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COMPARISON OF NUMERICAL TECHNIQUES IN SOLVING TRANSIENT ANALYSIS OF ELECTRICAL CIRCUITS

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ABSTRACT

In this paper, few numerical methods are proposed to solve transient analysis of electrical circuits namely Euler's method, Heun's method and Runge-Kutta method. In order to solve transient analysis, numerical methods for utilization in the companion of analytical method of transient circuit analysis are used to solve second-order differential equations which generated from circuit equations of a RLC circuit. The fourth-order Runge-Kutta method is found out the best numerical technique to solve the transient analysis due to its high accuracy of approximations.

Keywords: transient analysis, RLC circuit, euler method, heun's method, fourth-order runge-kutta method.

1. INTRODUCTION

Transient is a sudden application of source to a circuit or a brief increase in current or voltage in a circuit which cause damage to some sensitive components and instruments [2]. Transient analysis is an analysis of transient response of an electrical circuit which is occurred due to switching [3]. The main reason is transient analysis is applied to ensure the normal operation of high speed electronic information system [4]. Transient analysis of transmission line becomes more significant as the connection line of integrated circuit. It is important for an electrical circuit because it is commonly used to analyze the performance of the circuit.

There are many applications demand the result of transient response for systems such as non-uniform transmission line and Lightning Activity Monitoring System (LAMS) [5] due to the lightning event. The importance of transient analysis is to determine the performance and stability of a system.

Electrical circuit is defined as the interconnection of electric elements or electrical devices (Charles K. Alexander, 2009) and can be written mathematically using differential equations of first, second and upper order. Second-order circuit consists of resistors and the equivalent of two storage elements which are capacitor and inductor. RLC circuit is a typical example of second-order circuits which the three types of passive elements are present in the circuit. There are multiple applications for RLC circuit and some of the most important applications are oscillator and turners of radio or audio receiver. Analytical solutions in transient analysis are derived by analyzing the circuit using circuit equations and a transient response complete solution is formed. However, analytical solution of such RLC circuits in complex networks is very difficult and lengthy. Therefore, numerical methods offer a great solution to the system.

Table-1 shows the comparison over the three numerical methods to solve the RLC second order differential equations namely Euler's method, Heun method and Runge-Kutta method. Euler method is commonly used in particle dynamics simulation. It is the simplest integration method among the three methods. It has fast computational simulation but low degree of accuracy. Heun's method is applied on molecular dynamics simulation and computation of power system dynamics. This method is numerically more stable than Euler method and able to obtain accurate simulation results. However, the results obtained from Heun's method are less precise than Runge-Kutta method. Runge-Kutta method is utilised in many applications although it has higher computational time.

Table-1. Comparison over numerical methods.

Methods	Euler method	Heun's method	Runge-Kutta method
Application	• Particle dynamics simulation [6]	• Molecular dynamics simulation[7] • Computation of power system dynamics [8]	• Transient flow ideal gas [9] • Time-domain simulation of non-linear circuits [10]
Advantages	• Simplest integration method • Fast computational time	• High numerical stability • Accurate simulation results	• High accuracy • Stable • Reliable
Disadvantages	• Only for smooth particle moving • Low degree of accuracy	• More computation than EM method • Less precise compared to RK method	• Expensive to implement • Higher computational time

In this research, the three numerical methods are proposed to solve the transient analysis of electrical circuit. Comparison of which numerical methods offer the highest approximation to the analytical solution is done.

2. THE DERIVATION OF SECOND-ORDER CIRCUITS USING DIFFERENTIAL EQUATIONS

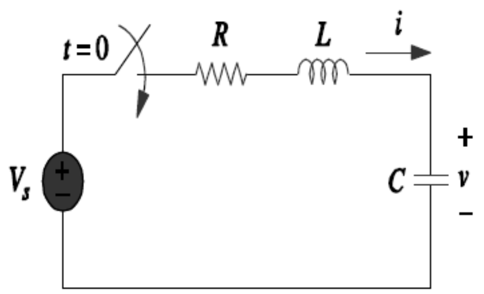

Figure-1. Series RLC circuit.

The differential equations of the RLC in Figure-1 is based on method of loop currents where the fundamental relationship between the current, $i(t)$and the individual circuit elements (Capacity, Inductance, Resistance) are given by;

Resistance: $v(t) = R\, i(t)$
Capacity: $i(t) = C\frac{dv}{dt}$ (1)
Inductance: $v(t) = L\frac{di}{dt}$

where
i = Current supplied by a capacitor
C = Capacitance of the capacitor
v = Voltage across the capacitor
t = A period of time

The Kirchhoff's Voltage Law (KVL) is applied around the loop for t>0 in which the voltage source of the circuit in Figure-1 is calculated by the following;

$$V_s = V_R + V_L + V_C \quad (2)$$

where
V_s = Voltage source of RLC circuit
V_R = Voltage across resistor
V_L = Voltage across inductor
V_C = Voltage across capacitor

The second-order differential equation of the RLC circuit with constant coefficients is generated from equation (1), which reflects the voltage loop circuit can be written as follows;

$$L\frac{d^2v}{dt^2} + R\frac{dv}{dt} + \frac{1}{C}v = \frac{V_s}{C} \quad (3)$$

where
v = Voltage
R = Resistance of resistor
L = Inductance of inductor
C = Capacitance of capacitor
V_s = Voltage source of RLC circuit

The RLC will be analyzed in order to determine its transient characteristics once the switch is closed. Equation (3) is solved using three numerical methods namely Euler's method, Heun method and Runge-Kutta method.

3. NUMERICAL METHODS

Numerical methods for ordinary differential equations are techniques used to find numerical approximations to solve ordinary differential equations. In this research, numerical methods for ordinary differential equations are utilised to solve the second-order differential equations that generated from the RLC circuit which shown as equation (3). The numerical methods used are Euler method, Heun's method and Fourth-order Runge-Kutta method.

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A. Formulation of Euler method

Euler method for second-order differential equation is written as [11, 12]:

$$y_{i+1} = y_i + hf(x_i, y_i, z_i) \tag{4}$$

$$z_{i+1} = z_i + hg(x_i, y_i, z_i) \tag{5}$$

where

i = 0, 1, 2 … n

h = Step size

B. Formulation of Heun's method

The adaptive Heun's method equation for solving second-order ordinary differential equation is shown as following [11, 12]:

$$y_{i+1} = y_i + \frac{1}{2}h(k_1 + k_2) \tag{6}$$

$$z_{i+1} = z_i + \frac{1}{2}h(l_1 + l_2) \tag{7}$$

where

$$k_1 = f(x_i, y_i, z_i) \tag{8}$$

$$l_1 = g(x_i, y_i, z_i) \tag{9}$$

$$k_2 = f(x_i + h, y_i + k_1 h, z_i + l_1 h) \tag{10}$$

$$l_2 = g(x_i + h, y_i + k_1 h, z_i + l_1 h) \tag{11}$$

C. Formulation of Runge-Kutta method

The adaptive Runge-Kutta method equation for solving second-order ordinary differential equation is shown as following [11, 12]:

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \tag{12}$$

$$z_{i+1} = z_i + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4) \tag{13}$$

where

$$k_1 = f(x_i, y_i, z_i) \tag{14}$$

$$l_1 = g(x_i, y_i, z_i) \tag{15}$$

$$k_2 = f(x_i + \tfrac{h}{2}, y_i + \tfrac{k_1 h}{2}, z_i + \tfrac{l_1 h}{2}) \tag{16}$$

$$l_2 = g(x_i + \tfrac{h}{2}, y_i + \tfrac{k_1 h}{2}, z_i + \tfrac{l_1 h}{2}) \tag{17}$$

$$k_3 = f(x_i + \tfrac{h}{2}, y_i + \tfrac{k_2 h}{2}, z_i + \tfrac{l_2 h}{2}) \tag{18}$$

$$l_3 = g(x_i + \tfrac{h}{2}, y_i + \tfrac{k_2 h}{2}, z_i + \tfrac{l_2 h}{2}) \tag{19}$$

$$k_4 = f(x_i + h, y_i + k_3 h, z_i + l_3 h) \tag{20}$$

$$l_4 = g(x_i + h, y_i + k_3 h, z_i + l_3 h) \tag{21}$$

4. NUMERICAL COMPARISON AND DISCUSSIONS

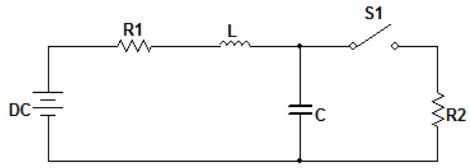

Figure-2. Series RLC circuit.

In this research, a series RLC circuit is set up as shown in Figure-2. Different types of transient analysis of RLC circuit would be conducted to study the nature of numerical methods on different conditions. The conditions mentioned are referred to underdamped response($R^2 - 4LC < 0$, critically damped response($R^2 - 4LC = 0$), and overdamped response($R^2 - 4LC > 0$). The total resistance is calculated by

$$R = R1 + R2 \tag{22}$$

The analytical solution to equation (3) is given by

$$v(t) = V_s + (A_1 + A_2 t)\exp\left(-\frac{R}{2L}t\right) \tag{23}$$

where A_1 and A_2 can be determined by applying the initial conditions.

The main purpose of this research is to conduct numerical simulations of equation (3) and compare the simulation results to the analytical solutions (23) in order to determine the accuracy of each numerical method.

A. Comparison between analytical solution and numerical methods

In order to obtain different types of response, the experiment is carried out with three different conditions. In each condition, the parameter of resistor R1 varies meanwhile the parameters of capacitor, C and inductor, L are fixed. The values of electrical elements are shown in Table-2.

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Table-2. The values of the electrical elements of series RLC circuit for three conditions.

Element	Value		
	Condition 1	**Condition 2**	**Condition 3**
Types of Response	Underdamped	Critically damped	Overdamped
DC voltage source, V_s	12 V	12 V	12 V
Resistor, $R1$	$< 4\ \Omega$	$4\ \Omega$	$> 4\ \Omega$
Resistor, $R2$	$1\ \Omega$	$1\ \Omega$	$1\ \Omega$
Inductor, L	1 H	1 H	1 H
Capacitor, C	0.25 F	0.25 F	0.25 F

Since the accuracy of the numerical methods will be affected by the time step size, the time step size will be constant for all the numerical methods used. In this experiment, the simulation time for all the conditions are from 0s to 20s with the step size of h= 0.2s. In order to compare the accuracy of the numerical methods, the computed points of the three numerical methods are taken at the specific time. The computed point of each method is compared with the analytic solution. Besides that, the maximum error, minimum error and average error are taken into account as another method to determine the accuracy of numerical methods.

Condition 1: Underdamped response$(R^2 - 4LC < 0)$

In condition 1, the resistance, $R1$ must be less than 4Ω in order to achieve underdamped response. In this part, the experiment is conducted by considering 1Ω as the value of resistor, $R1$ for the circuit. In order to obtain the best numerical method for transient analysis of RLC circuit, three numerical methods are compared with the analytical solution. The three numerical methods mentioned are Euler method, Heun's method and Fourth-order Runge-Kutta method (RK-4). The initial voltage and current for the RLC circuit is $v(0) = 6V$ and $i(0) = 6A$ respectively.

Figure-3 shows the transient analysis of RLC circuit using analytical solution, Euler method, Heun's method and RK-4 method where the resistance, $R1$ is 1Ω, inductance, L is 1H and capacitance, C is 0.25F. The minimum error, maximum error and average error between analytical solution and each numerical method are calculated and shown in Table-3.

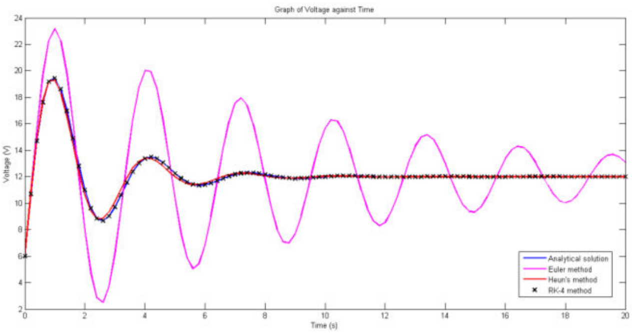

Figure-3. Transient analysis of RLC circuit using analytical solution, Euler method, Heun's method and RK-4 method ($R1$=1Ω, L=1H, C=0.25F).

Table-3. The minimum error, maximum error and average error between analytical solution and numerical methods (R1=1Ω, L=1H, C=0.25F).

Error	Numerical methods		
	Euler method	**Heun's method**	**RK-4 method**
Minimum	0.00%	0.00%	0.00%
Maximum	71.02%	4.27%	0.04%
Average	21.80%	0.69%	0.01%

Based on Table-3, the minimum errors for all three numerical techniques are 0.00% which indicated that there is no difference between the computed points of analytical solution and all the three numerical techniques. The minimum errors for all numerical methods are 0.00% because of the initial point of the entire line graphs are on the same point which is 6V. The maximum error of transient analysis using Heun's method is 4.27% and the average error is 0.69%. By using RK-4 method in solving transient analysis of electric circuit, the maximum error is 0.04% and the average error is 0.01%. As seen from Figure-4.1, Euler method has the highest maximum error and average error which are about 71.02% and 21.80% respectively.

According to Figure-3, it is clearly shown that when a computed point is taken at 1s from each line graph,

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Euler method has the worst accuracy of numerical simulation on the transient analysis of RLC circuit which is about 80.78%. The accuracy of the numerical simulation can be improved by using other numerical methods which are Heun's method and RK-4 method. By using Heun's method, the error between analytical solution and numerical method is reduced about 18.47%. The error between analytical solution and numerical method is reduced the most when using RK-4 method which the reduction of error is about 19.21%. Hence, the RK-4 method is selected for solving the other conditions.

Condition 2: Critically damped response($R^2 - 4LC = 0$)

In critically damped response, the resistance, $R1$ must be equal to 4Ω in order to fulfill the condition. The experiment is conducted by taking 4Ω as the value of resistor, $R1$ for the circuit. In this part, results of RK-4 method are compared with the analytical solution. The initial voltage and current for the RLC circuit is 2.4V and 2.4A respectively.

Figure-4 shows the transient analysis of RLC circuit using analytical solution and RK-4 method where the resistance, $R1$ is 4Ω, inductance, L is 1H and capacitance, C is 0.25F. The minimum error, maximum error and average error between analytical solution and RK-4 method are shown in Table-4.

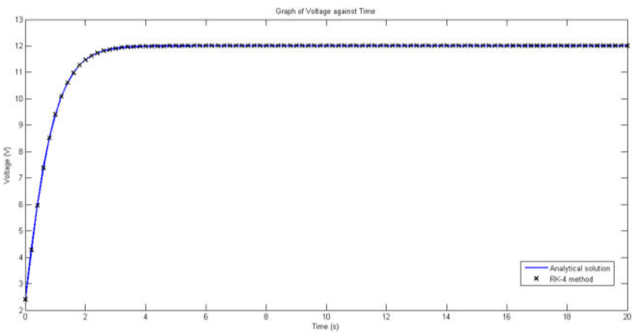

Figure-4. Transient analysis of RLC circuit using analytical solution and RK-4 method ($R1$=4Ω, L=1H, C=0.25F).

Table-4. The minimum error, maximum error and average error between analytical solution and RK-4 method ($R1$=4Ω, L=1H, C=0.25F).

Error	Numerical methods
	RK-4 Method
Minimum	0.00%
Maximum	0.03%
Average	0.00%

By using Fourth-order Runge-Kutta method as a numerical simulation on transient analysis of electrical circuit, the maximum error obtained is about 0.03% and the average error is about 0.00% (see Table 4). The RK-4 method is considered as a high accuracy numerical method which the accuracy is up to 99.97% in this critically damped case.

Condition 3: Overdamped response($R^2 - 4LC > 0$),

In condition 3, the resistance, $R1$ must be more than 4Ω in order to achieve overdamped response. The experiment is conducted by taking 10Ω as the value of resistor, $R1$ for the circuit. In this part, results of RK-4 method are compared with the analytical solution. The initial voltage and current for the RLC circuit is 1.0909V and 1.0909A respectively.

Figure-5 shows the transient analysis of RLC circuit using analytical solution and RK-4 method where the resistance, $R1$ is 10Ω, inductance, L is 1H and capacitance, C is 0.25F. The minimum error, maximum error and average error between analytical solution and RK-4 method are shown in Table-5.

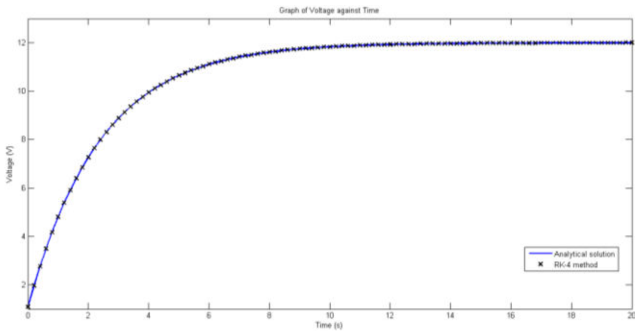

Figure-5. Transient analysis of RLC circuit using analytical solution and RK-4 method ($R1$=10Ω, L=1H, C=0.25F).

Table-5. The minimum error, maximum error and average error between analytical solution and RK-4 method ($R1$=10Ω, L=1H, C=0.25F).

Error	Numerical methods
	RK-4 Method
Minimum	0.00%
Maximum	0.17%
Average	0.00%

By using RK-4 method as a numerical simulation on transient analysis of electrical circuit, the maximum error and the average error obtained are about 0.17% and 0.00% respectively (see table 5). The accuracy of RK-4 in solving this condition is nearly 99.83% for this case.

B. Comparison of RK-4 method with different step size, h

In order to determine the relationship between Fourth-order Runge-Kutta method and step size, the experiment is carried out using Fourth-order Runge-Kutta

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method with different step sizes, h. In this part, the RLC circuit with underdamped response is utilised. The values of electrical elements are shown in Table-6. The initial voltage and initial current of the RLC circuit are 6V and 6A respectively.

Table-6. The values of the electrical elements of series RLC circuit.

Element	Value
DC voltage source	12 V
Resistor, $R1$	1 Ω
Resistor, $R2$	1 Ω
Inductor, L	1 H
Capacitor, C	0.25 F

In this part, the experiment is repeated three times by considering 0.5, 0.2 and 0.1 as the time step size, h. The total time for the transient analysis of RLC circuit is set to 20s. Since number of steps, n is used in MATLAB numerical code, the time step size is changed into the form of number of steps, n while doing the experiment. The number of steps for time step size of 0.5, 0.2 and 0.1 are 40, 100 and 200 respectively.

Transient analysis of RLC circuit using analytical solution and RK-4 method with step size of 0.5, 0.2 and 0.1 are shown in Figure-4.7, Figure-4.8 and Figure-4.9 correspondingly. Table-4.12 shows the minimum error, maximum error and average error between analytical solution and Fourth-order Runge-Kutta method with step size of 0.5, 0.2 and 0.1.

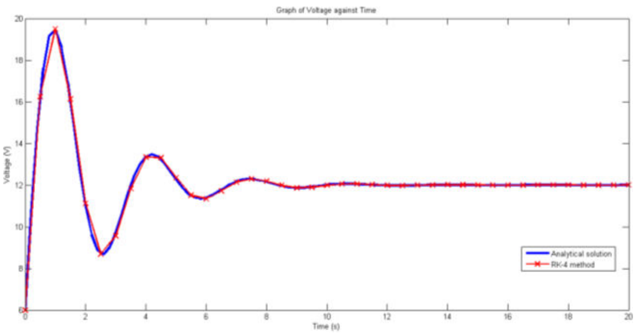

Figure-6. Transient analysis of RLC circuit using analytical solution and RK-4 method (time step size, h=0.5 or number of steps, n=40).

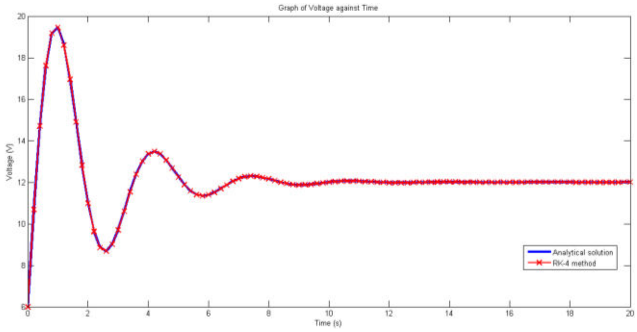

Figure-7. Transient analysis of RLC circuit using analytical solution and RK-4 method (time step size, h=0.2 or number of steps, n=100).

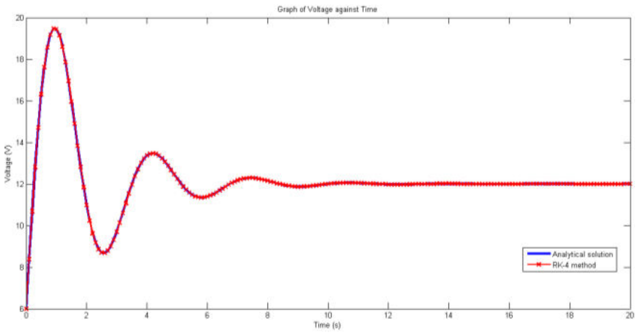

Figure-8. Transient analysis of RLC circuit using analytical solution and RK-4 method (time step size, h=0.1 or number of steps, n=200).

Table-7. The minimum error, maximum error and average error between analytical solution and numerical methods ($R1$=1Ω, L=1H, C=0.25F).

Error	Time step size, h		
	0.5	**0.2**	**0.1**
Minimum	0.00%	0.00%	0.00%
Maximum	1.50%	0.04%	0.00%
Average	0.25%	0.01%	0.00%

According to Table-7, the minimum errors for all three step sizes are 0.00% which indicated that there is no difference between the computed points of analytical solution and all the three numerical techniques. The maximum error of transient analysis using Fourth-order Runge-Kutta method with step size of 0.5 is 1.50% and the average error is 0.25%. By using step size of 0.2 in RK-4 method, the maximum error is 0.04% and the average error is 0.01%. As seen from Figure-8 and Table-7, Fourth-order Runge-Kutta method with step size of 0.1 has the lowest maximum.

As reported in results from Figure-6, Figure-7 and Figure-8, it is clearly shown that when a computed point is taken at 1s from each figure, step size of 0.5 has the worst accuracy of numerical simulation on the transient analysis of RLC circuit which is about 99.76%.

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The accuracy of the numerical simulation can be improved by decreasing the time step size or increasing the number of steps. In this research, the time step size is decreased to *h*=0.2 and *h*=0.1 in order to improve the accuracy of RK-4 method. When the time step size is decreased to *h*=0.2, the error between analytical solution and RK-4 method is reduced about 0.23%. The accuracy of numerical simulation on the transient analysis of the RLC circuit has the most improvement which is approximately 0.24% when the time step size is decreased to 0.1. As the number of steps increases or the time step size decreases, the accuracy of the numerical simulation on transient analysis using RK-4 method increases.

5. CONCLUSION AND RECOMMENDATION

The transient analysis of electrical circuit is analysed using analytical method and RK-4 method. With the usage of MATLAB, the process of obtaining results of transient analysis can be done systematically and convenient. The transient analysis is only done for series RLC circuit in three conditions which are underdamped response, critically damped response and overdamped response. In the first numerical simulation, three numerical methods are used to run the numerical simulation and the results are compared with the analytical solution. Throughout the numerical simulations, RK-4 method has the highest degree of accuracy which is up to 99.97%. In the second numerical simulation, transient analysis of RLC circuit is analysed using RK-4 with different time step size, *h*. In this simulation, the time step size is decreased gradually from 0.5 to 0.1 in order to determine the relationship of the accuracy of RK-4 method and time step size. By using step size of 0.1 in RK-4 method, the accuracy of this method is up to 100%. As the step size decimates, the absolute error also gets decimated. In other word, the accuracy of RK-4 method can be improved by decreasing the step size, h or increasing the number of steps, n. As a result, RK-4 method is a suitable method for solving transient analysis of electric circuit due to its high degree of accuracy and efficiency in solving second-order differential equations.

Based on the analysis above, RK-4 method would be the best numerical technique to solve a higher order of ordinary differential equations such as third-order differential equations or fourth-order differential equations. Besides, transient analysis could be conducted on more complex design electrical circuits.

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