VOL. 13, NO. 1, JANUARY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

A CONSISTENT METHODOLOGY FOR THE DEVELOPMENT OF INVERSE AND DIRECT KINEMATICS OF ROBUST INDUSTRIAL ROBOTS

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ABSTRACT

Our article contributes by presenting the mathematical foundation for offline programming of industrial robots oriented to the development of flexible manufacturing cells, the simulation of realistic robot that is approached by using the robot kinematics. For the implementation of the offline programming of a real industrial robot integrated in aflexible manufacturing cells, the model of direct kinematics, inverse kinematics and Jacobian is needed to obtain data of the industrial robot in its functions within the flexible manufacturing cells. This model is loaded as an additional module in modeling, simulation using MatLab software and programming software allowing to effectively check its results offline to enrich its content for future contributions.

Keyword: direct kinematics, inverse kinematics, industrial robot.

1. INTRODUCTION

In the last decade, there has been a growing motivation for developments in the area of industrial robotics, as in [1], [2], [3], presenting several methodologies to carry out its control [4], [5], [6]. An industrial robot is an open, programmable and reprogrammable kinematic chain, which makes it malleable in software; and allows it to accommodate its operational needs, which makes it malleable in terms of hardware. The robot ABB IRB 1600 is a robot of six axes created specifically for the industries that use the automation based on robotic systems and they have a wide structure specially adapted for its use, being able to communicate with external systems. It will be used to develop the direct and inverse kinematic model and the Jacobian model oriented to integrate with the flexible cells of industrial production.

The model of direct kinematics, inverse kinematics and the Jacobian model are based on concepts described in [7], [8], [9], [10]. It is not a literal copy, the author of this research, develops a methodology of kinematics based on homogeneous matrices of transformation (HMT) for the industrial robot IRB 1600 of ABB [11]. The modeling of kinematics with quaternions is a contribution of this research for this type of industrial robot.

2. METHODOLOGY

A. Model of direct kinematics with HMT

To apply the model of direct kinetics it is necessary to represent the coordinate system of the final effector or tool in relation to a coordinate system of the base of the robot, as follows:

$$T_n^0(q) = A_1^0(q_1) \times A_2^1(q_2) \times \dots \times A_n^{n-1}(q_n) \qquad (1)$$

Where q represents the variables of the joint; n, s, and a are vectors of the tool or tool's coordinate system and p is its position vector in relation to the base coordinate system. Figure-1 illustrates the tool center point coordinate system referring to the robot base system, is the representation of the direct kinematics. The following is a methodology to develop the model of direct kinematics based on the Denavit - Hartenberg system [2]:

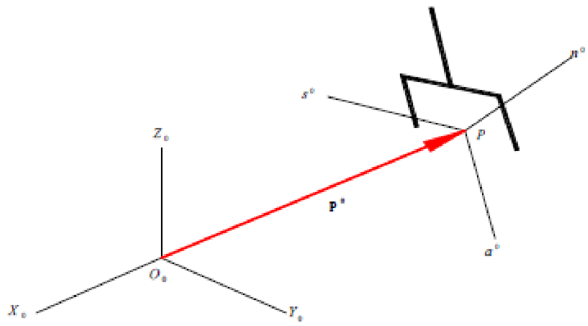

Figure-1. Rotation and translation of claws or tool according to the base coordinates of the robot.

Step 1: Define the directions of the axes $Z_a, Z_1, \dots Z_{n-1}$, From the base

Step 2: Define the origin O_0Of the base coordinate system on the axis Z_0.Axes X_0 and Y_0 chosen according to the rule of the right hand.

Step 3: Define the origin of $O_i$at the intersection Z_i_i with the common normal between the axes Z_{i-1}

VOL. 13, NO. 1, JANUARY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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and Z_iZ_i. If the axes $Z_{i-1}$and Z_i are parallel and the joint i is of revolution, then locate $O_i$so that $d_i = 0$; If the joint i is prismatic, locate $O_i$in a reference position to establish a mechanical limit.

Step 4: Select axis X_i along the normal common to the axes $Z_{i-1}$and $Z_i$with direction of the joint i towards the joint i + 1.

Step 5: Choose axis Y_i,so that with $X_i$the rule of the right hand is fulfilled.

Step 6: Choose the tool or clamp coordinate system where $X_n$is normal to Z_{n-1};); If the claw joint or tool is a revolution, then align $Z_n$with the direction Z_{n-1}.

Step 7: For i = 1,2 ... n construct Table-1 with the parameters $a_i, d_i, \alpha_i, \theta_i$.

Step 8: Based on the parameters in the previous table calculate the HMT A_$A_i^{i-1}(q_i)$ for i=1,2,…n.

Step 9: Define the model of the direct kinematics $T_n^0(q) = A_1^0(q_1) \times A_2^1(q_2) \times \ldots \times A_n^{n-1}(q_n)$ for the position and orientation of the system of coordinates of the claw or tool, in relation to the coordinate system of the base of the robot.

$$A_1^0(\theta_1) = \begin{bmatrix} c_1 & 0 & s_1 & a_1c_1 \\ s_1 & 0 & -c_1 & a_1s_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_2^1(\theta_2) = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_3^2(\theta_3) = \begin{bmatrix} c_3 & 0 & s_3 & a_3c_3 \\ s_3 & 0 & -c_3 & a_3s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3(\theta_4) = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_5^4(\theta_5) = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_6^5(\theta_6) = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

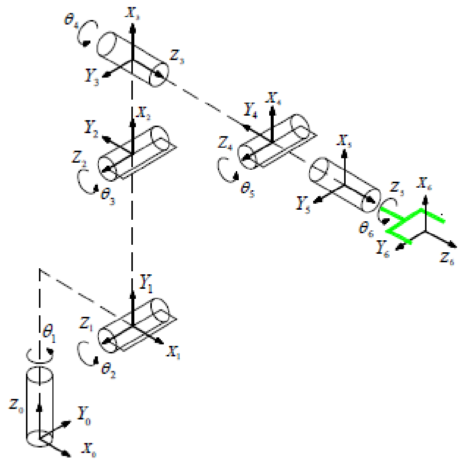

Figure-2. Coordinate systems.

Table-1. Parameters and variables of Denavit - Hartenberg.

Joint	Ai	Θi	Di	ai
1	90	0	$d1$	$a1$
2	0	90	0	$a2$
3	90	0	0	$a3$
4	-90	0	$d4$	0
5	90	0	0	0
6	0	0	$d6$	0

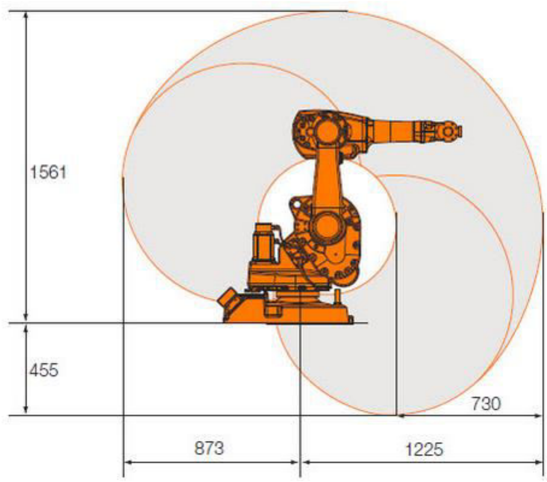

Figure-3. Work area. Lengths in millimeters.

The robot's direct kinematics model can be decomposed into two parts: (a) mechanical structure $\boldsymbol{T_3^0(q_A)}$ And (b) claw or tool $\boldsymbol{T_6^3(q_B)}$. The two models are integrated to obtain the final model of direct kinematics:

$$T_6^0(q) = T_3^0(q_A) \times T_6^3(q_B)$$

$$T_3^0(q_A) = A_1^0(\theta_1) \times A_2^1(\theta_2) \times A_3^2(\theta_3)$$

 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$T_6^3(q_B) = A_4^3(\theta_4) \times A_5^4(\theta_5) \times A_6^5(\theta_6)$

$$T_6^0(q) = \begin{bmatrix} n_x(q) & s_x(q) & a_x(q) & p_x \\ n_y(q) & s_y(q) & a_y(q) & p_y \\ n_z(q) & s_z(q) & a_z(q) & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

If the position and orientation of the robot tool claw or tool $O_n(X_n, Y_n, Z_n)$represented in expression (2) is not correct, an adjustment HMT is created because the mechanical system of the robot can Errors and the electronic system can generate errors in the assembly stage by the manufacturer, then, the robot needs calibration to adjust its parameters before entering into use.

B. Model of inverse kinematics with HMT

Reverse kinematics consists of finding possible seam configurations, corresponding to a specific position and orientation of the claw or tool. Their solution is important for the planning of the paths being complex because:

a) The equations to be developed in general are non-linear.
b) Multiple solutions can be found.
c) Infinite solutions may exist.
d) There may be unacceptable solutions due to the singularities of the robot.

The existence of three consecutive joints of revolution with axes intercepted at one point for all possible configurations is known as the solubility condition of Pieper [10]. Most industrial robots fulfill this condition, as shown in Figure-4 and Figure-5. For the IRB 1600 robot of ABB [1] at point W of Figure4, the axes $Z_3, Z_4\ e\ Z_5$corresponding to the coordinate system$O_4(X_4, Y_4, Z_4)$.

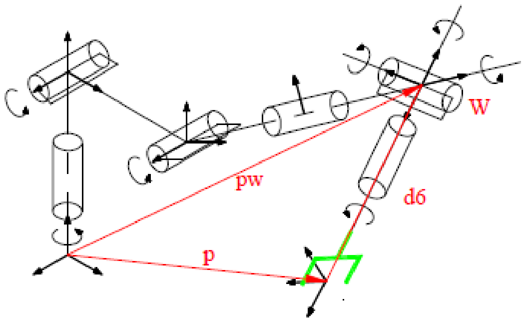

Figure-4. Pieper solubility condition [10].

A methodology for calculating inverse kinematics is to find the point of the robot configuration, which can be represented as a function of position and orientation of the claw or tool and as a function of a reduced number of joint variables. What it means to propose the problem of inverse kinematics in two parts: (a) Reverse kinematics of the mechanical structure and (b) Inverse kinematics of the spherical fist or wrist. This methodology can be developed with the following steps:

Step 1: Calculate the position of the hand or wrist $P_w(q_1, q_2, q_3)$
Step 2: Solve the inverse kinematics for (q_1, q_2, q_3)
Step 3: Calculate $R_3^0(q_1, q_2, q_3)$
Step 4: Calculate $R_6^3(\theta_4,\ \theta_5,\ \theta_6) = R_3^{0^T} R$
Step 5: Solve the inverse kinematics for the orientation $(\theta_4,\ \theta_5,\ \theta_6)$

For the robot IRB 1600 we have the following expression of direct kinematics:

$$T_6^0(q) = A_1^0(q_1) \times A_2^1(q_2) \times A_3^2(q_3) \times A_4^3(q_4) \times A_5^4(q_5) \times A_6^5(q_6) \quad (3)$$

Each of the HMTs on the right side of equation (4) represents the transformation of each joint. To apply Pieper's methodology [10] it is necessary to obtain the inverse of the HMT, that is, $A_i^{i-1}(q_i)$. So:

$$T = p^{R^T}$$
$$[T]^{-1} = \{[p][R]^T\}^{-1} = \{[R]^T\}^{-1}[-p] = [R]^T[-p]$$
$$[T]^{-1} = \begin{bmatrix} n_x & n_y & n_z & 0 \\ s_x & s_y & s_z & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -p_x \\ 0 & 1 & 0 & -p_y \\ 0 & 0 & 1 & -p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & n_y & n_z & -n^T \times p \\ s_x & s_y & s_z & -s^T \times p \\ a_x & a_y & a_z & -a^T \times p \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The idea of Pieper [10] is that the position W is a function $\theta_1, \theta_2\ e\ \theta_3$ and also of the position and orientation of the tool or tool coordinate system. The inverse kinematics methodology for the ABB IRB 1600 robot can be obtained from the position $W \equiv O_4$and can be written according to a geometric analysis:

$$P_w = P - d_6 . a \quad (5)$$
$$\begin{bmatrix} p_{wx} \\ p_{wy} \\ p_{wz} \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - d_6 \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$
$$P - d_6 . a = \{T_4^0\}_4 = P_w$$
$$A_1^0(\theta_1) . A_2^1(\theta_2) . A_3^2(\theta_3) . A_4^3(\theta_4) = P_w$$
$$\{A_2^1(\theta_2) . A_3^2(\theta_3) . A_4^3(\theta_4)\} = \{[A_1^0]^{-1} . P_w\}$$
$$\{A_3^2(\theta_3) . A_4^3(\theta_4)\} = \{[A_2^1]^{-1} . [A_1^0]^{-1} . P_w\}_4$$

VOL. 13, NO. 1, JANUARY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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$$\{A_4^3(\theta_4)\}_4 = \{[A_3^2]^{-1}.[A_2^1]^{-1}.[A_1^0]^{-1}.P_w\}_4 \tag{6}$$

From equations (5) to (6) the following simplified expressions are obtained:

$$\begin{aligned}
p_{wx} &= a_1c_1 + a_2c_{12} + a_3c_{123} - a_3c_1s_{23} \\
&\quad + d_4(c_{13}s_2 + c_{12}s_3) \\
p_{wy} &= a_1s_1 + a_2c_2s_1 + a_3c_{23}s_1 - a_3s_{123} \\
&\quad + d_4(c_3s_{12} + c_2s_{13}) \\
p_{wz} &= a_2s_2 + a_3c_2s_3 + d_1 + a_3c_3s_2 + d_4(-c_{23} + s_{23}) \quad (7)
\end{aligned}$$

$$\begin{aligned}
c_1p_{wx} + s_1p_{wy} &= d_4(c_3s_2 + c_2s_3) + a_2c_2 + a_3c_{23} \\
&\quad - a_3s_{23} \\
p_{wz} &= d_4(-c_{23} + s_{23}) + a_2s_2 + a_3s_3c_3 + a_3c_3s_3 \\
0 &= s_1p_{wx} - c_1p_{wy} \\
a_1c_2 + a_2 + a_2c_2 + a_3c_{23} - a_3s_{23} + d_1s_2 & \\
&= c_{12}p_{wx} + c_2s_1p_{wy} - d_1s_2 + s_2p_{wz} \\
-a_1s_2 + a_2s_2 + a_3c_3s_2 + a_3c_2s_3 + d_1c_2 & \\
&= c_2p_{wz} - c_1s_2p_{wx} - s_{12}p_{wy} \\
-a_3 + a_2c_3 + a_3(c_{23} - s_{23}) & \\
&= p_{wz}(c_3s_2 + c_2s_3) + c_3p_{wx}(c_{23} - s_{23}) \\
&\quad + p_{wy}s_3(c_{23} - s_{23}) \\
0 &= -c_3p_{wy} + p_{wx}s_3 \\
a_2s_3 + a_3(c_3s_2 + c_2s_3) + d_4 &= c_3p_{wx}(c_3s_2 + c_2s_3) + \\
p_{wy}s_3(c_3s_2 - c_2s_3) + p_{wz}(-c_{23} - s_{23}) & \quad (7)
\end{aligned}$$

The unknowns of the system $q = (q_1, q_2, q_3, q_4, q_5, q_6)^T$ are less than the number of equations, then the system has solution.

C. Model of direct kinematics with quaternions.

A methodology for the development of kinematics of the industrial robot arm based on quaternions, can be found in [12]. After obtaining the model of kinematics with HMT, a new coordinate system is constructed, shown in Figure 5, from which the direct kinematic model can be obtained with quaternions. In this next section, we will find the procedure to represent in symbolic form the steps that lead to obtain the mathematical model in quaternion:

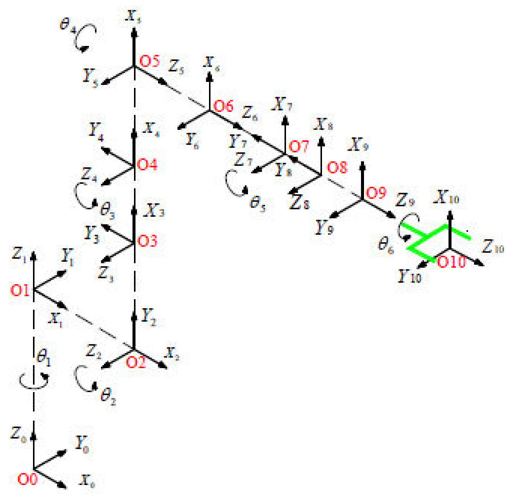

Figure-5. Coordinate system for the kinematic model with quaternions.

Step 1: $O_0 \rightarrow O_1T(Z_0, d_1)Rot(Z_0, \theta_1)$
Step 2: $O_1 \rightarrow O_2\boldsymbol{T}(X_1, a_1)\boldsymbol{Rot}(X_1, +90^\circ)$
Step 3: $O_2 \rightarrow O_3T(Y_2, a_2)Rot(Z_2, \theta_2)$
Step 4: $O_3 \rightarrow O_4\boldsymbol{T}(X_3, a_3)\boldsymbol{Rot}(Z_3, \theta_3)$
Step 5: $O_4 \rightarrow O_5T(Nulo)Rot(X_4, +90^\circ)$
Step 6: $O_5 \rightarrow O_6\boldsymbol{T}(Z_4, d_4)\boldsymbol{Rot}(Z_5, \theta_4)$
Step 7: $O_6 \rightarrow O_7\boldsymbol{T}(Nulo)\boldsymbol{Rot}(X_6, -90^\circ)$
Step 8: $O_7 \rightarrow O_8\boldsymbol{T}(Nulo)\boldsymbol{Rot}(Z_7, \theta_5)$
Step 9: $O_8 \rightarrow O_9\boldsymbol{T}(Nulo)\boldsymbol{Rot}(X_8, +90^\circ)$
Step 10: $O_9 \rightarrow O_{10}\boldsymbol{T}(Z_9, d_6)\boldsymbol{Rot}(Z_9, \theta_6)$

Where the displacements T are defined by the following position vectors:

$$r_1 = 0,0,d_1,\ r_2 = a_1,0,0,\ r_3 = 0,a_3,0,\ r_4 = a_3,0,0,\ r_5 = 0,0,0,\ r_6 = 0,0,d_4,\ r_7 = 0,0,0,\ r_8 = 0,0,0,$$
$$r_9 = 0,0,0,\ r_{10} = 0,0,d_6$$

And where Rot rotations are defined by the following quaternions, when $\dot{C}_\iota = \cos\frac{\theta_i}{2}\ y\ \dot{S}_\iota = \sin\frac{\theta_i}{2}$:

$$Q_1 = (\dot{C}_\iota, 0,0,\dot{S}_\iota), Q_2 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0,0\right), Q_3 = (\dot{C}_3, 0,0,\dot{S}_3), Q_4 = (\dot{C}_4, 0,0,\dot{S}_4), Q_5 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0,0\right),$$
$$Q_7 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0,0\right), Q_8 = (\dot{C}_8, 0,0,\dot{S}_8), Q_9 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0,0\right)\ y\ Q_{10} = (\dot{C}_{10}, 0,0,\dot{S}_{10}) \tag{8}$$

D. Calculation of position p

In this work the alternative of first translation is applied followed by rotation of the robot's claw relative to the coordinate system of the base. To compute the position p with expression (3) when i = 1, 2, ..., m and m = 10 we obtain a set of ten equations and making successive substitutions finally we obtain the following general expression:

$$[P_0] = [Q]_{i=1}^{m}.[P_m].[Q^*]_{m}^{i=1} = \sum_1^m \left(Q_1^{m-1}.r_m^i.Q^{*1}_{\ \ m-i}\right) \quad (9)$$

E. Rotation Calculation Rot

The relationships between the quaternions defining the Rot rotation of the claw coordinate system $O_{10}$with respect to the base coordinate system $O_0$by quaternion composition has the following general expression:

$$Rot = R_{i-1} = Q_1 R_1 \quad (10)$$

If i = 1,2, ..., m and m = 10 then we have a set of m equations and making successive substitutions, we obtain the following general form:

$$R_o = Q_1Q_2Q_3Q_4Q_5Q_6Q_7Q_8Q_9Q_{10}R_{10}$$

$$R_0 = Q_{12345678910}R_{10}$$

$$R_0 = \sum_i^m Q_I\, R_m \quad (11)$$

F. Algorithm for direct kinematics with quaternions

The algorithm to obtain the mathematical model with quaternions of direct kinematics has the following steps:

Step 1: Apply Denavit-Hartenberg convention to obtain the objective articulation coordinate systems.
Step 2: For consecutive and parallel joint axes, a quaternion is required to represent the displacement and rotation to include the joint variable.
Step 3: For consecutive intersecting or intersecting articulation axes, a transition coordinate system must be added to include the joint variable.
Step 4: Add to the target articulation systems the necessary transition systems from the base coordinate system to the claw coordinate system.
Step 5: Number from scratch and from base to claw, all resulting coordinate systems.
Step 6: Determine transition and rotation between consecutive coordinate systems, starting from the base.
Step 7: Write position vectors and rotation quaternions, starting and numbering them by the base.
Step 8: Define product multiplication order of quaternions:
First translation and then rotation of the coordinate system
First rotation and then translation of the coordinate system
Step 9: Develop product of quaternions to calculate position and orientation of the robot's claw.

G. Conversion of HMT to Quaternions and Vice-versa

The transformation of HMT to quaternions and vice versa can be deduced using intermediate or transition coordinate systems, displacements and rotations. Next, the final relations are represented, without considering their development. The representation of HMT T according to the components of a quaternion will be given through the following matrix, also known as direct relation:

$$T = 2\begin{bmatrix} q_0^2 + q_1^2 - \frac{1}{2} & q_1q_2 - q_3q_0 & q_1q_3 + q_2q_0 & 0 \\ q_1q_2 + q_3q_0 & q_0^2 + q_2^2 - \frac{1}{2} & q_2q_3 + q_1q_0 & 0 \\ q_1q_3 + q_2q_0 & q_2q_3 + q_1q_0 & q_0^2 + q_3^2 - \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

The inverse relationship can be obtained by equalizing the backwardness of the elements of the main diagonal of the matrix (30) with the matrix (11):

$$q_0 = \frac{1}{2}\sqrt{\left(n_x + o_y + a_z + 1\right)}$$

$$q_1 = \frac{1}{2}\sqrt{\left(n_x - o_y - a_z + 1\right)}$$

$$q_2 = \frac{1}{2}\sqrt{\left(-n_x + o_y - a_z + 1\right)}$$

$$q_3 = \frac{1}{2}\sqrt{\left(-n_x - o_y + a_z + 1\right)} \quad (12)$$

H. Methodology to obtain the Jacobian matrix J

In Figure-6 is shown infinitesimal translation and infinitesimal rotation of the claw or tool with a vector dr_6 and with a vector $d\Phi_6$Respectively. And it is written$V = (q).\dot{q}$ where:

$$V = \begin{bmatrix} V_L \\ V_A \end{bmatrix} y\ \dot{q} = \left[\dot{q1},\ \dot{q2},\ \dot{q3} \ldots\ \dot{qn}\right]^T \quad (12)$$

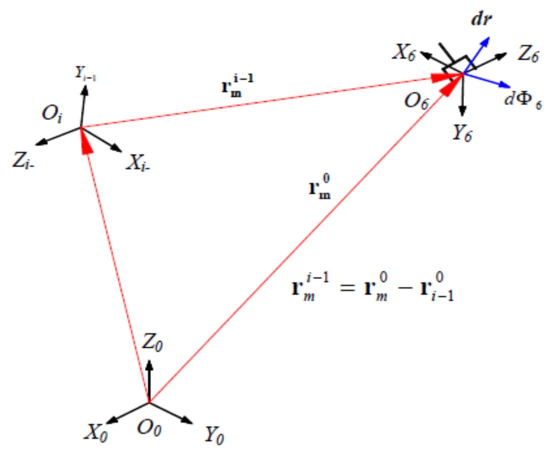

Figure-6. Direct differential kinematics.

The dimension of matrix J is 6x6 where the first three rows represent the linear velocity $V_L$and the last three lines represent the angular velocity V_A; Each column represents the linear velocity $J_{Li}$and the angular velocity J_{Ai}generated by each joint i. Then, you can write for the IRB 1600 of 6 degrees of freedom (D.O.F.):

$$J = \begin{bmatrix} J_{L1} & J_{L2} & J_{L3} & J_{L4} & J_{L5} & J_{L6} \\ J_{A1} & J_{A2} & J_{A3} & J_{A4} & J_{A5} & J_{A6} \end{bmatrix}$$

The resulting linear velocity $V_L$for the claw or tool is calculated with the following expression:

$$V_L = J_{L1}.\dot{q_1} + J_{L2}.\dot{q_2} + \cdots J_{Ln}.\dot{q_n} \quad (13)$$

If the i-th joint is of revolution, the effective angular velocity of rotation of the element is:

$$J_{L1}.\dot{q_1} = Z_{i-1}.\dot{\theta}_1 = \omega_i \quad (14)$$

Then, the angular velocity of the claw or tool is calculated with the following expression:

$$V_a = \omega_6 = J_{Ai}.\dot{q_1} + J_{A2}.\dot{q_2} + \cdots J_{Ai}.\dot{q_n} \quad (15)$$

When the joint is of revolution, the angular velocity induces an angular velocity in the claw or tool that is calculated as follows:

$$J_{Ai}.\dot{q_1} = \omega_i = Z_{i-1}.\dot{\theta}_1 \quad (16)$$

The expressions (13), (14), (15) and (16) allow to obtain the Jacobian matrix J of a robot n joints of revolution:

For a joint of revolution has the Jacobian matrix:

$$\begin{bmatrix} J_{Li} \\ J_{Ai} \end{bmatrix} = \begin{bmatrix} Z_{i-1} * r_6^{i-1} \\ Z_{i-1} \end{bmatrix} \quad (17)$$

The direction of the vector $Z_{i-1}$is represented by a unit vector $Z_{i-1}^{i-1} = [0 \quad 0 \quad 1]^T$with respect to the system coordinates $O_{i-1}(X_{i-1}, Y_{i-1}, Z_{i-1})$ and in relation to the base coordinate system. The following expression yields the composition of rotation matrices $R_j^{j-1}(q_j)$:

$$Z_{i-1}^0 = R_1^0(q_1).R_2^1(q_2) \ldots + R_{i-1}^{i-2}(q_{i-1}).Z_{i-1}^{i-1} \quad (18)$$

The position vector r_6^{i-1}in relation to the base coordinate system, is determined by HMT$A_j^{j-1}(q_j)r_6^{i-1}$or vector augmented 4x1 corresponding to rr_6^{i-1} e $r_{i-1}^{i-1} = [0 \quad 0 \quad 0 \quad 1]^T$. Then, it can be written:

$$r_6^{i-1} = A_1^0(q_1).A_2^1(q_2) \ldots A_n^{n-1}(q_n).r_6^{n-1} - A_1^0(q_1).A_2^1(q_2) \ldots A_{i-1}^{i-2}(q_{i-1}).r_{i-1}^{i-1} \quad (19)$$

The first term of the second member of the expression represent the position vector of O_0 a $O_6$and the second term represent the intermediate position vector O_0 a O_{i-1}

Knowing the velocities at each joint, the direct differential kinematics for the ABB IRB 1600 robot claw or tool can be calculated with the following expression:

$$V = J(q).\dot{q} \quad (20)$$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_{L1x} & J_{L2x} & J_{L3x} & J_{L4x} & J_{L5x} & J_{L6x} \\ J_{L1y} & J_{L2y} & J_{L3y} & J_{L4y} & J_{L5y} & J_{L6y} \\ J_{L1z} & J_{L2z} & J_{L3z} & J_{L4z} & J_{L5z} & J_{L6z} \\ J_{A1x} & J_{A2x} & J_{A3x} & J_{A4x} & J_{A5x} & J_{A6x} \\ J_{A1y} & J_{A2y} & J_{A3y} & J_{A4y} & J_{A5y} & J_{A6y} \\ J_{A1z} & J_{A2z} & J_{A3z} & J_{A4z} & J_{A5z} & J_{A6z} \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}$$

I. Methodology for the model of direct differential kinematics

Step 1: Define Gasket Type:

If the joint is prismatic apply: $\begin{bmatrix} J_{Li} \\ J_{Ai} \end{bmatrix} = \begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$

If the joint is of revolution apply: $\begin{bmatrix} J_{Li} \\ J_{Ai} \end{bmatrix} = \begin{bmatrix} Z_{i-1} * r_6^{i-1} \\ Z_{i-1} \end{bmatrix}$

Step 2: Calculate the vectors:

$$Z_{i-1}^0 = R_1^0(q_1).R_2^1(q_2) \ldots + R_{i-1}^{i-2}(q_{i-1}).Z_{i-1}^{i-1}$$
$$r_6^{i-1} = A_1^0(q_1).A_2^1(q_2) \ldots A_n^{n-1}(q_n).r_6^{n-1} - A_1^0(q_1).A_2^1(q_2) \ldots A_{i-1}^{i-2}(q_{i-1}).r_{i-1}^{i-1} \quad (21)$$

Step 3: Calculate the vector operation when a joint is a revolution: $Z_{i-1}^0 * r_6^{i-1}$

Step 4: Arrange the Jacobian matrix with the terms of Step 3:

$$\begin{bmatrix} J_{L1x} & J_{L2x} & J_{L3x} & J_{L4x} & J_{L5x} & J_{L6x} \\ J_{L1y} & J_{L2y} & J_{L3y} & J_{L4y} & J_{L5y} & J_{L6y} \\ J_{L1z} & J_{L2z} & J_{L3z} & J_{L4z} & J_{L5z} & J_{L6z} \\ J_{A1x} & J_{A2x} & J_{A3x} & J_{A4x} & J_{A5x} & J_{A6x} \\ J_{A1y} & J_{A2y} & J_{A3y} & J_{A4y} & J_{A5y} & J_{A6y} \\ J_{A1z} & J_{A2z} & J_{A3z} & J_{A4z} & J_{A5z} & J_{A6z} \end{bmatrix}$$

Step 5: Calculate the direct differential kinematics with the expression:

$$V = \begin{bmatrix} V_L \\ V_A \end{bmatrix} = J(q).\dot{q} \text{ , for} \dot{q} = (\dot{q_1}, \dot{q_2}, \dot{q_3}, \dot{q_4}, \dot{q_5}, \dot{q_6}) \quad (22)$$

J. Model of the inverse differential kinematics

With the expression $\dot{p} = J.\dot{q}$you can calculate the speed in the claw or tool as a function of the speeds of the joints. If the Jacobian matrix is not singular, that is, $|J| \neq 0$the inverse matrix J^{-1}exists. Therefore, we have:

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$$\dot{q} = J^{-1}.\dot{p} \tag{23}$$

That is the expression to calculate the speed in the known joints the speed in the claw or tool.

The Jacobian depends on the configuration that the robot assumes and changes the variables of the joints. Then, singularities can be defined to a set of points in the space of the joint where the Jacobian determinant is equal to zero $|J| = 0$. Considering the inverse of the Jacobian, where AdjJ is the attached matrix, we write:

$$J^{-1} = \frac{Adj\mathbf{J}}{|J|} \tag{24}$$

When the configuration of the robot approaches a singularity, some elements of J^{-1}assume excessively large values because AdjJ always considers finite values. Based on the above, it can be stated that the velocities required at the joints to move the claw or tool in some directions are too large, approaching infinity when $|J| = 0$ In this case limit, it can be said that the robot assumes an impossible configuration because the freedom of movement is restricted or there are singularities.

3. ANALYSIS OF RESULTS

Matlab is implemented in the direct and inverse kinematics of the robot IRB 1600 of the company ABB obtaining the following trajectories of position, speed and acceleration of the robot which are shown below.

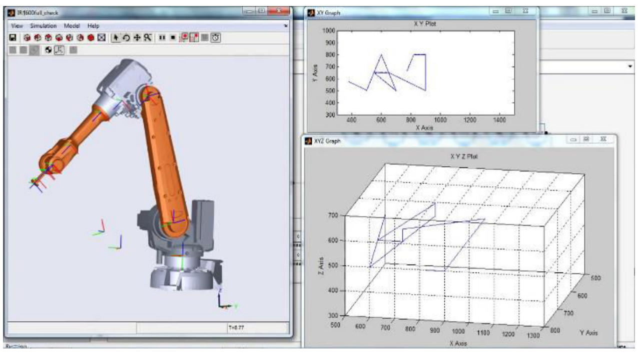

Figure-7. Simulink Simmechanics ABB IRB1600.

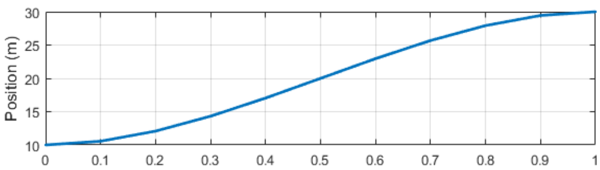

Figure-8. Position of the end effector of the industrial robot.

In Figure-9, we can observe the dynamical behavior of the robot. In it we can analyze that a change of position occurs from 10m to 30m in a time of 1 second. The position graph is curved, we can see that the slope is changing, which means that the speed is also changing. A changing speed implies acceleration. Then, the curvature in a graph means that the object is accelerating, that is, changing velocity, or in graphical terms, that its slope is changing.

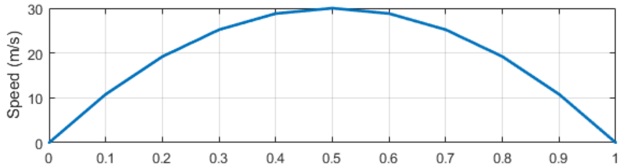

Figure-9. Speed of the end effector of the industrial robot.

The graph of the end effector speed of the industrial robot shows that it is concave down at the point t = 0.5 seconds reaching a value of 30m / s. We can observe that the function is monotone decreasing in the interval of time described previously. Given this we can infer that for this interval will have a negative slope. This fact is appreciable in the graph of acceleration (see Figure-11).

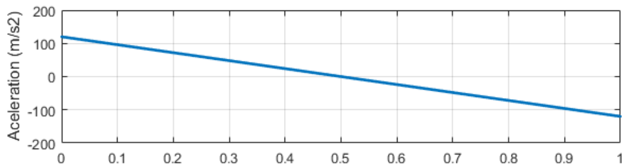

Figure-10. Acceleration of the end effector of the industrial robot.

Several changes are made in the trajectory of the final effector, demonstrating the effectiveness of the methodology applied for the development of reverse and direct kinetics of the robot IRB 1600. The following figure demonstrates the results obtained.

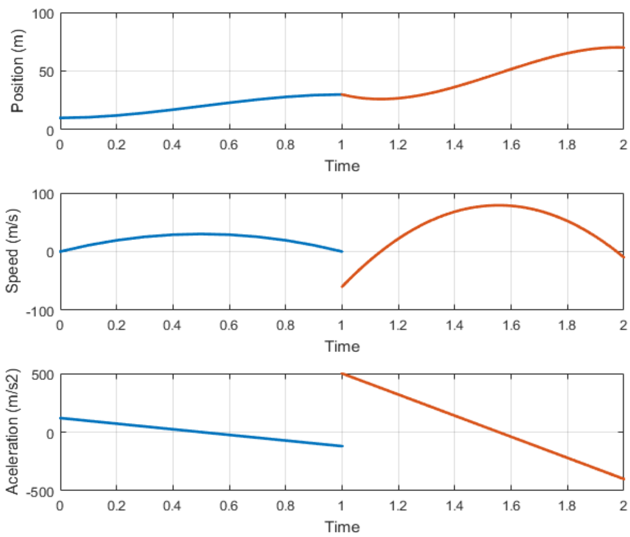

Figure-11. Adding different trajectories for the IRB 1600 robot and the results obtained in t = 2 seconds.

4. CONCLUSIONS

Within the methods of spatial location, we have the homogeneous transformation matrices which present the main advantage in that the position and the orientation are represented jointly in a very comfortable way. It presents serious inconveniences due to its high level of redundancy (6 D.O.F.) and therefore its high computational cost.

Using the method of the quaternions, a simpler and more efficient composition of rotations and translations is obtained. The main disadvantage is that it shows only the relative orientation.

The study of the mechanical systems can be done from the graphical interface of Simulink. Simmechanics allows you to work with block diagrams to simulate the movement of mechanical systems and measure the movement generated by mechanical actuation. It was determined by the simulation that the Robot IRB 1600 AAB has a range of 1.2 m and a speed in its parts of 180 ° / sec.

REFERENCES

[1] Ruthber Rodríguez Serrezuela, Adrián Fernando Chávarro Chavarro, Miguel Ángel Tovar Cardozo, Alejandro Leiva Toquicaand Luis Fernando Ortiz Martínez. 2017, April. Kinematic modelling of a robotic arm manipulator using Matlab.Journal of Engineering and Applied Sciences. 12(7): 2037-2045.

[2] Rojas J. H. C., Serrezuela R. R., López J. A. Q. & Perdomo K. L. R. 2016. LQR Hybrid Approach Control of a Robotic Arm Two Degrees of Freedom. International Journal of Applied Engineering Research. 11(17): 9221-9228.

[3] Serrezuela R. R., Chavarro A. F. C., Cardozo M. A. T. & Zarta J. B. R. 2016. An Optimal Control Based Approach to Dynamics Autonomous Vehicle. International Journal of Applied Engineering Research. 11(16): 8841-8847.

[4] Azhmyakov V., Rodriguez Serrezuela R., RiosGallardo A. M. & Gerardo Vargas W. 2014. An approximation based approach to optimal control of switched dynamic systems. Mathematical Problems in Engineering.

[5] Serrezuela R. R. & Chavarro A. F. C. 2016. Multivariable Control Alternatives for the Prototype Tower Distillation and Evaporation Plant. International Journal of Applied Engineering Research. 11(8): 6039-6043.

[6] Serrezuela R. R., Villar O. F., Zarta J. R. & Cuenca Y. H. 2016. The K-Exponential Matrix to solve systems of differential equations deformed. Global Journal of Pure and Applied Mathematics. 12(3): 1921-1945.

[7] Giraldo L. F., Delgado E. & Castellanos G. 2006. Cinemática inversa de un brazo robot utilizando algoritmos genéticos. Revista Avances en Sistemas e Informática. 3(1).

[8] Roberto M. T., Salvador C. R. J., Fernando H. R., Soto J. E. V. & Ortega J. C. P. 2005, November. Software de simulación de un robot manipulador marca mitsubishi rv-2aj. 4TO Congreso Nacional de Mecatrónica.

[9] Sánchez L. A. & Saavedra M. S. 2005. Matemáticas y robótica. Curso Interuniversitario “Sociedad, Ciencia, Tecnología Y Matemáticas.

[10] Morán O. D. & Monasterolo R. R. 2009. Enseñanza-Aprendizaje en Robótica: Construcción de Simuladores como Actividades de Comprensión. Formación universitaria. 2(4): 31-36.

[11] Navarro Narváez N. P. 2011. Modelado cinemático y dinámico de un manipulador de 5 grados de libertad articulado verticalmente.

[12] Azhmyakov V., Martinez J. C., Poznyak A. & Serrezuela R. R. 2015, July. Optimization of a class of nonlinear switched systems with fixed-levels control inputs. In American Control Conference (ACC), 2015 (pp. 1770-1775). IEEE.

[13] Azhmyakov V., Serrezuela R. R. & Trujillo L. G. 2014, October. Approximations based optimal control design for a class of switched dynamic systems. In Industrial Electronics Society, IECON 2014-40th Annual Conference of the IEEE (pp. 90-95). IEEE.

[14] Corke P. I. 1996. A robotics toolbox for MATLAB. IEEE Robotics& Automation Magazine. 3(1): 24-32.

[15] Yun X. & Bachmann E. R. 2006. Design, implementation, and experimental results of a quaternion-based Kalman filter for human body motion tracking. IEEE transactionson Robotics. 22(6): 1216-1227.

[16] Perez A. & McCarthy J. M. 2004. Dual quaternion synthesis of constrained robotic systems. Transactions-American Society of Mechanical

Engineers Journal of Mechanical Design. 126, 425-435.

[17] Bachmann E. R., Duman I., Usta U. Y., McGhee R. B., Yun X. P. & Zyda M. J. 1999. Orientation tracking for humans and robots using inertial sensors. In Computational Intelligence in Robotics and Automation, 1999. CIRA'99. Proceedings. 1999 IEEE International Symposium on (pp. 187-194). IEEE.

[18] Xian B., de Queiroz M. S., Dawson D. & Walker I. 2004. Task-space tracking control of robot manipulators via quaternion feedback. IEEE Transactions on Robotics and Automation. 20(1): 160-167.

[19] Adorno B. 2017. Robot Kinematic Modeling and Control Based on Dual Quaternion Algebra.

[20] Özgür E. & Mezouar Y. 2016. Kinematic modeling and control of a robot arm using unit dual quaternions. Robotics and Autonomous Systems. 77, 66-73.

[21] Farsoni S., Landi C. T., Ferraguti F., Secchi C. & Bonfè M. 2017. Compensation of Load Dynamics for Admittance Controlled Interactive Industrial Robots Using a Quaternion-Based Kalman Filter. IEEE Robotics and Automation Letters, 2(2), 672-679.

[22] Abaunza H., Castillo P., Victorino A. & Lozano R. 2017. Dual Quaternion Modeling and Control of a Quad-rotor Aerial Manipulator. Journal of Intelligent & Robotic Systems. 1-17.

[23] Yang X., Wu H., Li Y. & Chen B. 2017. A dual quaternion solution to the forward kinematics of a class of six-DOF parallel robots with full or reductant actuation. Mechanism and Machine Theory. 107, 27-36.

[24] Klafke G., Webster A., Agnol B. D., Pradel E., Silva J., de La Canal L. H., ... & Scheffer R. 2017. Multiple resistances to acaricides in field populations of Rhipicephalusmicroplus from Rio Grande do Sul state, Southern Brazil. Ticks and tick-borne diseases. 8(1): 73-80.