr>
<tr>
<td>9</td>
<td>2.610</td>
<td>2.676</td>
<td>-2.513</td>
<td>1403</td>
</tr>
<tr>
<td>10</td>
<td>2.755</td>
<td>2.883</td>
<td>-4.632</td>
<td>1386</td>
</tr>
</tbody>
</table>

Where:
- $I_L$ (exp) = the current from motor operation data condition
- $I_L$ (c) = the current from the calculation results by using the formulas created
- Error (%) = percentage error calculation results when compared against to the results of the experiment
- $N_r$ (RPM) = rotor speed (Rotation per minutes)

Table-2. The power factor during operation against the calculation results by using the formulas created.

<table>
<thead>
<tr>
<th>No.</th>
<th>$PF$ (exp)</th>
<th>$PF$ (c)</th>
<th>Error (%)</th>
<th>$N_r$ (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.43</td>
<td>0.364</td>
<td>15.395</td>
<td>1481</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.391</td>
<td>18.479</td>
<td>1476</td>
</tr>
<tr>
<td>3</td>
<td>0.54</td>
<td>0.444</td>
<td>17.704</td>
<td>1469</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
<td>0.506</td>
<td>12.793</td>
<td>1460</td>
</tr>
<tr>
<td>5</td>
<td>0.62</td>
<td>0.559</td>
<td>9.823</td>
<td>1452</td>
</tr>
<tr>
<td>6</td>
<td>0.66</td>
<td>0.600</td>
<td>9.030</td>
<td>1444</td>
</tr>
<tr>
<td>7</td>
<td>0.70</td>
<td>0.644</td>
<td>8.057</td>
<td>1434</td>
</tr>
<tr>
<td>8</td>
<td>0.73</td>
<td>0.688</td>
<td>5.767</td>
<td>1423</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.749</td>
<td>0.133</td>
<td>1403</td>
</tr>
<tr>
<td>10</td>
<td>0.77</td>
<td>0.789</td>
<td>-2.403</td>
<td>1386</td>
</tr>
</tbody>
</table>

Where:
- $PF$ (exp) = power factor from motor operation data condition
- $PF$ (c) = power factor from the calculation results by using the formulas created

Characteristics of the line current and power factor of the motor are shown in Table-1 and Table-2. By referring to Table-1 can be seen that the error of the calculation results when compared to the experimental results are below 5%. So, it can be said that the equivalent circuit and the formulas given have high accuracy for predicting the line current of the motor (accuracy level is about 95%). Then, by referring to Table-2, the formulas created have good result for calculating the power factor of the motor at the rotor speed below 1452 RPM (the error is below 10%). Therefore, the formulas created on the circuit are suitable for analyzing the power factor of the motor for high load condition (accuracy level is about 90%). If both tables are converted into graphs, the results can be seen in Figure-6 and Figure-7.

Figure-6. Comparison characteristic between current ‘$I_L$ (exp)’ against to current ‘$I_L$ (c)’.
From both Figure-6 and Figure-7 can be seen that the characteristics of the motor between calculation result and operation result are similar. So, the equivalent circuit with the formula created is suitable for predicting the characteristics of the line current and the power factor of the motor.

CONCLUSIONS

The results of this study show that the equivalent circuit and formulas created have high accuracy. The errors of the calculation results when compared to the experimental results are below 5%. So, the formulas created have high accuracy for predicting the line current of the motor (accuracy level is about 95%). Then, the formulas created also have a good result for calculating the power factor of the motor especially at the rotor speed below 1452 RPM (high load condition), where the errors are below 10%. Therefore, the formulas created on the circuit are also suitable for analyzing the power factor of the motor for high load condition (accuracy level is about 90%). The current and power factor characteristics of the calculation result are similar with the operating characteristics of the motor.

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