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PARAMETRIC IDENTIFICATION METHODS APPLIED TO AN ELECTROMECHANICAL PLANT

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ABSTRACT
The aim of this study is to identify the model for an electromechanical plant. This is done by the use of parametric identification methods. In this contribution, the Least Square Method (LS), the Instrumental Variables Method (IV) and the Prediction Error Method (PE) are used for identification. The identification is performed on input-output data generated by applying a PRBS signal to the motor of the electromechanical plant. Once the identification is done, the obtained models are validated by using sine waves with different frequencies as input signals.

Keywords: electromechanical system, Instrumental variables, least squares, prediction error.

1. INTRODUCTION
This work presents the parametric identification of the electromechanical plant illustrated in Figure-1. The electromechanical plant consists of 2 masses (m_1 and m_2), 3 springs (k_1, k_2 and k_3), 1 damper (c_1) and an electrical motor. The plant has only one input, the voltage applied to the motor ($u(t)$ expressed in Volts), and 2 outputs, the positions of the two masses ($y_1(t)$ and $y_2(t)$ expressed in counts or converted to m).

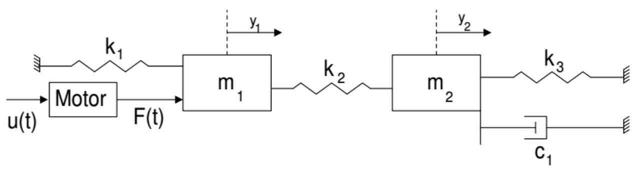

Figure-1. Schematic representation of the electromechanical plant.

It is assumed that the electrical motor dynamics can be neglected because it is fast compared to the mechanical dynamics, so that the motor can be represented by a pure static gain$K$:

The mechanical parameters are given in Table-1.

Table-1. Mechanical parameters.

m_1	1.7 kg
m_2	1.2 kg
k_1	800 N/m
k_2	800 N/m
k_3	450 N/m
c_1	9 N/(m/s)

Applying Newton's second law to the 2 masses of the electromechanical plant gives the following 2 equations:

$$m_1\ddot{y}_1(t) = F(t) - k_1y_1(t) + k_2\big(y_2(t) - y_1(t)\big)$$

$$m_2\ddot{y}_2(t) = k_2\big(y_1(t) - y_2(t)\big) - k_3y_2(t) - c_1\dot{y}_2(t)$$

As mentioned earlier, the force $F(t)$ provided by the motor is described by a pure static gain:

$$F(t) = Ku(t)$$

Switching to the Laplace transform of $y_1(t)$, $y_2(t)$ and $u(t)$ gives:

$$m_1s^2Y_1(s) = KU(s) - k_1Y_1(s) + k_2\big(Y_2(s) - Y_1(s)\big)$$

$$m_2s^2Y_2(s) = k_2\big(Y_1(s) - Y_2(s)\big) - k_3Y_2(s) - c_1sY_2(s)$$

Solving these 2 equations and substituting the numeric values for the parameters from Table-1 gives the continuous time transfer functions $G_1(s)$ and $G_2(s)$:

$$G_1(s) = \frac{Y_1(s)}{U(s)} = \frac{K(60s^2 + 450s + 62500)}{102s^4 + 765s^3 + 202250s^2 + 7.2\ 10^5s + 6.8\ 10^7}$$

$$G_2(s) = \frac{Y_2(s)}{U(s)} = \frac{40000K}{102s^4 + 765s^3 + 202250s^2 + 7.2\ 10^5s + 6.8\ 10^7}$$

From these continuous time transfer functions, the 2 eigen frequencies of the plant can be calculated. This is done by factorizing the denominator as follows

$$den\_G(s) = (s^2 + 2\zeta_1\omega_1 + \omega_1^2)(s^2 + 2\zeta_2\omega_2 + \omega_2^2)$$

Applying this factorization to the denominator of $G_1(s)$ and $G_2(s)$ gives the following eigen frequencies: $\omega_1 = 20.844$ rad/s and $\omega_2 = 39.171$ rad/s. For damping factors ζ_1 and/or ζ_2 smallerthan 0.7, the amplitude response reaches an extreme around the eigen frequencies ω_1 and/or ω_2. When the damping factor is much smaller

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than 0.7, the extreme lies exactly atthe eigenfrequency. For (6) and (7), the damping factors are respectively equal to 0.0829 and0.0516 so there is a peak in the amplitude response at each of the 2 eigen frequencies. This can be seen in the Bode plot of the 2 continuous time transfer functions (Figure-2).

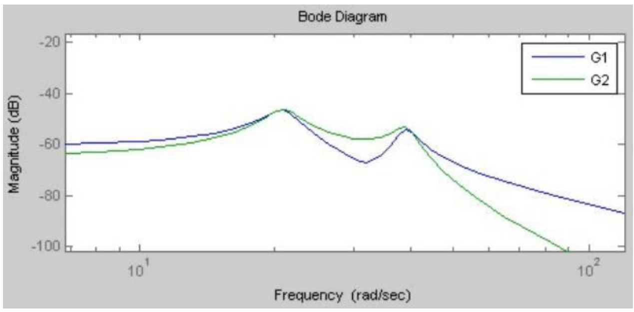

Figure-2. Bode diagram of the continuous time transfer function from the mathematical model.

2. MATERIALS AND METHODS

2.1 Sampling period and experiments

The sampling period is chosen by measuring the settling time of the system. This is done by applying a step-input to the steady-state system. After some time, the settling time, the system is again in steady-state. The sampling period is chosen so that there are about 50 samples during the settling time. For the electromechanical plant, the settling time is around1 s. This means that a sampling time of 20 ms is acceptable. However, a sampling period of 8.84 ms (2 times the servo period) will be used. This is because it is wanted to apply a sine wave to the system with frequencies equal to the eigen frequencies. For the biggest eigen frequency, this means a period of 160 ms, so a sampling time of 20 ms is too big for a good reconstruction of this sine wave.

For identification a random PRBS signal is applied to the plant. This PRBS signal consists of 923 segments with a segment time of 9 ms and gives 1 or 2 V as input to the motor. The PRBS signal is built so that there is at least 1 part of the signal where it is constant for some time so that steady-state is obtained. This is necessary for calculation of the static gain (K) of the motor.

When the experiment is performed, an output-file is created which contains the sample number, the time at which the sample was taken, the input-signal in Volts and the positions of the 2 masses (in counts). These last 2 variables are converted from "counts" to "meters" and the identification is performed with 3 identification methods: Least Squares Method (LS), Instrumental Variables Method (IV) and Prediction Error Method (PE). The order of the models must be predefined and is chosen based on the mathematical model. The number of poles is equal to 4 because it is a combined system of 2 second order differential equations. The number of zeros is not known, but must be smaller than 4. Therefore the number of zeros is chosen as 3.

2.2 Least squares method

Identification with the LS method is performed with the Matlab function *arx()*. The 2 resulting discrete time models are converted to continuous time models. These models are less accurate than the discrete time models, but they are easier to use for evaluation.

$$G_{1,arx}(s) = \frac{0.01102s^3 + 3.143s^2 + 272.8s + 5437}{s^4 + 68.56s^3 + 3046s^2 + 5.947\ 10^4 s + 8.833\ 10^5}$$

$$G_{2,arx}(s) = \frac{0.001314s^3 - 0.3234s^2 + 21.16s + 5989}{s^4 + 59.34s^3 + 3067s^2 + 5.096\ 10^4 s + 1.406\ 10^6}$$

The amplitude response for both models is given in Figure-3. For $G_{1,arx}(s)$ there is only 1 small peak, which is actually not more than a bump with a maximum at $\omega = 21$ rad/s. For $G_{2,arx}(s)$ there is only 1 clear peak at $\omega = 25.6$ rad/s. The reason why the 2 peaks cannot be seen at the eigenfrequencies, like in the mathematical model, are the damping factors.

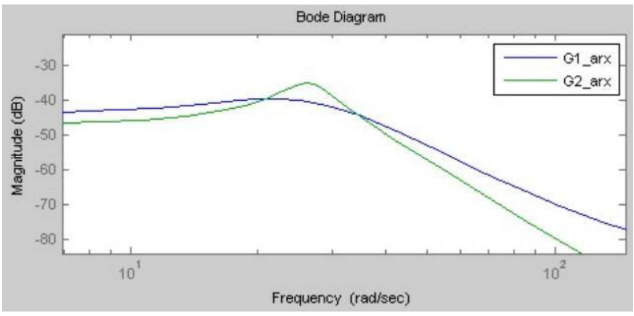

Figure-3. Bode diagram of the continuous time transfer function from the identified models with the LS method.

From Table-2 it can be seen that all the damping factors are close to 0.7 except for one. This is the reason why there is a peak in the amplitude response for $G_{2,arx}(s)$, as mentioned before. As result of these high damping factors, it can be said that the LS method doesnot give a good identification of the model.

Table-2. Eigen frequencies and damping factors for both identified models with the LS method.

Model	ζ_1	ω_1	ζ_2	ω_2
$G_{1,arx}(s)$	0.423	24.893	0.629	37.755
$G_{2,arx}(s)$	0.133	25.566	0.586	44.634

2.3 Instrumental variables method

Identification with the IV method is performed with the Matlab function *iv4()*. The 2 resulting discrete time models are converted to continuous time models:

$$G_{1,iv}(s) = \frac{0.011s^3 + 3.367s^2 + 242.8s + 3586}{s^4 + 64.55s^3 + 2777s^2 + 4.694\ 10^4 s + 5.888\ 10^5}$$

$$G_{2,iv}(s) = \frac{-9.488\ 10^{-5}s^3 - 0.03716s^2 - 15.94s + 4873}{s^4 + 14.34s^3 + 2680s^2 + 1.527\ 10^4 s + 1.155\ 10^6}$$

The amplitude response for both models is given in Figure-4. Just like with the LS method, there is only 1 bump with a maximum at $\omega = 18$ rad/s for $G_{1,iv}(s)$. For $G_{2,iv}(s)$ there are 2 clear peaks at 23.6 rad/s and 45.6 rad/s.

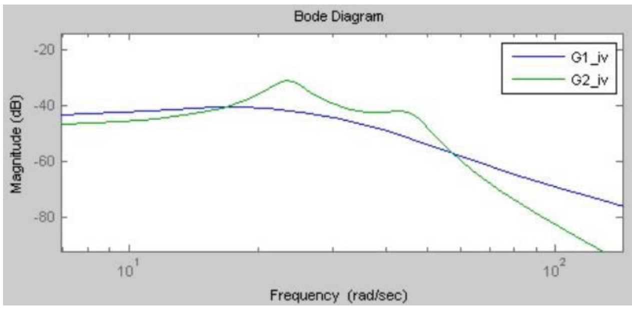

Figure-4. Bode diagram of the continuous time transfer function from the identified models with the IV method.

From Table-3 it can be seen that the damping factors for $G_{1,iv}(s)$ are close to 0.7, so that a clear peak is not obtained at the eigenfrequencies. For $G_{2,iv}(s)$ the damping factors are much smaller and there are 2 clear peaks. For this second model, the IV method gives better results than for the LS method, but it is still not a good identification.

Table-3. Eigen frequencies and damping factors for both identified models with the IV method.

Model	ζ_1	ω_1	ζ_2	ω_2
$G_{1,iv}(s)$	0.492	19.744	0.580	38.864
$G_{2,iv}(s)$	0.102	23.573	0.105	45.591

2.4 Prediction error method

Identification with the PE method is performed with the Matlab function *pem()*. The 2 resulting discrete time models are converted to continuous time models:

$$G_{1,pem}(s) = \frac{0.01027s^3 + 3.275s^2 + 40.52s + 3941}{s^4 + 9.772s^3 + 2150s^2 + 1.092\ 10^4 s + 7.661\ 10^5}$$

$$G_{2,pem}(s) = \frac{0.0002123s^3 - 0.002945s^2 + 11.01s + 3088}{s^4 + 12.69s^3 + 2097s^2 + 1.252\ 10^4 s + 7.474\ 10^5}$$

The amplitude response for both models is given in Figure-5. Just like with the mathematical method, there are 2 clear peaks for both models. For $G_{1,pem}(s)$ the peaks are at 21.4 rad/s and 40.8 rad/s, for $G_{2,pem}(s)$ they are at 21.7 rad/s and 39.8 rad/s.

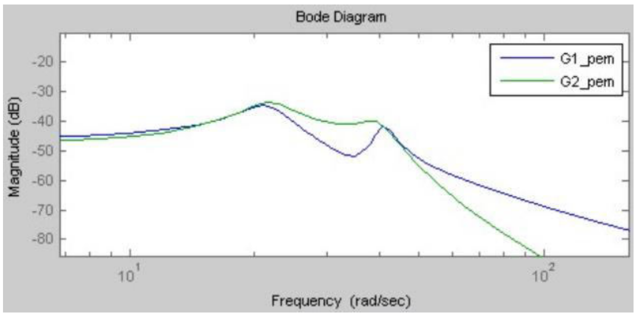

Figure-5. Bode diagram of the continuous time transfer function from the identified models with the PE method.

From Table-4 it can be seen that all the damping factors are small, so that there is a clear peak at every eigen frequency. Also, the eigen frequencies obtained with the PE method are close to the ones obtained from the mathematical model. Thus, it can be concluded that the PE method is a relatively good identification method.

Table-4. Eigen frequencies and damping factors for both identified models with the PE method.

Model	ζ_1	ω_1	ζ_2	ω_2
$G_{1,pem}(s)$	0.124	21.440	0.054	40.825
$G_{2,pem}(s)$	0.135	21.711	0.086	39.820

2.5 Estimation of the static gain

It was already mentioned that the motor can be represented by a pure static gain K. This static gain is unknown, but can be calculated from comparison of the amplituderesponse of the identified model and the mathematical model. From Figure-6 it can be seen that the identified model is vertically translated compared to the mathematical model. This vertical translation is the gain (in dB). Evaluation of the magnitudes of the mathematical model and the identified model with the PE method for both G_1 and G_2 gives 5.596 N/V and7.022 N/V as estimates for the gain. Thus, the average of these 2 estimates as estimation of the static gain is taken, which is 6.309 N/V.

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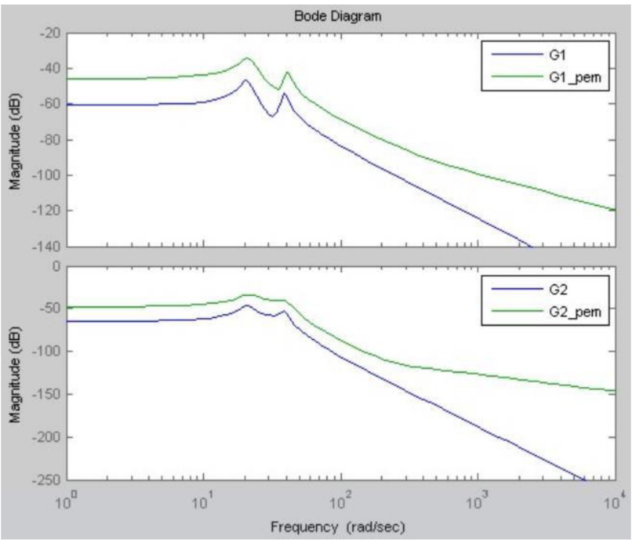

Figure-6. Bode diagrams of the continuous time transfer function from the identified models with the PE method and the mathematical model.

3. RESULTS AND DISCUSSIONS

It is clear that the PE method gives the best estimation of the mathematical models. It is the only identification method that gives good results for both eigen frequencies and damping factors. The LS Method gave very bad results both in the eigen frequencies and damping factors, while the IV Method was slightly better. It gave a better estimation of the damping factors, but only for the second model $G_{2,iv}(s)$, the first was as poor as for the LS Method.

The reason why the PE Method gives the best results is probably the "estimation" of the noise. The electromechanical plant is not an ideal system, like it was assumed for deriving the mathematical model. As one example, there is the friction between the masses and the ground. Another example is the spring which is also not ideal, like assumed in the mathematical model. All these non-ideal terms can be grouped and can be estimated as noise in the PE method. In the LS method and IV method, these non-ideal terms are not seen as noise but as the output of the system.

For the validation of the different models with experimental data, sine waves with different frequencies are used as input signals. Three experiments were considered; two of them in which the frequency of the sine waves should be equal to the eigen frequencies of the plant $\omega_1 = 20.844$ rad/s and $\omega_2 = 39.171$ rad/s, in the third one, the frequency of the sine wave was chosen in the interval(ω_1,ω_2), namely $\omega_3 = 28.274$ rad/s.

The output of these experiments is also sine waves with a different amplitude and phase shift, but with the same frequency. From this amplitude and phase shift, the magnitude and phase of the system at that specific frequency can be calculated as follows:

Input: $u(t) = A\sin(\omega t)$

Output: $y(t) = B\sin(\omega t + \varphi)$

Amplitude: $|G(j\omega)| = B/A$
Phase: $\angle G(j\omega) = \varphi$

The amplitude and phase shift of the output can be easily determined. To have a bigger accuracy, the average values of 10 periods are taken. The amplitude of the input sine wave is equal to 1 V.

The magnitude and phase was calculated for all three frequencies and are given in Table-5, Table-6 and Table-7.

Table-5. Magnitude and phase at frequency $\omega_1 = 20.844$ rad/s for different models and the experiment.

	Exp	Math	LSM	IVM	PEM
$\lvert G_1(j\omega)\rvert$	0.0071	0.0019	0.0044	0.0037	0.0060
$\angle G_1(j\omega)$	-126.1°	-123.7°	-154.7°	-146.9°	-93.3°
$\lvert G_2(j\omega)\rvert$	0.0079	0.0022	0.0035	0.0075	0.0105
$\angle G_2(j\omega)$	101.5°	87.1°	117.7°	139.3°	110.9°

Table-6. Magnitude and phase at frequency $\omega_2 = 39.171$ rad/s for different models and the experiment.

	Exp	Math	LSM	IVM	PEM
$\lvert G_1(j\omega)\rvert$	0.0161	0.0046	0.0103	0.0093	0.0186
$\angle G_1(j\omega)$	-78.3°	-85.1°	-63.4°	-75.9°	-70.2°
$\lvert G_2(j\omega)\rvert$	0.0167	0.0049	0.0105	0.0188	0.0209
$\angle G_2(j\omega)$	-108.4°	-99.6°	-64.7°	-55.6°	-81.3°

Table-7. Magnitude and phase at frequency $\omega_3 = 28.274$ rad/s for different models and the experiment.

	Exp	Math	LSM	IVM	PEM
$\lvert G_1(j\omega)\rvert$	0.0042	0.0007	0.0087	0.0067	0.0049
$\angle G_1(j\omega)$	-128.9°	-139.8°	-110.3°	-112.4°	-136.8°
$\lvert G_2(j\omega)\rvert$	0.0080	0.0014	0.0150	0.0135	0.0104
$\angle G_2(j\omega)$	-173.7°	-178.9°	-168.2°	-175.2°	-168.5°

The model obtained with the Least Squares Method (LSM) performs very poor, in magnitude and in phase. The model obtained with the Instrumental Variables Method (IVM) performs better, but for some conditions the results are very poor, like the phase$\angle G_2(j\omega)$ and the magnitude$|G_1(j\omega)|$. Like in the previoussection (which is evident), it is the Prediction Error Method (PEM) that gives the best model. But it has to bementioned that at the eigenfrequencies, the prediction of the phase is less accurate.

It is clear that the Prediction Error Method performs best for parametric identification for the electromechanical plant. It gave a good prediction of the eigen frequencies and of the Bode diagram. The Least Square Method performed very bad and gave no good results, not for the eigen frequencies, not for the Bode diagram. The Instrumental Variables Method was somewhere in between and performed well for some conditions (frequencies), but very poor for others.

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