



# NARROW BAND INTERFERENCE DETECTION IN OFDM SYSTEM USING COMPRESSED SENSING

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## ABSTRACT

Narrow band interference (NBI) is inevitable in multicarrier communications. The source of NBI may be intentional or non intentional. The performance of the receiver is degraded if the NBI is treated as noise. The bandwidth of the NBI or the number of carriers that are jammed by the interfering sources is very less compared to the total bandwidth of interest of the receiver. Such sparseness enables the application of sparse signal processing principles in the detection and cancellation of the narrow band interference. In this paper, we consider an orthogonal frequency division multiplexed (OFDM) based wireless system and few of its subcarriers are jammed or affected by the narrow band interference. We propose the techniques for cancellation of the NBI interference from the received signal based on compressive sensing (CS) framework and compare its performance with conventional techniques and as well as with the case where the NBI is treated as noise.

**Keywords:** interference detection, narrow band interference, OFDM,

## 1. INTRODUCTION

Orthogonal frequency division multiplexing is used in many wireless and wireline communication systems and is the main cause of narrow band interference. For example, UWB system faces interference in operating BW due to the other licensed system operating in same band. It generally happens in multiband OFDM ultra wide systems due to non intentional NBI. The major sources of NBI interference in wireless LANs are from the Bluetooth devices that are operating with the same frequency spectrum. Some of the wired systems including digital subscriber lines (DSL) and power line communications are affected by the impulse like pulses which cause radio frequency interference in the desired systems. There are cases of intentional narrow band interference creates hazardous the wireless nodes or networks used in several applications. In literature, the cancellation of the narrow band interference is not analyzed thoroughly. There have been few techniques been proposed so far. In one of the approaches proposed in [2] the color of the noise is removed by means of a prediction error filter which makes the narrow band interference flat in the band of interference. The length of PEF should be very long to whiten the NBI, in order to achieve so we have assumed it to be auto regressive process. The work done in [3] uses PEF to erase the affected tones with the other tones remain untouched using a error insertion mechanism. The approach used in this paper can be used for any spectral width. In the works done in [2] and [3] it is assumed that the narrow band interference creates only at isolated single frequencies. The work done in [4] based on the assumption that the first subcarrier is not jammed and to predict and the first subcarrier signal is received and detected which helps to decode the signals and error in the subsequent subcarriers. The major drawback in this approach is the propagation of interference estimate to all

the subsequent tones, with the help of decoding unit this approach has been generalized by means of introducing soft decision decoding techniques applied to orthogonal frequency division multiplexed transmissions. The algorithms proposed in this paper are based compressive sensing (CS) framework. A brief description of the CS background is provided in Section III and as well as in Section IV. The two algorithms proposed in this paper are based on compressed sensing theory and the review of which is presented in next section and the algorithm in [6], [7]. The two algorithms are derived in the section III and IV. In Sections V and VI, the simulation results and conclusion are presented

## 2. SYSTEM MODEL

### 2.1 Overview of compressive sensing theory

Compressive sensing theory asserts that it is possible to find unique solution from the underdetermined system of linear equations provided the certain conditions on the unknown vectors are satisfied. CS theory discussion in [6], [7] gives an idea of how to restore an unknown vector which is sparse in nature from a system of underdetermined linear measurements where the number of measurements is far less than the dimension of the unknown vector i.e.,  $M \ll N$ . The term "sparse" means that the number of non zero values of the vector is far less than its size. In some applications the unknown vector may approximately sparse, meaning, the significant values are far less than the actual dimension of the vector. The following optimization function can be used to recover the sparse vector which is based on  $l_1$ -norm minimization:

$$\min_{\tilde{x} \in \mathbb{C}^N} \|\tilde{x}\|_1 \text{ Subject to } B\tilde{x} = y \rightarrow \quad (1)$$



and  $\|\tilde{x}\|_1 \triangleq \sum_i |x_i|$  stands for the  $l_1$ -norm i.e., the sum of absolute values of the vector of  $x$  and  $H$  denotes the  $M \times N$  measurement matrix. The noise measurement is considered in the problem statement as follows:

$$y = Bx + w \rightarrow \quad (2)$$

where  $Z \in \mathbb{C}^M$  accounts for the corruption in the observation. Convex optimization program [8] is used to recover  $x$  in the noisy case.

$$\min_{\tilde{x} \in \mathbb{C}^N} \|\tilde{x}\|_1 \text{ subject to } \|y - B\tilde{x}\|_2 \leq \epsilon \rightarrow \quad (3)$$

where  $\|\cdot\|_2$  stands for the  $L_2$  norm, and the parameter  $\epsilon$  accounts for the effect of noise in the observation or corruption in the measurements. The solution of the above convex optimization program provides the non zero indices of the unknown vector and the estimate of their values. Now the unknown vector can be written as, by solving the objective function problem defined in (3) by which the non zero values of  $x$  are estimated. Once the indices corresponding to non zero values of the unknown vector is found, it can be interpreted as

$$x = Pl \rightarrow \quad (4)$$

where the vector  $l$  corresponds to the values of the significant indices of  $x$ . Moreover the matrix  $P$  is referred to as the *selection* matrix which has a single '1' in each column and rest all entries are zeros, and the numbers of the rows which has a '1' in at least one column is estimated and stored in index vector  $l$ .

$$y = BPl + z \rightarrow \quad (5)$$

Now the unknown index vector  $l$  can be recovered using least squares method as:

$$\tilde{l} = (P^H B^H B P)^{-1} P^H B^H y \rightarrow \quad (6)$$

## 2.2 Problem statement

The time domain received signal in case of orthogonal frequency division multiplexing systems becomes

$$y = Bx + p + s \rightarrow \quad (7)$$

where  $B$  is channel matrix,  $X$  is transmitted signal,  $P$  is NBI signal,  $S$  is zero-mean complex additive white Gaussian noise vector. The observation vector can be written in terms of the frequency domain representation of the narrow band interference, data signal and the transformed noise vector as:

$$y = BF^H X + F^H P + F^H S \rightarrow \quad (8)$$

with  $F$  denoting the discrete Fourier transformation of the data vector  $X$  matrix. The entries of  $J$ , can be represented as

$$p_k = \begin{cases} a_k, & f_l \leq k \leq f_h \\ 0, & \text{otherwise} \end{cases} \rightarrow \quad (9)$$

The objective is to estimate the indices of the subcarrier frequencies corresponding to the jamming signal amplitudes and their corresponding frequencies. The amplitude  $a_k$  denotes the  $k^{th}$  frequency jammer amplitude. The jammer width is denoted as  $f_h - f_l$  where the  $f_h, f_l$  denote the upper and lower frequencies of the jammer bandwidth. The objective is to detect the jammer frequencies and their corresponding amplitudes at those frequencies. It can be noted that the affected jammer bandwidth is quite narrow compared to the wideband signal of interest and hence the problem contains inherent sparsity in its nature. Though it is straightforward to treat the narrowband interference signal as noise, it will severely affect the performance of the receivers which makes the cancellation of the narrowband interference crucial.

Let us assume that the jammer bandwidth  $B$  is known at the receiver and let us define a pre multiplication matrix  $W$  in such a way that the product  $WB$  becomes null as follows:

$$\tilde{y} = By = WF^H P + Bs = BJ + \tilde{z} \rightarrow \quad (10)$$

where  $B = WF^H$  is measurement matrix and  $\tilde{z} = Bs$ . It is evident that the frequencies corresponding to jammer signals and their amplitudes can be estimated using least squares approach. Moreover, the rows of the premultiplication matrix should lie in the orthogonal complement of the column space of the jammer matrix to make  $WB = 0$ . Now the problem deduces to design the matrix  $W$  for a given receiver signal model. In the subsequent sections, we propose the design algorithms for two kinds of receiver signal models in different applications.

## 3. ZERO-PREFIXED OFDM SYSTEMS

### 3.1 NBI Cancellation

The received signal model in zero prefixed OFDM systems can be written as

$$y_{N+V} = B_{(N+V)} \times N^X N + F_{N+V}^H P_{N+V} + R_{N+V} \rightarrow \quad (11)$$

with the total number of subcarriers of the OFDM system being  $N$  and the guard length to avoid inter symbol interference is denoted by  $v$ . In order to find a suitable null space to design the matrix  $W$  the jammer matrix  $B$  should be tall. It should be noted the addition of the zero prefix makes the desired structure in the channel matrix. The channel matrix has the form  $B = \begin{bmatrix} B_o \\ 0_{(V-L+1) \times N} \end{bmatrix}$  with the



number of taps satisfies the condition  $L \leq v + 1$ , and the matrix  $B_0$  holds a Toeplitz structure of size  $N$ . By setting the NBI cancellation matrix as  $W = [W_0 N]$  we see that the product  $W_0 B_0 = 0$ , which leaves the choice of  $N$  can arbitrary. Thus the matrix  $W_0$  can be the matrix which acts like a projection operator onto the orthogonal complement of the column space of  $B$  as follows:

$$I_{N+W-1} - B_0 (B_0^H B_0)^{-1} B_0^H \rightarrow \quad (12)$$

It can be easily verified that the product  $W_0 B_0 = 0$ . As the value of  $N$  can be chosen with a choice that does not affect the design of the NBI cancellation matrix, the estimation and cancellation of the jammer frequency components is robust in this method.

Let us decompose the matrix  $B_0$  using QR factorization such that  $B_0 = QR$ , where  $Q = [Q_1 Q_2]$  and  $R = \begin{bmatrix} R_1 \\ 0_{(W-1) \times N} \end{bmatrix}$ . In this representation the matrices  $Q_1$  and  $Q_2$  are unitary with their sizes being  $(N+W-1) \times (N)$  and  $(N+W-1) \times (W-1)$ , respectively and whereas the matrix  $R_1$  is upper triangular of size  $N \times N$ . Now the matrix,  $W_0$  can be written as

$$W_0 = I_{N+W-1} - QR(R^H Q^H QR)^{-1} R^H Q^H \rightarrow \quad (13)$$

by noting that the  $Q^H Q = I_{N+W-1}$ , and  $R^H R = R_1^H R_1$  and by simple algebraic manipulations reveal that

$$W_0 = I_{N+W-1} - Q_1 Q_1^H \rightarrow \quad (14)$$

$Q_1$  can be formulated efficiently [13] based on the structure of  $B_0$ . The steps of NBI cancellation using compressive sensing framework can be summarized as given in the following steps:

- Design a premultiplication matrix whose columns are in the left null space of the channel matrix corresponding to desired signal.
- Multiplying the received signal with the above said matrix will force the data vector to zero.
- Detect the frequencies corresponding to jammer signal.
- Design an equalizer to estimate and cancel the narrow band interference signal.

### 3.2 Design of equalizers

As the narrow band interference is decoded and removed from the observation signal, the task is to detect the data signal vector. This can be accomplished using minimum means square error receiver to minimize the squared error in the detection. The equalizer matrix is designed based on the assumption that the residual signal after the NBI cancellation and the transformed color

noise are independent. Thus the equalizer matrix can be written as:

$$G = R_{XX} F_N B^H \cdot [B F_N^H R_{XX} F_N B^H + R_{ZZ} + F_{N+V}^H R_{J_{res} J_{res}} F_{N+V}]^{-1} \rightarrow \quad (20)$$

where  $R_{ZZ} \triangleq E[zz^H] = N_0 I_{N+v}$ ,  $R_{J_{res} J_{res}} \triangleq E[J_{res} J_{res}^H]$ . Assuming that there is no correlation between transmitted data, and have the same average energy  $\epsilon_X$ , then  $R_{XX} = \epsilon_X I_N$ . It should be noted that the design choice of  $G$  using this method necessitates the matrix inversion operation of size  $(N+v)$  and hence involves intense computational complexity. Therefore it is proposed to apply the alternate equalizers which follows as:

$$G_t = \epsilon_X V^H D^H \left[ \frac{N \epsilon_X}{N+v} D D^H + R_{ZZ} + R_{J_{res} J_{res}} \right]^{-1} F_{N+V} \rightarrow \quad (21)$$

Where  $V = F_{N+V} [F_N \quad O_{N \times v}]^H$ . The matrix  $D$  contains the entries of the  $(N+v)$ -point DFT computation. Though, the term  $R_{J_{res} J_{res}}$  brings more computational problem in the matrix inversion as given above it can be circumvented by noting the structure of the matrix  $R_{J_{res} J_{res}}$  which can be expressed as

$$R_{J_{res} J_{res}} = S E \left[ (d - \hat{d})(d - \hat{d})^H \right] S^H \rightarrow \quad (22)$$

The structure of the matrix  $S$  forces the matrix  $R_{J_{res} J_{res}}$  to contain the non zero elements only at the sub matrix and all other entries are zero. The non zero values of the of are  $R_{J_{res} J_{res}}$  are equal to the main diagonal elements of the matrix fall along the main diagonal of  $R_{J_{res} J_{res}}$ . The problem can be solved by two methods.

In the first method, it is proposed to keep only the diagonal elements of the matrix and to discard the other entries i.e., the off diagonal values of the matrix  $R_{J_{res} J_{res}}$ , whereas in the second method it is considered to apply the equalizers as follows. The carriers that correspond to jammer frequencies are equalized with more number of taps than the carriers which are not affected with single tap equalizers. The multi tap equalizers can be designed with low complexity by minimizing the number of taps in that by exploiting the fact that the narrow band interference contains only less number of significant taps. For the multi tap equalizers with less number of taps and with the structure of  $R_{J_{res} J_{res}}$ , the error in the equalized signal can be written as:

$$d - \hat{d} = - \underbrace{(S^H B^H C^{-1} B S)^{-1} S^H B^H C^{-1} W z}_{t=k} = K z \rightarrow \quad (23)$$

Hence,

$$R_{J_{res} J_{res}} = S K E[zz^H] K^H S^H = N_0 S K K^H S^H \rightarrow \quad (24)$$



Recalling

$C = N_o W W^H$ , we can simplify  $KK^H$  as follows,

$$KK^H = \frac{1}{N_o} (S^H A^H C^{-1} B S)^{-1}$$

that

#### 4. SIMO-BASED APPROACH-

The another method to make the channel matrix  $B$  to be tall i.e., it has more number of rows than its columns is accomplished by single input multi output system model as follows. In order to make the discussion simple and elegant, we consider the case of the receive equipped with two antennas. We define the channel from the transmitter and to the two receivers as  $B_1$  and  $B_2$  and each of the channel matrices  $B_i$  has the form  $B_i = \begin{cases} B_{io} \\ O_{(v-W+1) \times N} \end{cases}$  for  $i = 1, 2$ . Thus we rewrite the received signal by horizontally concatenating the received vectors in the received antenna 1 and 2 as:

$$\begin{aligned} Y_{2(N+v)} &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} X_N + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} p \\ p \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_H X_N + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} F_{N+v}^H \\ F_{N+v}^H \end{bmatrix} \rightarrow \end{aligned} \quad (26)$$

and it is assumed that the two receive antennas confront the equal interference and the additive noise statistics remain the same in both of these receiver antennas.

$$W_{(N+L-1) \times 2(N+v)} = [I_{N+L-1} N_1 - B_{1o} B_{2o}^+ N_2] \rightarrow \quad (27)$$

where  $(.)^\dagger$  stands for the pseudo inverse of non square matrices. To make  $WH = 0$  the matrices  $N_1$  and  $N_2$  are designed as  $N$ . Thus the received signal becomes

$$\begin{aligned} \tilde{Y}_{(N+L-1)} &= W_{(N+L-1) \times 2(N+v)} Y_{2(N+v)} \\ &= W \underbrace{\begin{bmatrix} F_{N+v}^H \\ F_{N+v}^H \end{bmatrix}}_B J + \\ &\quad \underbrace{W \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}}_S \rightarrow \end{aligned} \quad (28)$$

Compared to other methods to bring the desired structure in the channel matrix, the single input multi output (SIMO) model has an additional advantage of diversity in terms of the different spatial channels for the two receiver antennas. However it should be noted that the spatial diversity obtained herein depends on the separation of the antennas in the receiver and also on the spatial correlation between the two channels of the receiver antennas.

#### 5. SIMULATION RESULTS

We present the numerical results of the wireless system based on OFDM transmission. The simulation

provides the performance of an orthogonal frequency division multiplexing wireless transmitter and receiver model. The bits used here are coded with QPSK phase modulation scheme. The bit error rate performance of the system is analyzed against the various lengths used for zero prefixing in the OFDM system and the result is shown in Figure-1. The comparison of performance is also shown under the performance of the receiver when it knows the frequencies of the jammer signal and then the jammer amplitudes are estimated using WLS. The latter schemes, simulation of both the proposed equalization method. The performance of the free of interference scenario and the scenario where interference exists and is ignored at the receiver is also shown in Figure-2. The two approaches discussed as the one with exploiting the zero prefix values in the time domain OFDM transmission and the other one with additional diversity gains by means of the spatial correlation obtained through the two receive antennas. These algorithms have estimated the location of jammed subcarrier and simulation has shown that performance achieved is very near to the interference free scenario when the jammer width is very large.

Figure-1 shows the performance characteristics when the jammer location is exactly known to us with the proposed ZPbased approach and it can be verified that the proposed approach perfectly estimates and cancels the jammer frequencies in an optimal way. Though, there is some performance loss in estimating the jammer amplitudes using weighted least squares, compared to the scenario where there is no interference. The loss is not much over a wide range of jammer width and falls drastically when the jammer width is over 15 subcarriers. However for the case of jammer frequencies for more than 5, I but the second approach attain significant achievement compared to the zero prefixed based transmission schemes. Further, Figure-1 clearly shows the PEF-based method in [3]. Practical possibilities are not so good, the jammer covariance matrix and weights of the PEF taps are calculated with correct jammer locations and. In general it is not mandatory to have the information about the frequencies of the jammed signal carriers a priori. But this additional information may improve the performance of the detector in it has the knowledge of the jammed frequencies in particularly during the cases where the width of jammed subcarriers is quite large. Figure-2 shows that the SIMO based model performs better when combined with spatial diversity and antenna selection is deployed at the receiver. Moreover it is necessary to equalize the interference cancelled signal. Similar to other approaches the algorithm we proposed efficiently locates the jammer frequencies and its performance can be enhanced if the perfect channel state information is available at the receiver. Figure-2 demonstrates that our proposed approach can cancel the jammer bandwidth upto 25% the OFDM symbol bandwidth and efficiently improves the bit error rate performance.

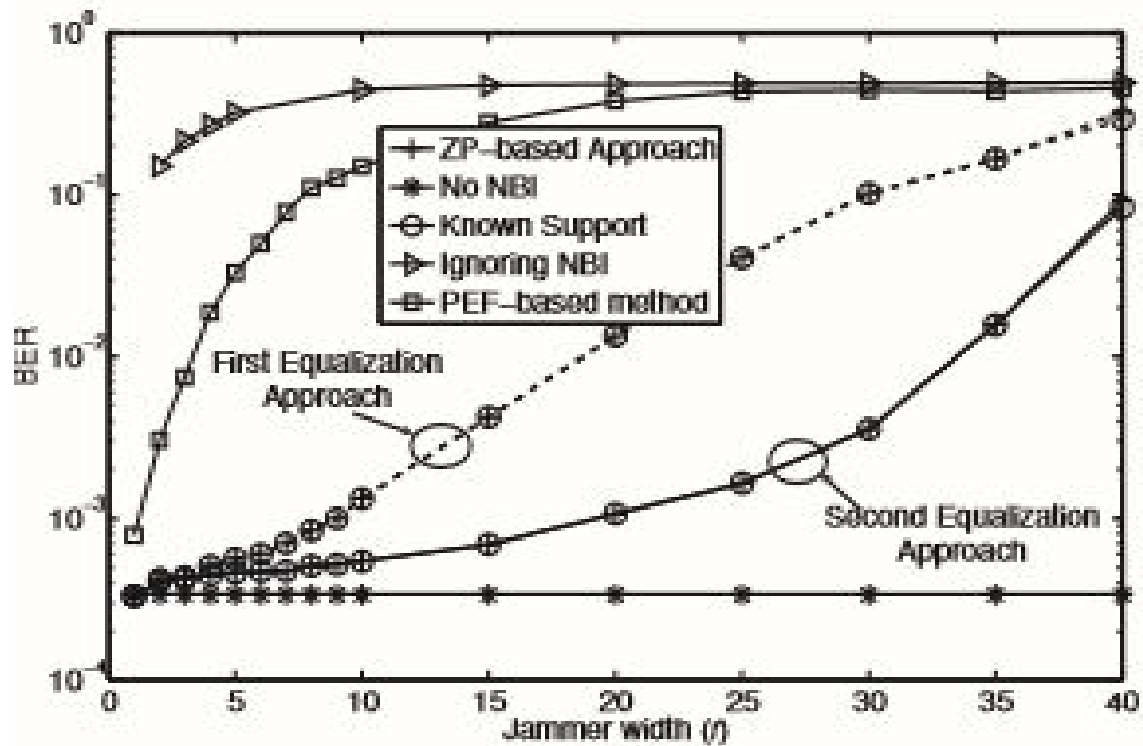


Figure-1. BER versus jammer width for ZP approach.

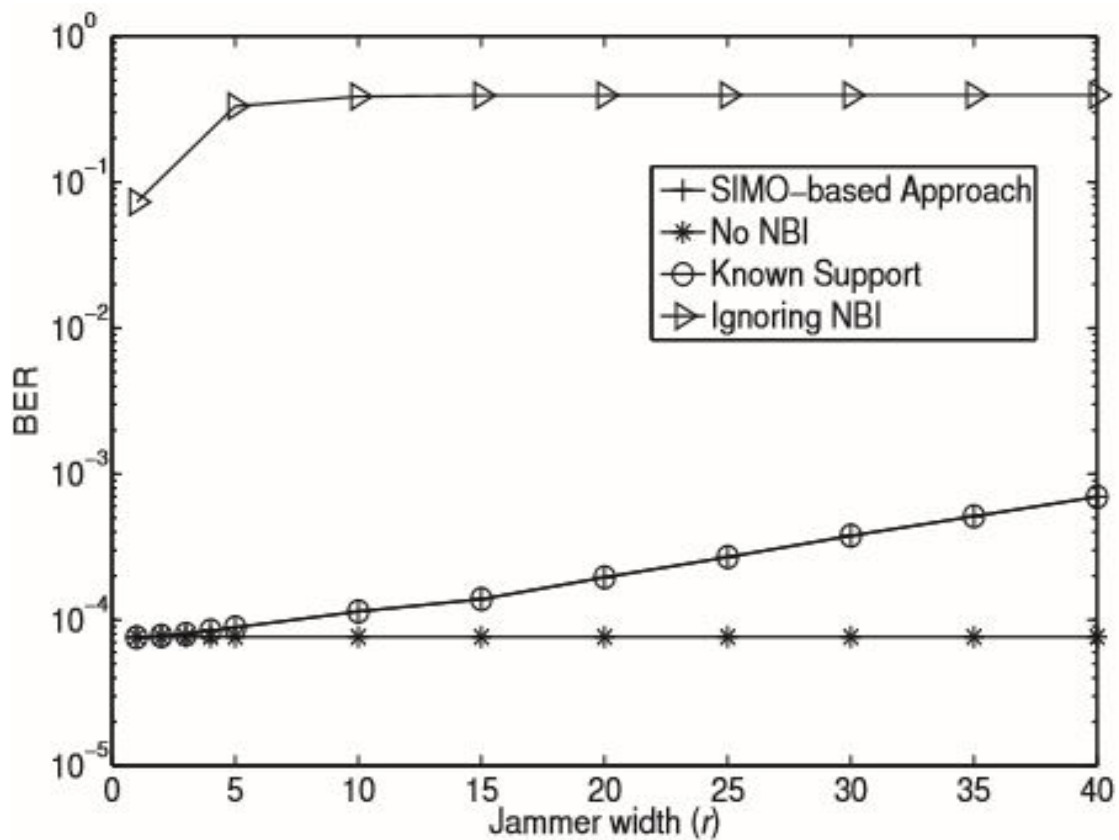


Figure-2. BER versus jammer width for SIMO system.





## 6. CONCLUSIONS

To reduce the NBI in OFDM systems, there were two approaches discussed in the picture. Further it is shown that the multi antenna receiver can improve the detection performance by exploiting the spatial diversity and selection combining and thereby enhances the bit error rate performance of the receiver. The another approach makes use of the zero prefixed sequence to bring out the desired structure in the resulting channel matrix to facilitate the premultiplication matrix with its columns in the null space of the channel matrix. These algorithms have estimated the location of jammed subcarrier and simulation has shown that performance achieved is very near to the interference frees scenario when the jammer width is very large.

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