



LOADING CHARACTERISTICS OF GOLD CODES IN A SPREAD-SPECTRUM SYSTEM

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ABSTRACT

Properties of spreading codes set a limit on the performance of spread-spectrum systems. In this paper, we investigate loading characteristics of even- and odd-degree Gold codes in a multi-user spread-spectrum system, from a few simultaneous users to full system load. Software simulations are carried out on the system performance for transmission of random QPSK symbols through the channel. The performance of up to about a thousand users is considered. Results for six different code lengths, ranging from 31-chip to 1023-chip Gold codes, are reported. The results show that the odd-degree Gold codes give better multiple-access performance than their even-degree counterparts. Whereas the bit-error-rate (BER) of the odd-degree codes exhibited relatively marginal loss in performance when the system was loaded, their even-degree counterparts degraded rapidly in performance, resulting in early emergence of an error floor, culminating in premature system saturation. The results show that the odd-degree Gold codes have better properties that make them more suitable for multiple-access applications.

Keywords: spread-spectrum system, direct-sequence code division multiple access (DS-CDMA), Gold codes, error floor, system saturation.

1. INTRODUCTION

The advent of spread spectrum wireless communication, alternatively known as code-division multiple-access (CDMA), dates back to the late 1940s. It was developed originally for the military as a means of establishing secure, jam-resistant communications. Over the years, spread spectrum technology has found its way into the larger society because of some other properties that makes it attractive for commercial and civil use. Today, the technology has become a significant worldwide communication technique.

Recently, a relatively new technique called interleaved division multiple access (IDMA) was proposed in literature [1-4] as an alternative to CDMA. The technique (IDMA) has been attracting much attention because it is commonly believed to be a promising candidate for future wireless technology. In IDMA, signal spreading and matrix multiplication are avoided, thereby minimising transmission bandwidth and computational requirements [4-7]. Because of these important benefits of IDMA, CDMA is sometimes considered as being outdated and irrelevant to future wireless systems. A careful consideration shows that this is not the case. This is briefly explained as follows. CDMA has certain important characteristics which are absent in IDMA. For example, CDMA possesses jam-resistant properties not available in IDMA. Apart from this, CDMA has covert transmission capability not obtainable in IDMA. Furthermore, CDMA signals are low-level signals, spread out over a wideband. This makes it possible for CDMA systems to co-exist over the same bandwidth alongside with other transmission technologies like the frequency-division multiple access whose energy is concentrated over a narrowband. CDMA remains a major technology not only in wireless telephony, but also in global positioning systems (GPS). Viterbi [8] indicates that as at 2002, over one hundred

million consumers use devices that employ spread spectrum technology to provide wireless personal communication or position-location or both. Statistics [9] also show that the number of CDMA subscribers grew from about 7.8 million in 1997 to about 577 million in 2010.

The performance of a spread spectrum system depends greatly on the properties of the code sequences used for the system. The type and properties of codes set bounds on the capability of the system; improper code selection leads to poor system performance. This fact applies not only to the basic CDMA system, but also other emerging technologies like the space-time coded multicarrier CDMA system.

The significance of spreading codes has made the search for better spreading codes an important research subject. For example in [10,11], Hsiao-Hwa Chen, et al proposed the design of complementary codes for CDMA systems, and later proposed an interference-cancellation method for the same [12]. Another example is in [13], where M. Pal and S. Chattopadhyay proposed an algorithm for generating orthogonal minimum cross-correlation spreading codes.

Although there exist vast amount of published work on spread spectrum systems, as well as works relating to the design, properties, performance and generation of spreading codes [13-17], papers that comparatively examine the loading characteristics of even- and odd-degree Gold codes in a multi-user CDMA system are not available to the best of our knowledge. We seek to fill this gap. In this paper, we investigate the performance of even- and odd-degree Gold codes for a multi-user system, from a few simultaneous users to full system load. The performance for up to about a thousand users is considered. Results for six different code lengths, ranging from 31-chip to 1023-chip Gold codes, are reported. The



outcome shows that whereas the odd-degree codes exhibited relatively marginal loss in performance when loaded, the performance of their even-degree counterparts degraded rapidly when subjected to similar conditions. The results show that the odd-degree Gold codes are better candidates for multiple-access applications, and that the use of even-degree Gold codes for such applications should be avoided.

The work reported in this paper forms part of a larger study. Other aspects of the work have been reported in conferences [18-20] and in accredited journals [21, 22].

In the rest of this paper, system model is contained in Section 2, followed by an itemisation of the research approach in Section 3. Results are presented in Section 4, discussion in Section 5, and lastly, conclusion in Section 6.

2. SYSTEM MODEL

For a DS-CDMA system, the spread spectrum signal transmitted by a user k can be expressed as:

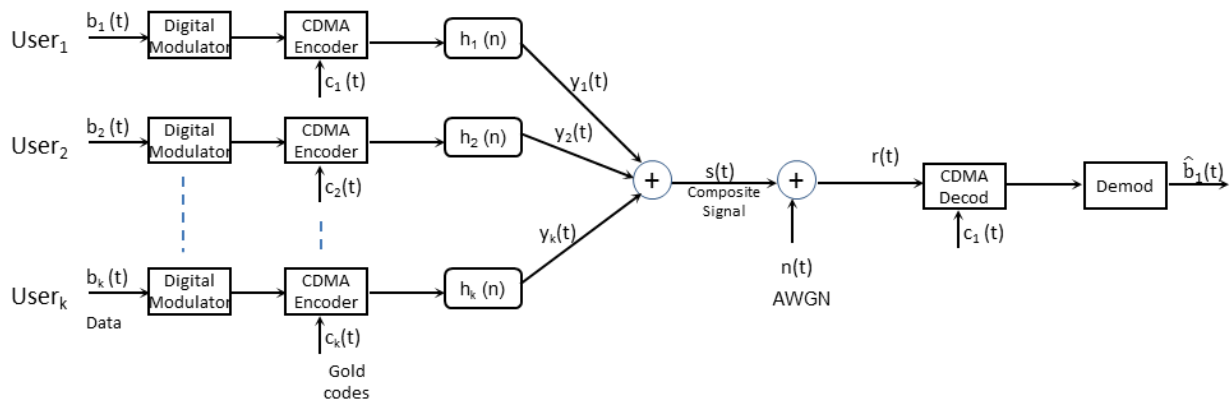


Figure-1. Model of a multi-user DS-CDMA system.

$$h_k(t) = \sum_{l=1}^L \beta_{kl} e^{j\gamma_{kl}} \delta(t - \tau_{kl}) \quad (3)$$

At the receiving end, the received signal $r_k(t)$ for the user is obtained by convolving $s_k(t)$ with $h_k(t)$ and adding noise so that:

$$r_k(t) = \int_{-\infty}^{\infty} s_k(\tau) h_k(t - \tau) d\tau + n(t) \quad (4)$$

where $n(t)$ represents the channel noise. Substituting the expressions for $s_k(t)$ and $h_k(t)$ into this integral, and using relevant properties of the Dirac delta function $\delta(t)$ gives:

$$r_k(t) = \sum_{l=1}^L A\beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c(t - \tau_{kl}) - \theta_{kl}) + n(t) \quad (5)$$

For a multi-user system comprising K users (Figure-1), the received signal $r(t)$ is a linear superposition of the signals for the users, and is given by:

$$s_k(t) = A c_k(t) b_k(t) \cos(\omega_c t + \theta_k) \quad (1)$$

where $b_k(t)$ is the user binary data, $c_k(t)$ is spreading code and ω_c is carrier frequency. The spreading code $c_k(t)$ for the user can be denoted as:

$$c_k(t) = \sum_{i=1}^N c_k^i P_c(t - iT_c), \quad c_k^i \in \{-1, 1\} \quad (2)$$

where N is length of the code, and P_c is a rectangular pulse having a duration T_c .

Let the wireless communication channel be represented by multiple paths having a real positive gain β_l , propagation delay τ_l and phase shift γ_l , where l is the path index. The channel impulse response $h_k(t)$ for user k and L independent paths can be modelled as:

$$r(t) = \sum_{k=1}^K \sum_{l=1}^L A\beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c(t - \tau_{kl}) - \theta_{kl}) + n(t) \quad (6)$$

where k is user index, l is path index, L is number of paths, β_{kl} is path gain for user k and path l , τ_{kl} is propagation delay, and γ_{kl} is phase shift. Let user-1 be the reference user. Assuming coherent demodulation, the receiver output $z(m)$ for m^{th} bit during the bit duration T_b of the user is given by:

$$\begin{aligned} z_1(m) &= \int_{mT_b}^{(m+1)T_b} r(t) c_1(t) \cos \omega_c t dt \\ &= \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=1}^K \sum_{l=1}^L A\beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c(t - \tau_{kl}) - \theta_{kl}) + n(t) \right\} c_1(t) \cos \omega_c t dt \end{aligned}$$



$$\begin{aligned}
 &= \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{l=1}^L A\beta_{1l} e^{j\gamma_{1l}} c_1(t - \tau_{1l}) b_1(t - \tau_{1l}) \cos(\omega_c(t - \tau_{1l}) - \theta_{1l}) \right\} c_1(t) \cos \omega_c t dt \\
 &+ \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=2}^K \sum_{l=1}^L A\beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c(t - \tau_{kl}) - \theta_{kl}) \right\} c_1(t) \cos \omega_c t dt \\
 &\quad + \int_{mT_b}^{(m+1)T_b} n(t) c_1(t) \cos \omega_c t dt \\
 &= z_{11} + z_{12} + z_{13} \quad (7)
 \end{aligned}$$

where:

$$\begin{aligned}
 z_{11} &= \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{l=1}^L A\beta_{1l} e^{j\gamma_{1l}} c_1(t - \tau_{1l}) b_1(t - \tau_{1l}) \cos(\omega_c(t - \tau_{1l}) - \theta_{1l}) \right\} c_1(t) \cos \omega_c t dt, \\
 z_{12} &= \int_{mT_b}^{(m+1)T_b} \left\{ \sum_{k=2}^K \sum_{l=1}^L A\beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t - \tau_{kl}) \cos(\omega_c(t - \tau_{kl}) - \theta_{kl}) \right\} c_1(t) \cos \omega_c t dt
 \end{aligned}$$

And

$$z_{13} = \int_{mT_b}^{(m+1)T_b} n(t) c_1(t) \cos \omega_c t dt$$

z_{11} represents the desired signal for the reference user, z_{12} is interference term, and z_{13} is noise term.

Gold codes are a type of pseudo-noise (PN) sequences derived from combination of certain pairs of m -sequences called *preferred sequences*, implemented using linear feedback shift registers (LFSR). A Gold code has a period $N = 2^n - 1$, where n is the length of the shift register. The period N also represents the length of the Gold code. Gold codes exhibit three-valued cross-correlation function [23-25] with values $\{-1, -t(n), t(n)-2\}$, where:

$$t(n) = \begin{cases} 2^{(n+1)/2} + 1, & n \text{ odd} \\ 2^{(n+2)/2} + 1, & n \text{ even} \end{cases} \quad (8)$$

3. METHODOLOGY

Software simulations were carried out for the transmission of random QPSK symbols in an additive white Gaussian noise (AWGN) channel having zero mean and unit variance, using Gold code-encoding for odd-degree and even-degree Gold codes, for degree $n = 5, 6, \dots, 10$. The Gold codes were generated from appropriate combinations of preferred pairs of m -sequences, obtained from software implementation of linear feedback shift registers for the generator pairs shown by Table-1.

At the receiving end, recovered data were compared with the original transmitted data for the determination of bit-error-rate (BER). Parametric BER graphs were generated for various code sets.

For each code length, BER performance was simulated for a set of users, ranging from a few simultaneous users to full system load. Independent data symbols were used for the users. The data symbols of each user were encoded with a distinct, user-specific Gold code. At the receiving end, the same code was used for the decoding of the user's signal, for a recovery of the original transmitted data stream. Simulations for the various code sets were carried out under the same set of conditions, thereby ensuring consistency of conditions. For this work, perfect synchronization and perfect power control were assumed. Effects of interleavers and error correction codes are not considered.

Table-1. Generator polynomials for the Gold codes.

n	$P_l^n(x)$	*Generator polynomial	N
5	$P_1^5(x)$	$x^5 + x^2 + 1$	31
	$P_2^5(x)$	$x^5 + x^4 + x^3 + x^2 + 1$	
6	$P_1^6(x)$	$x^6 + x^5 + 1$	63
	$P_2^6(x)$	$x^6 + x^5 + x^4 + x + 1$	
7	$P_1^7(x)$	$x^7 + x^6 + 1$	127
	$P_2^7(x)$	$x^7 + x^4 + 1$	
8	$P_1^8(x)$	$x^8 + x^7 + x^6 + x + 1$	255
	$P_2^8(x)$	$x^8 + x^7 + x^5 + x^3 + 1$	
9	$P_1^9(x)$	$x^9 + x^5 + 1$	511
	$P_2^9(x)$	$x^9 + x^8 + x^7 + x^2 + 1$	
10	$P_1^{10}(x)$	$x^{10} + x^7 + 1$	1023
	$P_2^{10}(x)$	$x^{10} + x^9 + x^8 + x^5 + 1$	

* $P_1^n(x)$ and $P_2^n(x)$ are the generator polynomials of the preferred pair used for obtaining corresponding set of Gold codes of degree n .

4. RESULTS

This section presents results of simulations carried out on the system performance, starting with results for a single user.

4.1 Performance for a single user

Figure-2 shows graphs of BER for a single user. The right-most curve on this figure is that of uncoded data transmission for a single user. The figure shows the close agreement between results of analysis and simulation for the uncoded data transmission. For the purpose of comparison, this curve will be retained on all results to be presented in this paper.

Clearly, Figure-2 shows that longer Gold codes give better error-rate performance. The figure also shows that with reference to the uncoded data transmission, at a BER of 10^{-4} , the codes provide coding gain of about 14.8,



17.4, 20.6, 23.8, 26.9 and 29.9 dB when $N = 31, 63, 127, 255, 511$ and 1023 respectively. From this we see that there is an average of about 3.03 dB-step in coding gain between adjacent code lengths, which can be explained in terms of the ratio of the process gain (or spreading factor) of the codes.

4.2 Performance of even-degree Gold codes

Having examined the performance for a single user, we shall now consider results for even-degree Gold codes, for $n = 6, 8$ and 10 , corresponding to code length $N = 63, 255$ and 1023 respectively.

Performance of degree-6 Gold codes: Here we consider results for degree-6 Gold codes, which corresponds to the code length $N = 63$. Figure-3 shows results for the codes. This figure shows that as the number of users increases, the system BER becomes worse. Furthermore, when full-load is approached, the rate of the performance degradation becomes rapid, resulting in emergence of error floor. This behaviour is expected, in connection with increase in multiple-access interference (MAI) when the number of users increases. As at 10 users (about one-sixth full-load), the performance can be seen to flatten out to a BER of about 10^{-1} .

Performance of degree-8 Gold codes: We shall now consider results for degree-8 Gold codes, which corresponds to $N = 255$. Figure-4 shows the results. As with the previous figure, this figure shows that BER becomes worse when the number of users increases, ultimately resulting in emergence of error floor. However, comparing Figure-3 with Figure-4 shows that the 255-chip code is able to support more users. This is in agreement with fundamental theory, the 255-chip code being a longer code. Whereas only 10 users cause the BER of a 63-chip code to flatten out to a BER of 10^{-1} , it takes about 100 users (two-fifth full-load) to make the performance a 255-chip code to degrade to the same BER.

Performance of degree-10 Gold codes: We shall now consider results for degree-10 Gold codes, which corresponds to $N = 1023$. Figure-5 shows the results for the codes. A look at this figure shows a behaviour similar to that observed in the other even-degree codes that have just been considered: there is degradation in BER, resulting in emergence of error floor when number of users is elevated. For the 1023-chip codes however, the results show that it takes about 500 users (about half full-load) for the BER to degrade to about 10^{-1} .

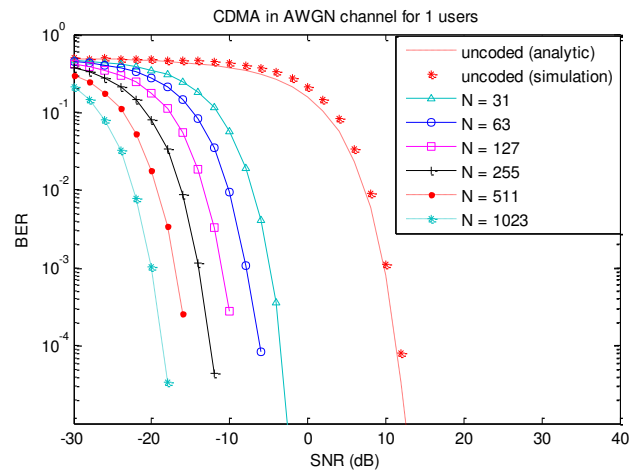


Figure-2. Bit-error-rate for a single user for uncoded and coded data transmission.

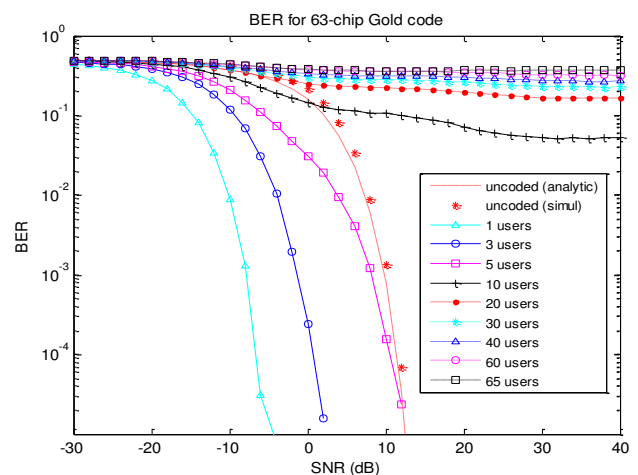


Figure-3. Performance of degree-6 Gold codes.

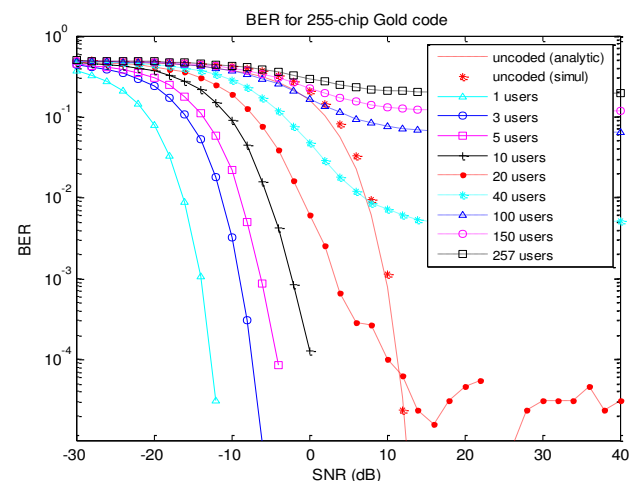


Figure-4. Performance of degree-8 Gold codes.

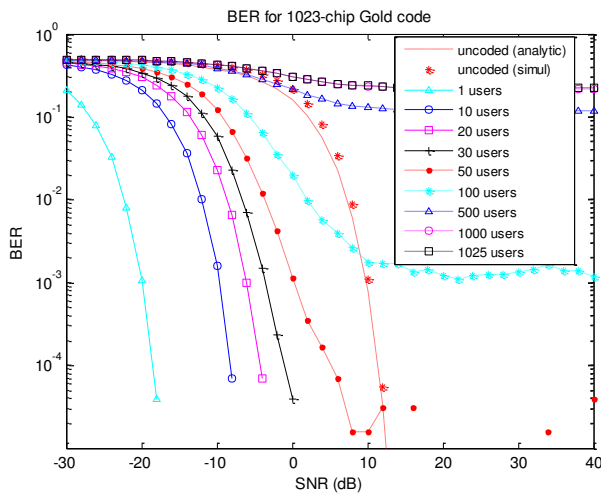


Figure-5. Performance of degree-10 Gold codes.

4.3 Performance of odd-degree Gold codes

Having examined the performance of even-degree codes, we shall now consider results for odd-degree Gold codes, for $n = 5, 7$ and 9 , corresponding to code length $N = 31, 127$ and 511 respectively.

Performance of degree-5 Gold codes: Here we consider results for degree-5 Gold codes, Figure-6. A look at this figure shows that the performance of this code set is strikingly different from the ones that have been considered thus far. The differences are enumerated as follows:

- Performance degradation is not rapid but gradual for this current set of codes.
- When the system is heavily loaded, the codes maintain their high-SNR steep slope.
- There is no serious system saturation when the system is heavily loaded, even up to full system load.
- For this code set, the BER performance curves exhibit no error floor.
- When fully loaded, the code performance is still as good as that of an uncoded, single-user system.

The performance of this set of codes is surprisingly better than expected. We shall now consider results for other sets of odd-degree codes.

Performance of degree-7 and degree-9 Gold codes: Figure-7 shows results for degree-7 Gold codes. By comparing this figure with the previous one, it is obvious that the two set of codes have similar performance characteristics, enumerated in the previous subsection. The same behaviour can be seen in Figure-8 for degree-9 Gold codes. These indicate that the desirable behaviour

that were enumerated in the previous subsection is general to the odd-degree Gold codes considered here.

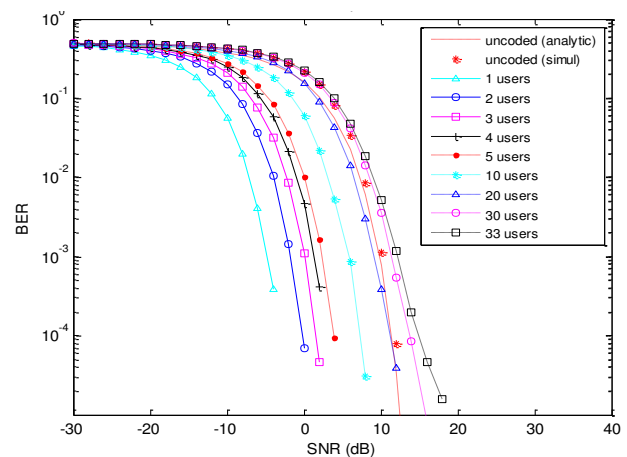


Figure-6. Performance of degree-5 Gold codes.

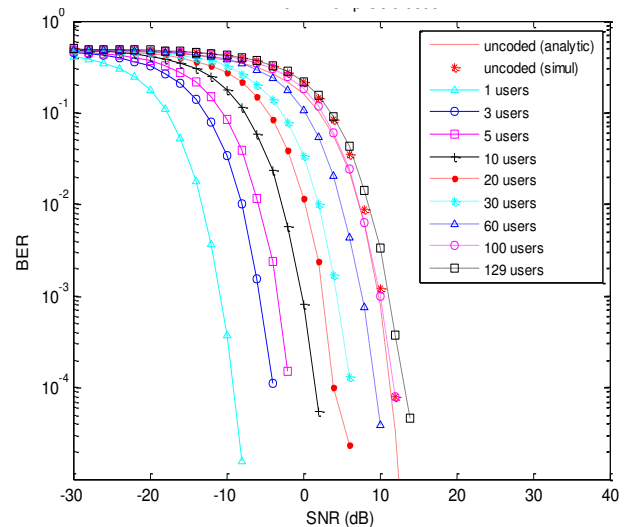


Figure-7. Performance of degree-7 Gold codes.

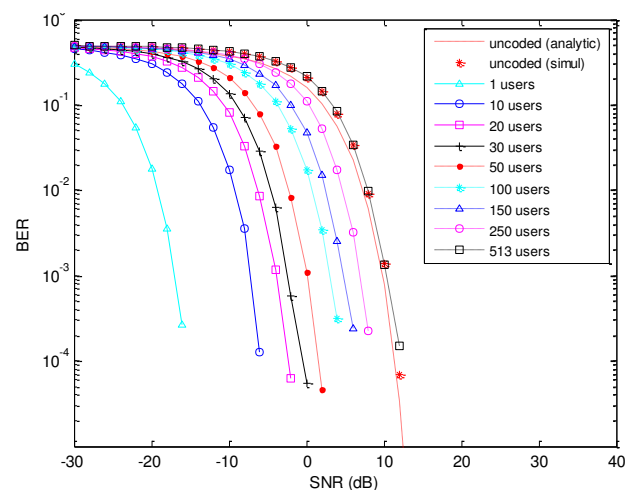


Figure-8. Performance of degree-9 Gold codes.



5. DISCUSSIONS

Results presented in previous section confirm that, generally, the system BER becomes worse when the number of users increases. This trend agrees with fundamental concepts, in connection with increasing MAI that results from elevated number of users. However, the sharp difference in the rate of degradation for the different code lengths demands some consideration. Being from the same family, all the codes would normally be expected to exhibit similar behaviour. In contrast, the results show that this is not the case.

Results show that for the degree 6, 8 and 10 Gold codes, BER degraded rapidly when the number of users increases. This led to the emergence of error floor and system saturation. Now, these three sets of codes are even-degree codes, which implies that this behaviour is peculiar to the even-degree Gold codes.

In contrast to the above, the results show that BER of the degree 5, 7 and 9 Gold codes exhibited relatively marginal degradation in performance when the system was loaded. The codes suffered less dB-loss, and maintained their steep, high-SNR slopes regardless of elevated number of users. Now, these set of codes are odd-degree codes, meaning that the positive characteristics are peculiar to the odd-degree Gold codes. Hence, we conclude that in a multi-user environment, the odd-degree Gold codes have better performance than even-degree Gold codes.

There is yet a further observation. Ordinarily for a given family of spreading codes, a longer code is expected to give better performance than a shorter code. In contrast, the results show that when the system was loaded, the 31-chip Gold codes gave better performance than the 63-chip Gold codes. Whereas the 31-chip code exhibited no error floor when the system was loaded, the 63-chip code did. The same pattern can be seen among the remaining odd-degree and even-degree codes (Figure-9). This implies that a longer code will not necessarily give a better performance.

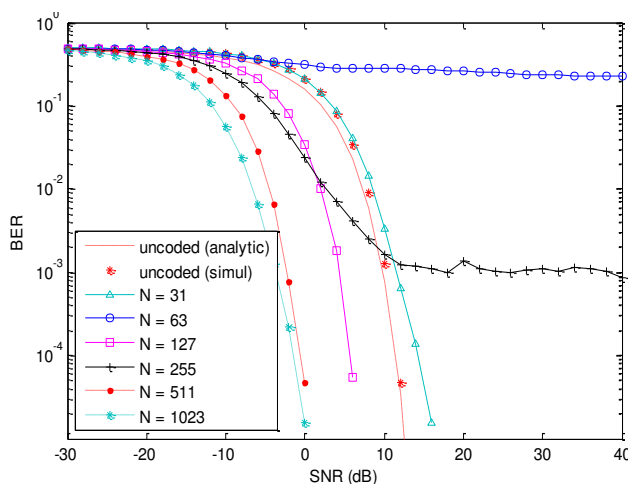


Figure-9. Results for 30 users showing an example of inferior performance of 63-chip and 255-chip Gold codes, when compared to their shorter 31-chip counterpart.

6. CONCLUSIONS

This paper investigated the performance of even- and odd-degree Gold codes in a multi-user spread spectrum system. Results for six different sets of codes, ranging from 31-chip to 1023-chip Gold codes were reported. The work considered light to heavy system loading, involving up to about a thousand simultaneous users. Results showed that when the system was loaded, the odd-degree Gold codes performed better than their even-degree counterparts. Whereas the latter exhibited error floor and system saturation when the system was loaded, no such performance degradation was noticeable in the former. The results suggest that odd-degree Gold codes are better suited for multiple-access applications than their even-degree counterparts.

The work reported in this paper is foundational to the development of a proposed space-time coded multi-carrier CDMA system in a frequency-selective channel.

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