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EFFECTS OF NON-UNIFORM TEMPERATURE GRADIENTS ON SURFACE TENSION DRIVEN TWO COMPONENT MAGNETOCONVECTION IN A POROUS-FLUID SYSTEM

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ABSTRACT

The Hydrothermal growth of crystals is mathematically modelled as the onset of Surface tension driven double diffusive magneto convection in a two-layer system comprising an incompressible two component, electrically conducting fluid saturated porous layer over which lies a layer of the same fluid in the presence a vertical magnetic field. Both the upper boundary of the fluid layer and the lower boundary of the porous layer are rigid and insulating to both heat and mass. At the interface the velocity, shear stress, normal stress, heat, heat flux, mass and mass flux are assumed to be continuous conducive for Darcy-Brinkman model. The resulting eigenvalue problem is solved exactly for both parabolic and inverted parabolic temperature profiles and analytical expressions of the Thermal Marangoni Number are obtained. Effects of variation of different physical parameters on the Thermal Marangoni Number for both profiles are compared.

Keywords: surface tension, magnetic field, temperature profiles, Marangoni number.

1. INTRODUCTION

Double-diffusive convection is a mixing process driven by the interaction of two fluid components which diffuse at different rates. Leading expert Timour Radko [1] presents the first systematic overview of the classical theory of double-diffusive convection in a coherent narrative, bringing together the disparate literature in this developing field. Double diffusion convection holds importance in natural processes and engineering applications. Double diffusive convection occurs in sea water, the mantle flow in the earth's crust as well as in many engineering and physical problems such as in contaminant transport in saturated soils, food processing, and the spread of pollutants, also appears in the modeling of solar ponds. Marangoni convection resulting from the local variation of surface tension due to a non-uniform temperature distribution is an interesting fluid mechanical problem. In the present investigation the double diffusive convection in a composite layer horizontally bounded by rigid walls presence of vertical magnetic field is considered and the resulting eigenvalue problem is solved exactly. The effects of variation of the important physical parameters and its thermal Marangoni number are obtained. The idea of using magnetic field is to dampenmelt turbulence and thereby improve microscopic homogeneity of the crystal has been first introduced independently by Utech and Flemmings [2] and Chedzey and Hurle [3]. In addition to damping out the turbulence and thereby removing the dopant striations, the magnetic field can be used to control the growth conditions at various stages in the growth process.

Though some literature is available for single layers of fluid/porous layers, but in composite layers, there are very few. Double diffusive convection in composite layers has wide applications in crystal growth and solidification of alloys. Inspite of its wide applications not much work has been done in this area. Recently Siddheshwar and Pranesh [4] have obtained the effects of a non- uniform temperature gradient and magnetic field on the onset of convection driven by surface tension in a horizontal layer of Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid / isothermal boundary have been considered. A linear stability analysis is performed. The Galerkin technique is used to obtain the eigen values. Shivakumara *et al* [5] have investigated the onset of surface tension driven convection in a two layer system comprising an incompressible fluid saturated porous layer over which lies a layer of the same fluid. The critical Marangoni number is obtained for insulating boundaries both by Regular Perturbation technique and also by exact method. S.P.M. Isa *et al* [6] have investigated the effect of magnetic field on the onset of Marangoni convection in a horizontal layer with a free-slip bottom heated from below and cooled from above with non-uniform basic temperature gradient is considered. Melviana Johnson Fu *et al* [7] have obtained the effect of a non-uniform basic temperature gradient and magnetic field on the onset of Marangoni convection in a horizontal micropolar fluid layer using the Rayleigh-Ritz technique. S.P.M. Isa *et al* [8] investigated the effect of non-uniform temperature gradient and magnetic field on Marangoni convection in a horizontal fluid layer heated from below and cooled from above with a constant heat flux. A linear stability analysis is performed. Norihan Md. Arifin *et al* [9] studied the effect of a non-uniform basic temperature gradient and magnetic field on the onset of Benard-Marangoni convection in a horizontal micropolar fluid layer using the Rayleigh-Ritz technique. Joseph *et al* [10] have obtained effects of electric field and non-uniform basic temperature gradient on the onset of Rayleigh-

ARPN Journal of Engineering and Applied Sciences

Benard-Marangoni convection in a micropolarfluid is studied using the Galerkin technique. R. Sumithra and Manjunatha. N [11, 12] have obtained the closed form solution of the problem of surface tension driven magneto convection is investigated in a two layer system comprising an incompressible electrically conducting fluid saturated porous layer over which lies a layer of the same fluid in the presence of a vertical magnetic field.

2. FORMULATION OF THE PROBLEM

Consider a horizontal two-component, electrically conducting fluid saturated isotropic sparsely packed porous layer of thickness d_m underlying a two component fluid layer of thickness d with imposed magnetic field intensity H_0 in the vertical z – direction. The lower surface of the porous layer is rigid and the upper surface of the fluid layer is free with surface tension effects depending on both temperature and concentration. Both the boundaries are kept at different constant temperatures and salinities. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z – axis, vertically upwards. The continuity, solenoidal property of the magnetic field, momentum, energy, species concentration and magnetic induction are, for the fluid layer

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\nabla \cdot \vec{H} = 0 \tag{2}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}\cdot\nabla\right)\vec{q}\right] = -\nabla P + \mu\nabla^2\vec{q} + \mu_p\left(\vec{H}\cdot\nabla\right)\vec{H} \tag{3}$$

$$\frac{\partial T}{\partial t} + \left(\vec{q}\cdot\nabla\right)T = \kappa\nabla^2 T \tag{4}$$

$$\frac{\partial C}{\partial t} + \left(\vec{q}\cdot\nabla\right)C = k_c\nabla^2 C \tag{5}$$

$$\frac{\partial \vec{H}}{\partial t} = \nabla\times\vec{q}\times\vec{H} + \nu_m\nabla^2\vec{H} \tag{6}$$

For the porous layer,

$$\nabla_m \cdot \vec{q}_m = 0 \tag{7}$$

$$\nabla_m \cdot \vec{H} = 0 \tag{8}$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial \vec{q}_m}{\partial t} + \frac{1}{\varepsilon^2}\left(\vec{q}_m\cdot\nabla_m\right)\vec{q}_m\right] = -\nabla_m P_m + \mu_m\nabla_m^2\vec{q}_m - \frac{\mu}{K}\vec{q}_m - \mu_p\left(\vec{H}\cdot\nabla_m\right)\vec{H} \tag{9}$$

$$A\frac{\partial T_m}{\partial t} + \left(\vec{q}_m\cdot\nabla_m\right)T_m = \kappa_m\nabla_m^2 T_m \tag{10}$$

$$\varepsilon\frac{\partial C_m}{\partial t} + \left(\vec{q}_m\cdot\nabla_m\right)C_m = k_m\nabla_m^2 C_m \tag{11}$$

$$\varepsilon\frac{\partial \vec{H}}{\partial t} = \nabla_m\times\vec{q}_m\times\vec{H}_m + \nu_{em}\nabla_m^2\vec{H}_m \tag{12}$$

Where $\vec{q} = (u,v,w)$ is the velocity vector, $\vec{H}$ is the magnetic field, ρ_0 is the fluid density, t is the time, μ is the fluid viscosity, $P = p + \frac{\mu_p H^2}{2}$ is the total pressure, μ_p is the magnetic permeability, T is the temperature, κ is the thermal diffusivity of the fluid, k_c is the solute diffusivity of the fluid, C is the concentration or the salinity field, $\nu_m = \frac{1}{\mu_p\sigma}$ is the magnetic viscosity, K is the permeability of the porous medium, $A = \frac{\left(\rho_0 C_p\right)_m}{\left(\rho C_p\right)_f}$ is the ratio of heat capacities, C_p is the specific heat, ε is the porosity, $\nu_{em} = \frac{\nu_m}{\varepsilon}$ is the effective magnetic viscosity and the subscripts 'm' and 'f' refer to the porous medium and the fluid respectively.

The basic state is quiescent, in the fluid layer

$$\left[u,v,w,P,T,C,\vec{H}\right] = \begin{bmatrix} 0,0,0,P_b(z),T_b(z), \\ C_b(z),H_0(z) \end{bmatrix} \tag{13}$$

and in the porous layer

$$\left[u_m,v_m,w_m,P_m,T_m,C_m\right] = \begin{bmatrix} 0,0,0,P_{mb}(z_m), \\ T_{mb}(z_m),C_{mb}(z_m) \end{bmatrix} \tag{14}$$

where the subscript 'b' denotes the basic state. The temperature and species concentration distributions $T_b(z)$, $T_{mb}(z_m)$ and $C_b(z)$, $C_{mb}(z_m)$ respectively are found to be

$$-\frac{\partial T_b}{\partial z} = \frac{\left(T_0 - T_u\right)}{d}h(z) \quad \text{in } 0 \le z \le d \tag{15}$$

$$-\frac{\partial T_{mb}}{\partial z_m} = \frac{\left(T_l - T_0\right)}{d_m}h_m(z_m) \quad \text{in } 0 \le z_m \le d_m \tag{16}$$

$$C_b(z) = C_0 - \frac{\left(C_0 - C_u\right)z}{d} \quad \text{in } 0 \le z \le d \tag{17}$$

$$C_{mb}(z_m) = C_0 - \frac{\left(C_l - C_0\right)z_m}{d_m} \quad \text{in } 0 \le z_m \le d_m \tag{18}$$

ARPN Journal of Engineering and Applied Sciences

where $T_0=\dfrac{\kappa d_m T_u+\kappa_m dT_l}{\kappa d_m+\kappa_m d}$, $C_0=\dfrac{k_c d_m C_u+k_{cm} dC_l}{k_c d_m+k_{cm} d}$ are the interface temperature and concentrations, $h(z)$ and $h_m(z_m)$ are temperature gradients in fluid and porous layer respectively.

We superimpose infinitesimal disturbances on the basic state as,

$$\left[\vec{q},P,T,C,\vec{H}\right]=\begin{bmatrix}0,P_b(z),T_b(z),\\ C_b(z),H_0(z)\end{bmatrix}+\left[\vec{q}',P',\theta,S,\vec{H}'\right] \tag{19}$$

and

$$\left[\vec{q}_m,P_m,T_m,C_m,\vec{H}\right]=\begin{bmatrix}0,P_{mb}(z_m),T_{mb}(z_m),\\ C_{mb}(z_m),H_0(z_m)\end{bmatrix}+\left[\vec{q}'_m,P'_m,\theta_m,S_m,\vec{H}'\right] \tag{20}$$

where the primed quantities are the perturbed once over their equilibrium counterparts. Now Equations. (19) and (20) are substituted into the Equations. (1) to (12) and are linearized in the usual manner. Next, the pressure term is eliminated from (3) and (9) by taking curl twice on these two equations and only the vertical component is retained. The variables are then non dimensionalized using $\dfrac{d^2}{\kappa}$, $\dfrac{\kappa}{d}$, T_0-T_u, C_0-C_u and H_0 as the units of time, velocity, temperature, species concentration and the magnetic field in the fluid layer and $\dfrac{d_m^2}{\kappa_m}$, $\dfrac{\kappa_m}{d_m}$, T_l-T_0, C_l-C_0 as the corresponding characteristic quantities in the porous layer.

The separate length scales are chosen for the two layers (see F. Chen and C.F. Chen [13], D.A. Nield [14]), so that each layer is of unit depth such that $(x,y,z)=d\left(x',y',z'\right)$ and $(x_m,y_m,z_m)=d_m\left(x_m',y_m',z_m'-1\right)$.

The following dimensionless equations (after neglecting the primes) are: in, $0\le z\le 1$

$$\frac{1}{\Pr}\frac{\partial\nabla^2 w}{\partial t}=\nabla^4 w+Q\tau_{fm}\frac{\partial\nabla^2 H_z}{\partial z} \tag{21}$$

$$\frac{\partial\theta}{\partial t}=w\,h(z)+\nabla^2\theta \tag{22}$$

$$\frac{\partial S}{\partial t}=w+\tau\nabla^2 S \tag{23}$$

$$\frac{\partial H_z}{\partial t}=\frac{\partial w}{\partial t}+\tau_{fm}\nabla^2 H_z \tag{24}$$

In $0\le z_m\le 1$

$$\frac{\beta^2}{\Pr_m}\frac{\partial\nabla_m^2 w_m}{\partial t}=\hat{\mu}\beta^2\nabla_m^4 w_m-\nabla_m^2 w_m+Q_m\tau_{mm}\beta^2\frac{\partial\nabla_m^2 H_{zm}}{\partial z_m} \tag{25}$$

$$A\frac{\partial\theta_m}{\partial t}=w_m\,h_m(z_m)+\nabla_m^2\theta_m \tag{26}$$

$$\varepsilon\frac{\partial S_m}{\partial t}=w_m+\tau_{pm}\nabla_m^2 S_m \tag{27}$$

$$\varepsilon\frac{\partial H_{zm}}{\partial t}=\frac{\partial w_m}{\partial t}+\tau_{mm}\nabla_m^2 H_{zm} \tag{28}$$

For the fluid layer $\Pr=\dfrac{\nu}{\kappa}$ is the Prandtl number, $Q=\dfrac{\mu_p H_0^2 d^2}{\mu\kappa\tau_{fm}}$ is the Chandrasekhar number, $\tau_{fm}=\dfrac{\nu_{mv}}{\kappa}$ and $\tau=\dfrac{k_c}{\kappa}$ are the diffusivity ratios. For the porous layer, $\Pr_m=\dfrac{\varepsilon\nu_m}{\kappa_m}$ is the Prandtl number, $\beta^2=\dfrac{K}{d_m^2}=Da$ is the Darcy number, β is porous parameter, $\hat{\mu}=\dfrac{\nu_m}{\nu}$ is the viscosity ratio, $Q_m=\dfrac{\mu_p H_0^2 d_m^2}{\mu\kappa_m\tau_{mm}}=Q\varepsilon\hat{d}^2$ is the Chandrasekhar number, $\tau_{mm}=\dfrac{\nu_{em}}{\kappa_m}$ and $\tau_{pm}=\dfrac{k_{cm}}{\kappa_m}$ are the diffusivity ratios in the porous layer and $h(z)$, $h_m(z_m)$ are the non-dimensional temperature gradients with $\int_0^1 h(z)\,dz=1$, $\int_0^1 h_m(z_m)\,dz=1$ and W, W_m are dimensionless vertical velocities in fluid and porous layers respectively.

Introducing the normal mode solutions of the form,

$$\begin{bmatrix}w\\ \theta\\ S\\ H\end{bmatrix}=\begin{bmatrix}W(z)\\ \Theta(z)\\ \Sigma(z)\\ H(z)\end{bmatrix}f(x,y)e^{nt} \tag{29}$$

and

$$\begin{bmatrix}w_m\\ \theta_m\\ S_m\\ H_m\end{bmatrix}=\begin{bmatrix}W_m(z_m)\\ \Theta_m(z_m)\\ \Sigma_m(z_m)\\ H_m(z_m)\end{bmatrix}f_m(x_m,y_m)e^{n_m t} \tag{30}$$

with $\nabla_2^2 f+a^2 f=0$ and $\nabla_{2m}^2 f_m+a_m^2 f_m=0$, where a and a_m are the nondimensional horizontal wave numbers, n and n_m are the frequencies. Since the dimensional horizontal wave numbers must be the same for the fluid and porous layers, we must have $\frac{a}{d}=\frac{a_m}{d_m}$ and hence $a_m=\hat{d}a$.

Substituting Equations. (29) and (30) into the Equations. (21) to (28) and denoting the differential operator $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial z_m}$ by D and D_m respectively, an eigenvalue problem consisting of the following ordinary differential equations is obtained,

In $0\le z\le 1$,

$$\left(D^2-a^2+\frac{n}{\Pr}\right)\left(D^2-a^2\right)W=-Q\tau_{fm}D\left(D^2-a^2\right)H \quad (31)$$

$$\left(D^2-a^2+n\right)\Theta+W\,h(z)=0 \quad (32)$$

$$\left[\tau\left(D^2-a^2\right)+n\right]S+W=0 \quad (33)$$

$$\left[\tau_{fm}\left(D^2-a^2\right)+n\right]H+DW=0 \quad (34)$$

In $0\le z_m\le 1$

$$\left[\left(D_m^2-a_m^2\right)\hat{\mu}\beta^2+\frac{n_m\beta^2}{\Pr_m}-1\right]\left(D_m^2-a_m^2\right)W_m$$
$$=Q_m\tau_{mm}\beta^2D_m\left(D_m^2-a_m^2\right)H_m \quad (35)$$

$$\left(D_m^2-a_m^2+An_m\right)\Theta_m+W_m\,h_m(z_m)=0 \quad (36)$$

$$\left[\tau_{pm}\left(D_m^2-a_m^2\right)+n_m\varepsilon\right]S_m+W_m=0 \quad (37)$$

$$\left[\tau_{mm}\left(D_m^2-a_m^2\right)+n_m\varepsilon\right]H_{mz}+DW_m=0 \quad (38)$$

Since the principle of exchange instability holds for surface tension driven convection in fluid layer and porous layer, we assume that it holds good even for the present configuration as well and eliminating the magnetic field in Equations. (31) and (35). The eigen value problem becomes

in $0\le z\le 1$,

$$\left(D^2-a^2\right)^2W=QD^2W \quad (39)$$

$$\left(D^2-a^2\right)\Theta+W\,h(z)=0 \quad (40)$$

$$\tau\left(D^2-a^2\right)S+W=0 \quad (41)$$

In $0\le z_m\le 1$

$$\left[\left(D_m^2-a_m^2\right)\hat{\mu}\beta^2-1\right]\left(D_m^2-a_m^2\right)W_m=Q_m\beta^2D_m^2W_m \quad (42)$$

$$\left(D_m^2-a_m^2\right)\Theta_m+W_m\,h_m(z_m)=0 \quad (43)$$

$$\tau_{pm}\left(D_m^2-a_m^2\right)S_m+W_m=0 \quad (44)$$

3. BOUNDARY CONDITIONS

The upper boundary of the fluid layer and the lower boundary of the porous layer are rigid and insulating to heat and mass. At the interface the velocity, shear stress, normal stress, heat, heat flux, mass and mass flux are assumed to be continuous conducive for Darcy-Brinkman model. All the boundary conditions are nondimensionalized and then subjected to normal mode expansion and are

$$\begin{aligned}
&D^2W(1)+M\,a^2\Theta(1)+M_s\,a^2S(1)=0,\\
&W(1)=0,\ D\Theta(1)=0,\ DS(1)=0\\
&\hat{T}W(0)=W_m(1),\ \hat{T}\hat{d}DW(0)=D_mW_m(1),\\
&W_m(0)=0,\ DW_m(0)=0,\\
&D_m\Theta_m(0)=0,\\
&\hat{T}\hat{d}^2\left(D^2+a^2\right)W(0)=\hat{\mu}\left(D_m^2+a_m^2\right)W_m(1),\\
&\hat{T}\hat{d}^3\beta^2\left(D^3W(0)-3a^2DW(0)\right)\\
&=-D_mW_m\left(1\right)+\hat{\mu}\beta^2\left(D_m^3W_m\left(1\right)-3a_m^2D_mW_m\left(1\right)\right),\\
&\Theta(0)=\hat{T}\Theta_m(1),\ D\Theta(0)=D_m\Theta_m(1),\\
&S(0)=\hat{S}\,S_m(1),\ DS(0)=D_mS_m(1),\ D_mS_m(0)=0
\end{aligned} \quad (45)$$

where

$\hat{S}=\frac{C_l-C_0}{C_0-C_u}$, $\hat{T}=\frac{T_l-T_0}{T_0-T_u}$, $M=-\frac{\partial\sigma_t}{\partial T}\frac{\left(T_0-T_u\right)d}{\mu\kappa}$ is the Thermal Marangoni number, $M_s=-\frac{\partial\sigma_t}{\partial C}\frac{\left(C_0-C_u\right)d}{\mu\kappa}$ is the solute Marangoni number.

The Equations. (39) to (44) are to be solved with respect to the boundary conditions (45).

4. METHOD OF SOLUTION

The solutions of W and W_m are found to be,

$$\begin{aligned}W\left(z\right)&=A_1Cosh\left(\delta z\right)+A_2Sinh\left(\delta z\right)\\&+A_3Cosh(\xi z)+A_4Sinh(\xi z)\end{aligned} \quad (46)$$

$$\begin{aligned}W_m\left(z_m\right)&=A_5Cosh\left(C_4z_m\right)+A_6Sinh\left(C_4z_m\right)\\&+A_7Cosh\left(C_5z_m\right)+A_8Sinh\left(C_5z_m\right)\end{aligned} \quad (47)$$

Where $\delta=\frac{\sqrt{Q}-\sqrt{Q+4a^2}}{2}$, $\xi=\frac{\sqrt{Q}+\sqrt{Q+4a^2}}{2}$, $C_4=\sqrt{\frac{C_1+C_3}{2}}, C_5=\sqrt{\frac{C_1-C_3}{2}}$ and $A_i^{'}s(i=1,2,3,......8)$ are constants and the expressions for $W(z)$ and $W_m(z_m)$ are appropriately written as

$$W\left(z\right)=A_1\begin{bmatrix}Cosh\left(\delta z\right)+a_1Sinh\left(\delta z\right)\\+a_2Cosh(\xi z)+a_3Sinh(\xi z)\end{bmatrix} \quad (48)$$

ARPN Journal of Engineering and Applied Sciences

$$W_m\left(z_m\right)=A_1\begin{bmatrix}a_4Cosh\left(C_4z_m\right)+a_5Sinh\left(C_4z_m\right)\\+a_6Cosh\left(C_5z_m\right)+a_7Sinh\left(C_5z_m\right)\end{bmatrix} \qquad (49)$$

where $a_i^{'}s(i=1,2,3,......7)$ are determined using the velocity boundary conditions of (45), we get

$$a_1=\frac{\Delta_{33}}{\Delta_{32}},\ a_2=-\frac{1}{\Delta_{29}}\left[a_1\,\Delta_{30}+\Delta_{31}\right],\ a_3=\frac{a_2\,\Delta_{22}+\Delta_{23}-a_1\,\Delta_{21}}{\Delta_{20}},$$

$$a_4=\frac{a_2\,\Delta_{18}+\Delta_{19}}{\Delta_{17}},\ a_5=\frac{\hat{T}\left(1+a_2\right)-a_4\Delta_1}{\Delta_2},\qquad a_6=-a_4,$$

$$a_7=-\frac{a_5\,c_4}{c_5},\ a_8=\hat{T}a_{10}\cosh a_m+\hat{T}a_{11}\sinh a_m+\Delta_{35},$$

$$\Delta_1=Coshc_4-Coshc_5,\ \Delta_2=Sinhc_4-\frac{c_4}{c_5}Sinhc_5,$$

$$\Delta_3=c_4\,Sinhc_4-c_5\,Sinhc_5,$$

$$\Delta_4=c_4\,Coshc_4-c_4\,Coshc_5,$$

$$\Delta_5=\hat{\mu}\left[\left(c_4^2+a_m^2\right)Coshc_4-\left(c_5^2+a_m^2\right)Coshc_5\right],$$

$$\Delta_6=\hat{\mu}\left[\left(c_4^2+a_m^2\right)Sinhc_4-\left(c_5^2+a_m^2\right)\frac{c_4}{c_5}Sinhc_5\right],$$

$$\Delta_7=\hat{T}\,\beta^2\hat{d}^3\left(\delta^3-3a^2\delta\right),\quad \Delta_8=\hat{T}\,\beta^2\hat{d}^3\left(\xi^3-3a^2\xi\right),$$

$$\Delta_9=\left(-1-3a_m^2\hat{\mu}\beta^2\right)\left(c_4\,Sinhc_4-c_5\,Sinhc_5\right),$$

$$\Delta_{10}=\left(-1-3a_m^2\hat{\mu}\beta^2\right)\left(c_4\,Coshc_4-c_4\,Coshc_5\right),$$

$$\Delta_{11}=\hat{\mu}\beta^2\left(c_4{}^3\,Sinhc_4-c_5{}^3\,Sinhc_5\right),$$

$$\Delta_{12}=\hat{\mu}\beta^2\left(c_4{}^3\,Coshc_4-c_4c_5{}^2\,Coshc_5\right),$$

$$\Delta_{13}=\Delta_9+\Delta_{11},\quad \Delta_{14}=\Delta_{10}+\Delta_{12},$$

$$\Delta_{15}=\hat{T}\,\hat{d}^2\left(\delta^2+a^2\right),\quad \Delta_{16}=\hat{T}\,\hat{d}^2\left(\xi^2+a^2\right),$$

$$\Delta_{17}=\Delta_5-\frac{\Delta_1\,\Delta_6}{\Delta_2},\quad \Delta_{18}=\Delta_{16}-\hat{T}\,\frac{\Delta_6}{\Delta_2},$$

$$\Delta_{19}=\Delta_{15}-\hat{T}\,\frac{\Delta_6}{\Delta_2},\quad \Delta_{20}=\hat{T}\hat{d}\xi,$$

$$\Delta_{21}=\hat{T}\hat{d}\delta,\quad \Delta_{22}=\frac{\Delta_{18}\,\Delta_3}{\Delta_{17}}+\hat{T}\,\frac{\Delta_4}{\Delta_2}-\frac{\Delta_{18}\,\Delta_1\,\Delta_4}{\Delta_2\,\Delta_{17}},$$

$$\Delta_{23}=\frac{\Delta_{19}\,\Delta_3}{\Delta_{17}}+\hat{T}\,\frac{\Delta_4}{\Delta_2}-\frac{\Delta_{19}\,\Delta_1\,\Delta_4}{\Delta_2\,\Delta_{17}},$$

$$\Delta_{24}=\frac{\Delta_{18}\,\Delta_{13}}{\Delta_{17}}+\hat{T}\,\frac{\Delta_{14}}{\Delta_2}-\frac{\Delta_{18}\,\Delta_1\,\Delta_{14}}{\Delta_2\,\Delta_{17}},$$

$$\Delta_{25}=\frac{\Delta_{19}\,\Delta_{13}}{\Delta_{17}}+\hat{T}\,\frac{\Delta_{14}}{\Delta_2}-\frac{\Delta_{19}\,\Delta_1\,\Delta_{14}}{\Delta_2\,\Delta_{17}},$$

$$\Delta_{26}=Cosh\xi+\frac{\Delta_{22}}{\Delta_{20}}Sinh\xi,$$

$$\Delta_{27}=Sinh\delta-\frac{\Delta_{21}}{\Delta_{20}}Sinh\xi,$$

$$\Delta_{28}=Cosh\delta+\frac{\Delta_{23}}{\Delta_{20}}Sinh\xi,\ \Delta_{29}=\frac{\Delta_8\,\Delta_{22}}{\Delta_{20}}-\Delta_{24},$$

$$\Delta_{30}=\Delta_7-\frac{\Delta_8\,\Delta_{21}}{\Delta_{20}},\ \Delta_{31}=\frac{\Delta_8\,\Delta_{23}}{\Delta_{20}}-\Delta_{25},$$

$$\Delta_{32}=\Delta_{27}-\frac{\Delta_{30}\,\Delta_{26}}{\Delta_{29}},\ \Delta_{33}=\frac{\Delta_{26}\,\Delta_{31}}{\Delta_{29}}-\Delta_{28},$$

5. LINEAR TEMPERATURE PROFILE

Here taking

$h(z)=1$ and $h_m(z_m)=1$ (50) Substituting equation (50) into the heat equations (40) and (43), the expressions for Θ and Θ_m are obtained as

$$\Theta(z)=A_1\left[a_{20}\,Coshaz+a_{21}Sinhaz+g_1(z)\right] \qquad (50)$$

$$\Theta_m(z_m)=A_1\left[a_{22}\,Cosha_mz_m+a_{23}Sinha_mz_m+g_{m1}(z_m)\right] \qquad (51)$$

where

$$g_1(z)=-\begin{bmatrix}\frac{1}{\delta^2-a^2}\left(a_1Sinh\delta z+Cosh\delta z\right)\\+\frac{1}{\xi^2-a^2}\left(a_3Sinh\xi z+a_2\,Cosh\xi z\right)\end{bmatrix}$$

$$g_{m1}(z_m)=-\begin{bmatrix}\frac{1}{c_4^2-a_m{}^2}\left(a_4Coshc_4z_m+a_5\,Sinhc_4z_m\right)\\+\frac{1}{c_5^2-a_m{}^2}\left(a_6Coshc_5z_m+a_7\,Sinhc_5z_m\right)\end{bmatrix} \qquad (52)$$

where $a_i^{'}s(i=20,...23)$ are constants to be determined by using the temperature boundary conditions of (45), we get

$$a_{20}=\hat{T}\hat{\mu}\left(a_{22}\cosh a_m+a_{23}\sinh a_m\right)-\Delta_{52},$$

$$a_{21}=\frac{a_{22}\;a_m\sinh a_m+a_{23}\,a_m\cosh a_m-\Delta_{53}}{a},$$

$$a_{22}=-\frac{\Delta_{57}}{\Delta_{56}},\ a_{23}=\frac{\Delta_{55}}{a_m},$$

$$\Delta_{52}=\hat{T}\begin{pmatrix}\frac{c_4\left(a_4\,Sinhc_4+a_5\,Coshc_4\right)}{\left(c_4^2-a_m^2\right)}+\\\frac{c_5\left(a_6\,Sinhc_5+a_7\,Coshc_5\right)}{\left(c_5^2-a_m^2\right)}\end{pmatrix}-\left(\frac{1}{\left(\delta^2-a^2\right)}+\frac{a_2}{\left(\xi^2-a^2\right)}\right),$$

$$\Delta_{53}=\frac{c_4\left(a_4\,Sinhc_4+a_5\,Coshc_4\right)}{\left(c_4^2-a_m^2\right)}$$
$$+\frac{c_5\left(a_6\,Sinhc_5+a_7\,Coshc_5\right)}{\left(c_5^2-a_m^2\right)}-\frac{\delta a_1}{\left(\delta^2-a^2\right)}-\frac{\xi a_3}{\left(\xi^2-a^2\right)},$$

$$\Delta_{54}=\frac{\delta}{\left(\delta^2-a^2\right)}\left(\,Sinh\delta+a_1\,Cosh\delta\right)+$$
$$\frac{\xi}{\left(\xi^2-a^2\right)}\left(a_2\,Sinh\xi+a_3\,Cosh\xi\right),$$

$$\Delta_{55}=\frac{a_5c_4}{\left(c_4^2-a_m^2\right)}+\frac{c_5a_7}{\left(c_5^2-a_m^2\right)},$$

$$\Delta_{56}=\hat{T}a\,Sinha\;Cosha_m+a_m\,Cosha\,Sinha_m,$$

$$\Delta_{57}=\frac{\Delta_{55}}{a_m}\left(\hat{T}a\,Sinha\;Sinha_m+a_m\,Cosha\;Cosha_m\right)-$$
$$\left(a\,Sinha\,\Delta_{52}+\Delta_{53}\,Cosha+\Delta_{54}\right),$$

ARPN Journal of Engineering and Applied Sciences

The species concentration equations (41) and (44), the expressions for S and S_m are obtained as

$$S(z) = A_1\left[a_{16}\,Coshaz + a_{17}Sinhaz + f_2(z)\right] \tag{53}$$

$$S_m(z_m) = A_1\left[a_{18}\,Cosha_m z_m + a_{19}Sinha_m z_m + f_{m2}(z_m)\right] \tag{54}$$

Where

$$f_2(z) = -\frac{1}{\tau}\begin{bmatrix} \dfrac{\left(a_1 Sinh\delta z + Cosh\delta z\right)}{\left(\delta^2 - a^2\right)} \\ + \dfrac{\left(a_3 Sinh\xi z + a_2\,Cosh\xi z\right)}{\left(\xi^2 - a^2\right)} \end{bmatrix}$$

$$f_{m2}(z_m) = -\frac{1}{\tau_{pm}}\begin{bmatrix} \dfrac{1}{\left(c_4^2 - a_m{}^2\right)} \\ \begin{pmatrix} a_4 Coshc_4 z_m \\ + a_5\,Sinhc_4 z_m \end{pmatrix} \\ + \dfrac{1}{\left(c_5^2 - a_m{}^2\right)} \\ \begin{pmatrix} a_6 Coshc_5 z_m \\ + a_7\,Sinhc_5 z_m \end{pmatrix} \end{bmatrix}$$

where $a_i^{'}s(i = 16,...19)$ are constants to be determined by using the concentration boundary conditions of (45), we get

$$a_{16} = \hat{S}\left(a_{18}\cosh a_m + a_{19}\sinh a_m\right) - \Delta_{47},$$

$$a_{17} = \frac{a_{18}\,a_m \sinh a_m + a_{19}\,a_m \cosh a_m - \Delta_{48}}{a},$$

$$a_{18} = -\frac{\Delta_{51}}{\Delta_{50}},\quad a_{19} = \frac{\Delta_{49}}{a_m},$$

$$\Delta_{46} = \frac{1}{\tau}\left[\frac{\delta\left(\,Sinh\delta + a_1 Cosh\delta\right)}{\delta^2 - a^2} + \frac{\xi\left(a_2 Sinh\xi + a_3\,Cosh\xi\right)}{\xi^2 - a^2}\right],$$

$$\Delta_{47} = \frac{\hat{S}}{\tau_{pm}}\begin{bmatrix} \dfrac{\left(a_5\,Sinhc_4 + a_4\,Coshc_4\right)}{\left(c_4^2 - a_m^2\right)} \\ + \dfrac{\left(a_7\,Sinhc_5 + a_6\,Coshc_5\right)}{\left(c_5^2 - a_m^2\right)} \end{bmatrix} - \frac{1}{\tau}\left(\frac{1}{\left(\delta^2 - a^2\right)} + \frac{a_2}{\left(\xi^2 - a^2\right)}\right),$$

$$\Delta_{48} = \frac{1}{\tau_{pm}}\begin{bmatrix} \dfrac{c_4\left(a_4\,Sinhc_4 + a_5\,Coshc_4\right)}{\left(c_4^2 - a_m^2\right)} \\ + \dfrac{c_5\left(a_6\,Sinhc_5 + a_7\,Coshc_5\right)}{\left(c_5^2 - a_m^2\right)} \end{bmatrix} - \frac{1}{\tau}\left(\frac{a_1\delta}{\left(\delta^2 - a^2\right)} + \frac{a_3\xi}{\left(\xi^2 - a^2\right)}\right),$$

$$\Delta_{49} = \frac{1}{\tau_{pm}}\left[\frac{a_5 c_4}{\left(c_4^2 - a_m^2\right)} + \frac{c_5 a_7}{\left(c_5^2 - a_m^2\right)}\right],$$

$$\Delta_{50} = \hat{S}a\,Sinha\;Cosha_m + a_m\,Cosha\,Sinha_m,$$

$$\Delta_{51} = \frac{\Delta_{49}}{a_m}\left(\hat{S}a\,Sinha\;Sinha_m + a_m\,Cosha\;Cosha_m\right) - \left(a\,Sinha\,\Delta_{47} + \Delta_{48}\,Cosha + \Delta_{46}\right),$$

Now the thermal Marangoni number is obtained by the boundary condition $(45)^1$ as

$$M_1 = -\frac{\left(\Lambda_1 + \Lambda_2\right)}{\Lambda_3} \tag{55}$$

$$\Lambda_1 = \delta^2\,Cosh\delta + a_1\,\delta^2 Sinh\delta + a_2\xi^2 Cosh\xi + a_3\xi^2 Sinh\xi$$

$$\Lambda_2 = M_s a^2\left\{a_{16}Cosha + a_{17}Sinha - \left(\begin{matrix} \dfrac{\left(Cosh\delta + a_1 Sinh\delta\right)}{(\delta^2 - a^2)} + \\ \dfrac{\left(a_2 Cosh\xi + a_3 Sinh\xi\right)}{(\xi^2 - a^2)} \end{matrix}\right)\right\}$$

$$\Lambda_3 = a^2\begin{bmatrix} a_{20}Cosha + a_{21}Sinha \\ -\dfrac{\left(a_1 Sinh\delta + Cosh\delta\right)}{\left(\delta^2 - a^2\right)} - \dfrac{\left(a_3 Sinh\xi + a_2 Cosh\xi\right)}{(\xi^2 - a^2)} \end{bmatrix}$$

6. PARABOLIC TEMPERATURE PROFILE

Following Sparrow *et al* [15], we consider a parabolic temperature profile of the form

$$h\left(z\right) = 2z \quad and \quad h_m(z_m) = 2z_m \tag{56}$$

Substituting equation (56) into the heat equations (40) and (43), the expressions for Θ and Θ_m are obtained as

$$\Theta(z) = A_1\left[a_8\,Coshaz + a_9 Sinhaz + f_1(z)\right] \tag{57}$$

$$\Theta_m(z_m) = A_1\left[a_{10}\,Cosha_m z_m + a_{11}Sinha_m z_m + f_{m1}(z_m)\right] \tag{58}$$

Where
$$f_1(z) = -\begin{bmatrix} \dfrac{2z}{\delta^2 - a^2}\left(a_1 Sinh\delta z + Cosh\delta z\right) - \\ \dfrac{4\delta}{\left(\delta^2 - a^2\right)^2}\left(Sinh\delta z + a_1 Cosh\delta z\right) + \\ \dfrac{2z}{\xi^2 - a^2}\left(a_3 Sinh\xi z + a_2\,Cosh\xi z\right) - \\ \dfrac{4\xi}{\left(\xi^2 - a^2\right)^2}\left(a_2 Sinh\xi z + a_3\,Cosh\xi z\right) \end{bmatrix}$$

$$f_{m1}(z_m) = -\begin{bmatrix} \dfrac{2z_m}{c_4^2 - a_m{}^2} \\ \left(a_4 Coshc_4 z_m + a_5\,Sinhc_4 z_m\right) - \\ \dfrac{4c_4}{\left(c_4^2 - a_m{}^2\right)^2} \\ \left(a_5 Coshc_4 z_m + a_4\,Sinhc_4 z_m\right) + \\ \dfrac{2z_m}{c_5^2 - a_m{}^2} \\ \left(a_6 Coshc_5 z_m + a_7\,Sinhc_5 z_m\right) - \\ \dfrac{4c_5}{\left(c_5^2 - a_m{}^2\right)^2} \\ \left(a_7 Coshc_5 z_m + a_6\,Sinhc_5 z_m\right) \end{bmatrix}$$

where $a_i^{'}s(i = 8,...11)$ are constants to be determined by using the temperature boundary conditions of (45), we get

ARPN Journal of Engineering and Applied Sciences

$$a_8 = \hat{T}a_{10}\cosh a_m + \hat{T}a_{11}\sinh a_m + \Delta_{35},$$

$$a_9 = \frac{a_{10}a_m \sinh a_m + a_{11}a_m \cosh a_m + \Delta_{36}}{a},\ a_{10} = \frac{\Delta_{39}}{\Delta_{38}},\ \ a_{11} = -\frac{\Delta_{37}}{a_m},$$

$$\Delta_{34} = \frac{2(\delta^2+a^2)}{\left(\delta^2-a^2\right)^2}\left(a_1 Sinh\delta + Cosh\delta\right) - \frac{2\delta}{\delta^2-a^2}\left(Sinh\delta + a_1 Cosh\delta\right) - \frac{2}{\xi^2-a^2}\left((\xi a_2 + a_3)Sinh\xi + (\xi a_3 + a_2)Cosh\xi\right) + \frac{4\xi^2}{\left(\xi^2-a^2\right)^2}\left(a_3 Sinh\xi + a_2 Cosh\xi\right),$$

$$\Delta_{35} = \hat{T}\left(\frac{4c_4\left(a_4 Sinhc_4 + a_5 Coshc_4\right)}{\left(c_4^2-a_m^2\right)^2} + \frac{4c_5\left(a_6 Sinhc_5 + a_7 Coshc_5\right)}{\left(c_5^2-a_m^2\right)^2}\right) - \left(\frac{4\delta a_1}{\left(\delta^2-a^2\right)^2} + \frac{4\xi a_3}{\left(\xi^2-a^2\right)^2}\right) - 2\hat{T}\left(\frac{\left(a_5 Sinhc_4 + a_4 Coshc_4\right)}{\left(c_4^2-a_m^2\right)} + \frac{\left(a_7 Sinhc_5 + a_6 Coshc_5\right)}{\left(c_5^2-a_m^2\right)}\right),$$

$$\Delta_{36} = -\frac{2(\delta^2+a^2)}{\left(\delta^2-a^2\right)^2} - \left(\frac{2(\xi^2+a^2)}{\left(\xi^2-a^2\right)^2}\right)a_2 - \frac{2c_4}{\left(c_4^2-a_m^2\right)}\left(a_4 Sinhc_4 + a_5 Coshc_4\right) + \left(\frac{2(c_4{}^2+a_m^2)}{\left(c_4^2-a_m^2\right)^2}\right)\left(a_5 Sinhc_4 + a_4 Coshc_4\right) - \frac{2c_5}{\left(c_5^2-a_m^2\right)}\left(a_6 Sinhc_5 + a_7 Coshc_5\right) + \left(\frac{2(c_5{}^2+a_m^2)}{\left(c_5^2-a_m^2\right)^2}\right)\left(a_7 Sinhc_5 + a_6 Coshc_5\right),$$

$$\Delta_{37} = \left(\frac{2(c_5{}^2+a_m^2)}{\left(c_5^2-a_m^2\right)^2}\right)a_6 + \left(\frac{2(c_4{}^2+a_m^2)}{\left(c_4^2-a_m^2\right)^2}\right)a_4,$$

$$\Delta_{38} = \hat{T}a\,Sinha\;Cosha_m + a_m\,Cosha\,Sinha_m,$$

$$\Delta_{39} = \frac{\Delta_{37}}{a_m}\left(\hat{T}a\,Sinha\;Sinha_m + a_m\,Cosha\;Cosha_m\right) - \left(a\,Sinha\,\Delta_{35} + \Delta_{36}\,Cosha + \Delta_{34}\right),$$

Now the thermal Marangoni number is obtained by the boundary condition $(45)^{1}$ as

$$M_2 = -\frac{\left(\Lambda_1 + \Lambda_2\right)}{\Lambda_4} \tag{59}$$

$$\Lambda_4 = a^2\left[\begin{array}{l} a_8 Cosha + a_9 Sinha - \\ \dfrac{2\left(Cosh\delta + a_1 Sinh\delta\right)}{(\delta^2-a^2)} + \dfrac{4\delta\left(a_1 Cosh\delta + Sinh\delta\right)}{\left(\delta^2-a^2\right)^2} \\ -\dfrac{2\left(a_2 Cosh\xi + a_3 Sinh\xi\right)}{(\xi^2-a^2)} + \dfrac{4\xi\left(a_3 Cosh\xi + a_2 Sinh\xi\right)}{(\xi^2-a^2)^2} \end{array}\right]$$

7. INVERTED PARABOLIC TEMPERATURE PROFILE

For the inverted parabolic temperature profile we have

$$h(z) = 2(1-z) \quad and \quad h_m(z_m) = 2(1-z_m) \tag{60}$$

Substituting equation (60) into the heat equations (40) and (43), the expressions for Θ and Θ_m are obtained as

$$\Theta(z) = A_1\left[a_{12}\,Coshaz + a_{13}Sinhaz + f_3(z)\right] \tag{61}$$

$$\Theta_m(z_m) = A_1\left[a_{14}\,Cosha_m z_m + a_{15}Sinha_m z_m + f_{m3}(z_m)\right] \tag{62}$$

Where

$$f_3(z) = -\left[\begin{array}{l} \dfrac{2(1-z)}{\delta^2-a^2}\left(a_1 Sinh\delta z + Cosh\delta z\right) + \\ \dfrac{4\delta}{\left(\delta^2-a^2\right)^2}\left(Sinh\delta z + a_1 Cosh\delta z\right) + \\ \dfrac{2(1-z)}{\xi^2-a^2}\left(a_3 Sinh\xi z + a_2\,Cosh\xi z\right) + \\ \dfrac{4\xi}{\left(\xi^2-a^2\right)^2}\left(a_2 Sinh\xi z + a_3\,Cosh\xi z\right) \end{array}\right]$$

$$f_{m3}(z_m) = -\left[\begin{array}{l} \dfrac{2(1-z_m)}{c_4^2-a_m{}^2}\left(a_4 Coshc_4 z_m + a_5\,Sinhc_4 z_m\right) \\ +\dfrac{4c_4}{\left(c_4^2-a_m{}^2\right)^2}\left(a_4\,Sinhc_4 z_m + a_5 Coshc_4 z_m\right) \\ +\dfrac{2(1-z_m)}{c_5^2-a_m{}^2}\left(a_6 Coshc_5 z_m + a_7\,Sinhc_5 z_m\right) \\ +\dfrac{4c_5}{\left(c_5^2-a_m{}^2\right)^2}\left(a_6\,Sinhc_5 z_m + a_7 Coshc_5 z_m\right) \end{array}\right]$$

where $a_i^{'}s(i=12,...15)$ are constants to be determined by using the temperature boundary conditions of (45), we get

$$a_{12} = \hat{T}a_{14}\cosh a_m + \hat{T}a_{15}\sinh a_m + \Delta_{41},$$

$$a_{13} = \frac{a_{14}\, a_m \sinh a_m + a_{15}\, a_m \cosh a_m + \Delta_{42}}{a},$$

$$a_{14} = \frac{\Delta_{45}}{\Delta_{44}}, \quad a_{15} = -\frac{\Delta_{43}}{a_m}, \quad \Delta_{40} = \left[-\frac{2(\delta^2+a^2)}{\left(\delta^2-a^2\right)^2}\right]\left(a_1\,Sinh\delta + Cosh\delta\right) + \left[-\frac{2(\xi^2+a^2)}{\left(\xi^2-a^2\right)^2}\right]\left(a_3 Sinh\xi + a_2\,Cosh\xi\right),$$

$$\Delta_{41} = \frac{2}{\left(\delta^2-a^2\right)} + \frac{4\delta a_1}{\left(\delta^2-a^2\right)^2} + \frac{2a_2}{\left(\xi^2-a^2\right)} + \frac{4\xi a_3}{\left(\xi^2-a^2\right)^2} - \frac{4c_4\hat{T}}{\left(c_4^2-a_m^2\right)^2}\left(a_4\,Sinhc_4 + a_5\,Coshc_4\right) - \frac{4c_5\hat{T}}{\left(c_5^2-a_m^2\right)^2}\left(a_6\,Sinhc_5 + a_7\,Coshc_5\right),$$

$$\Delta_{42} = \frac{2\delta a_1}{\left(\delta^2-a^2\right)} + \frac{2(\delta^2+a^2)}{\left(\delta^2-a^2\right)^2} + \frac{2a_3\xi}{\left(\xi^2-a^2\right)} + \left(\frac{2(\xi^2+a^2)}{\left(\xi^2-a^2\right)^2}\right)a_2 - \left(\frac{2(c_4{}^2+a_m^2)}{\left(c_4^2-a_m^2\right)^2}\right)\left(a_5\,Sinhc_4 + a_4\,Coshc_4\right) - \left(\frac{2(c_5{}^2+a_m^2)}{\left(c_5^2-a_m^2\right)^2}\right)\left(a_7\,Sinhc_5 + a_6\,Coshc_5\right),$$

$$\Delta_{43} = -\frac{2c_4a_5}{\left(c_4^2-a_m^2\right)} - \frac{2c_5a_7}{\left(c_5^2-a_m^2\right)} - \left(\frac{2\left(c_4^2+a_m^2\right)}{\left(c_4^2-a_m^2\right)^2}\right)a_4 - \left(\frac{2\left(c_5^2+a_m^2\right)}{\left(c_5^2-a_m^2\right)^2}\right)a_6,$$

$$\Delta_{44} = \hat{T}a\,Sinha\;Cosha_m + a_m\,Cosha\,Sinha_m,$$

$$\Delta_{45} = \frac{\Delta_{43}}{a_m}\left(\hat{T}a\,Sinha\;Sinha_m + a_m\,Cosha\;Cosha_m\right) - \left(a\,Sinha\,\Delta_{41} + \Delta_{42}\,Cosha + \Delta_{40}\right),$$

Now the thermal Marangoni number is obtained by the boundary condition $(45)^1$ as

$$M_3 = -\frac{\left(\Lambda_1+\Lambda_2\right)}{\Lambda_5} \tag{63}$$

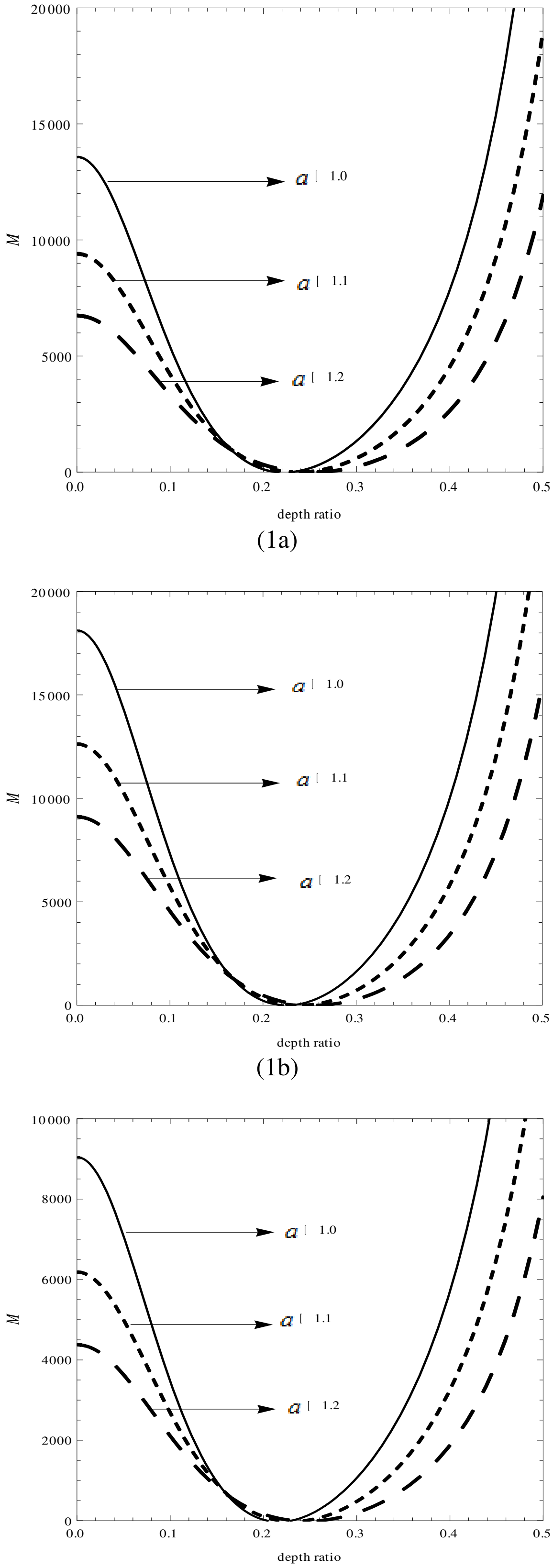
where

$$\Lambda_5 = a^2\left[\begin{array}{l} a_{12}Cosha + a_{13}Sinha - \dfrac{4\delta\left(Sinh\delta + a_1Cosh\delta\right)}{\left(\delta^2-a^2\right)^2} \\ -\dfrac{4\xi\left(a_2Sinh\xi + a_3Cosh\xi\right)}{(\xi^2-a^2)^2} \end{array}\right]$$

8. RESULTS AND DISCUSSIONS

The Thermal Marangoni numbers M_1, M_2 and M_3 obtained as a functions of the parameters are drawn versus the depth ratio $\hat{d}$ and the results are represented graphically showing the effects of the variation of one physical quantity, fixing the other parameters. The fixed values of the parameters are $Q = 200, \hat{T} = \hat{S} = 1.0,\ a = 1.0, \varepsilon = 0.7, \beta = 0.2,$

$M_{s1} = M_{s2} = 10, \tau = \tau_{pm} = 0.25$ and $\hat{\mu} = 2.0$.The effects of the parameters $a, \beta, Q, \hat{\mu}, \varepsilon$ and M_s on the Thermal Marangoni number are obtained and portrayed in the figures 1 to 6 respectively.



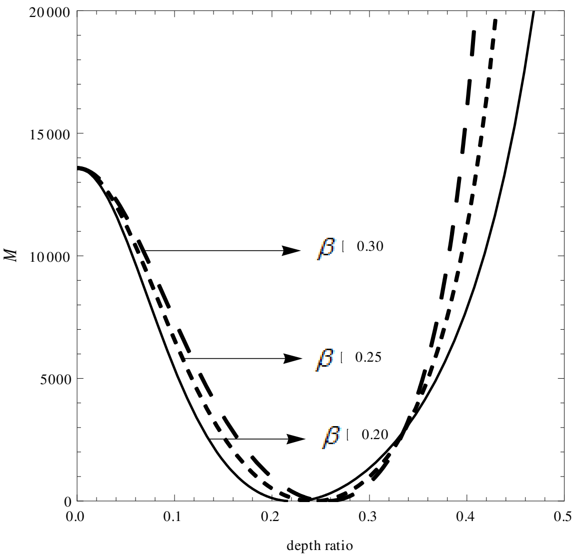
(1a)

(1b)

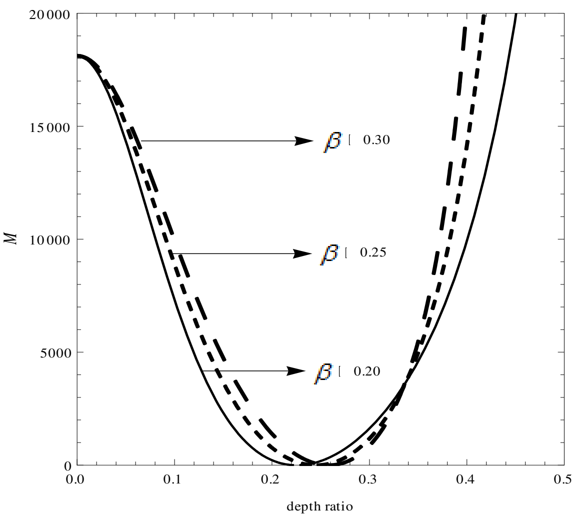
(1c)

Figure-1a, 1b& 1c. The effects of a horizontal wave number on the thermal Marangoni number M.

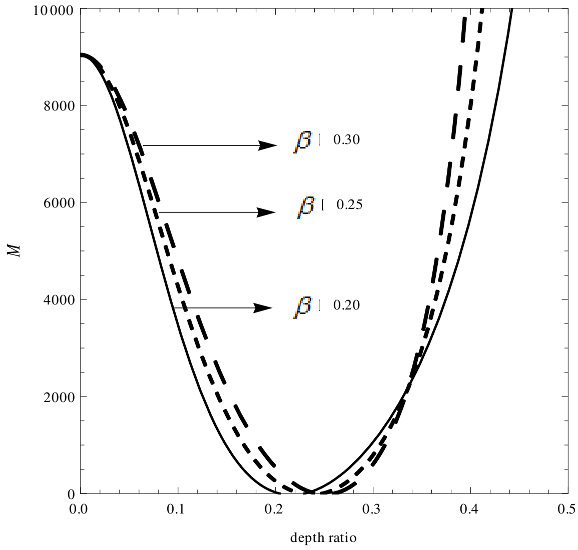
The effects of a horizontal wave number on the Thermal Marangoni numbers for both the parabolic and inverted parabolic profiles $M_1, M_2 \& M_3$ are shown in Figure-1a, 1b and 1c respectively for $a = 1.0, 1.1$ and 1.2. From the figures it is clear that the Thermal Marangoni number for the parabolic profile is more than that for the inverted parabolic profile. At the value of $0.2 \le \hat{d} < 0.3$, the effect of both the profiles are neutral and no effect of the horizontal wave number a on the thermal Marangoni number. The curves for the three wave numbers both the profiles are converging up to the value of $\hat{d} = 0.2$, whereas the three curves are diverging for the values of the depth ratio $\hat{d} \ge 0.2$. For both the profiles, when the value of a, the horizontal wave number is increased, the Thermal Marangoni numbers decrease and its effect is to destabilize the system. That is, its effect is to advance surface tension driven convection.



(2a)



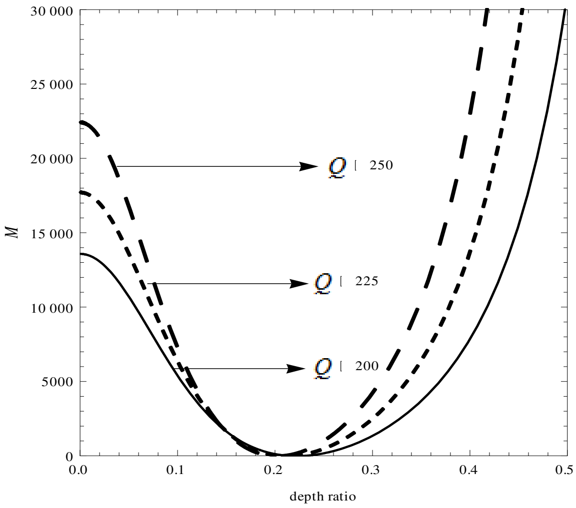
(2b)



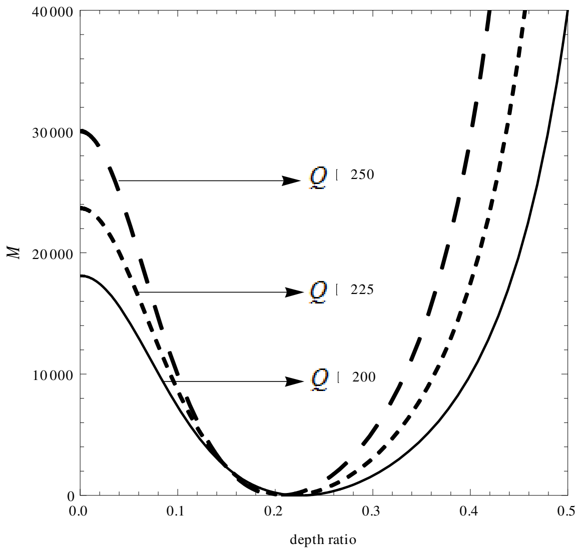
(2c)

Figure-2a, 2b & 2c. The effects of β on the thermal Marangoni number M.

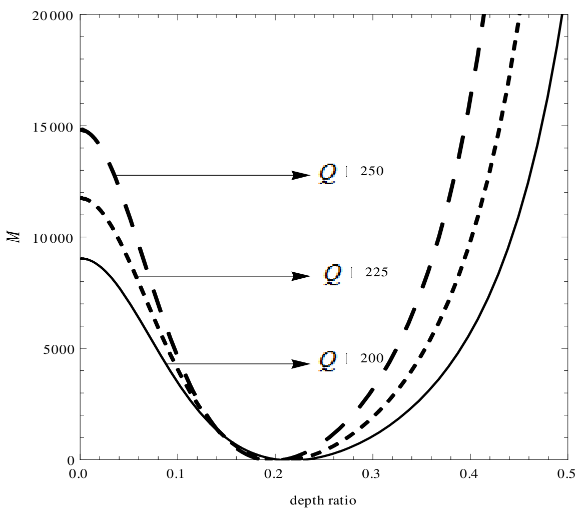
The effects of the porous parameter β on the thermal Marangoni numbers for the both the profiles are exhibited in the Figure-2a, 2b and 2c. The curves are for $\beta = 0.20, 0.25, 0.30$. The curves diverge for smaller values of the depth ratio, converge near $\hat{d} = 0.24$ and again diverge and converge at $\hat{d} = 0.32$ and, as the depth ratio is further increased the curves diverge. For smaller values of depth ratio, increase in the value in the value of the porous parameter increases the thermal Marangoni number, whereas for values of the depth ratio $0.24 \le \hat{d} \le 0.34$, the increase in the value of the porous parameter is to decrease the thermal Marangoni number and again for values of $\hat{d} \ge 0.34$ the behaviour again reverses. So, the onset of surface tension driven convection can either be made faster or delayed by choosing an appropriate value of the porous parameter depending on the depth ratio. In other words increasing the permeability of the porous matrix one can destabilize and also stabilize the fluid layer system, this may be due to the presence of magnetic field.



(3a)



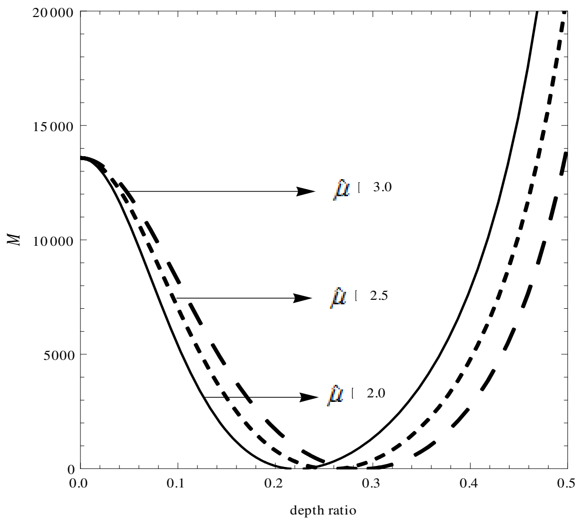
(3b)



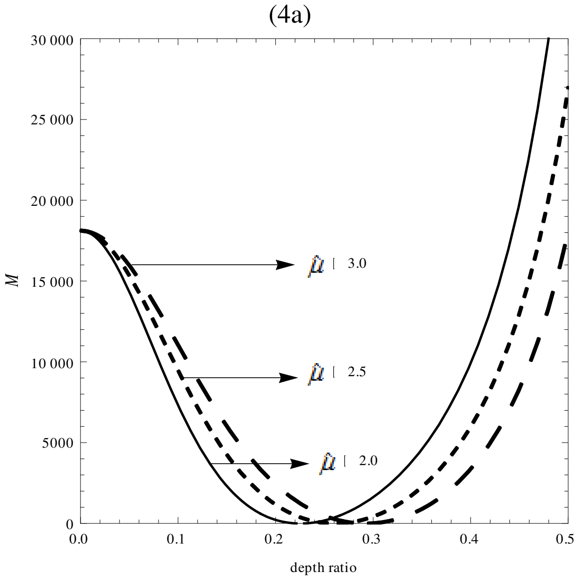
(3c)

Figure-3a, 3b & 3c. The effects of Q on the thermal Marangoni number M.

Figure-3 exhibits the effects of the magnetic field on the onset of convection by the Chandrasekhar number Q. From the figures it is clear that the Thermal Marangoni number for the parabolic profile is more than that for the inverted parabolic profile for a fixed value of depth ratio. At the values of $\hat{d} = 0.2$ *to* 0.24, the effects of both the profiles are neutral and there is no effect of the Q on the thermal Marangoni number. The curves for the three Chandrasekhar numbers for both the profiles are converging up to the value of $\hat{d} = 0.24$, whereas the three curves diverge for the values of the depth ratio $\hat{d} \geq 0.2$. For both the profiles, when the value of the Chandrasekhar number is increased, the Thermal Marangoni numbers increase and hence stabilize the system. That is the Marangoni convection is delayed for the smaller values of $\hat{d}$ that is for values of $\hat{d} \leq 0.2$ and $\hat{d} \geq 0.24$.



(4a)



(4b)

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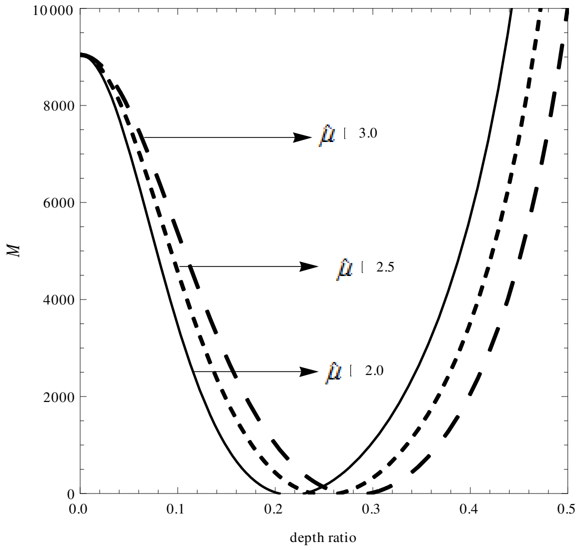
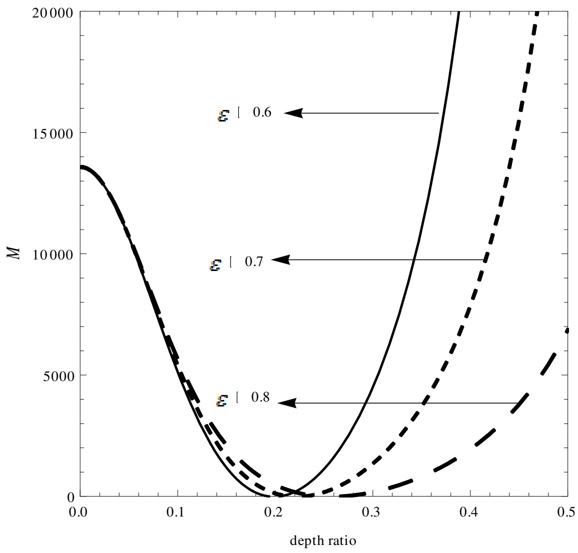


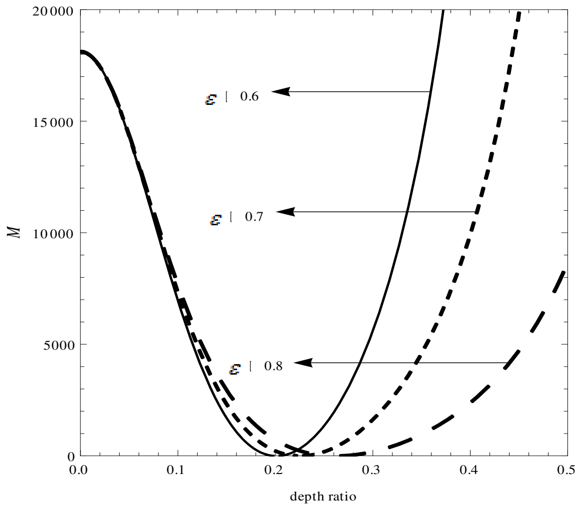
(4c)

Figure-4a, 4b& 4c. The effects of $\hat{\mu}$ on the thermal Marangoni number M.

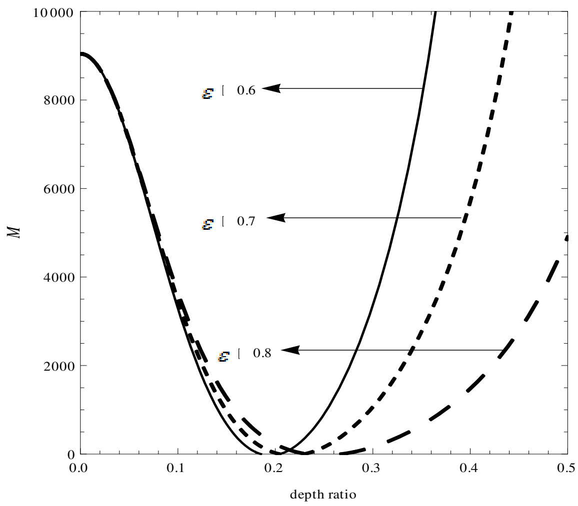
The effects of the viscosity ratio $\hat{\mu}$, which is the ratio of the effective viscosity of the porous matrix to the fluid viscosity are displayed in Figures 4a, 4b and 4c. The curves diverge and again converge between the values of depth ratio $0 \le \hat{d} \le 0.24$. The curves are diverging for the values of the depth ratio $\hat{d} \ge 0.24$ for both the profiles and the behaviour of the change in the viscosity ratio reverses. Increase in the value of the viscosity ratio increases the thermal Marangoni number for the values of depth ratio $0 \le \hat{d} \le 0.24$. Whereas the same decreases the thermal Marangoni number for $\hat{d} \ge 0.24$. The effect of the viscosity ratio is to stabilize the system for smaller values of the depth ratio, while the effect of the same is to destabilize the system for later values of the depth ratio.



(5a)



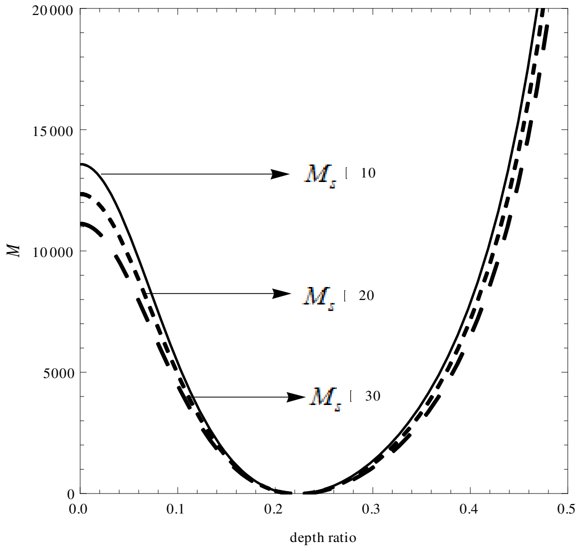
(5b)



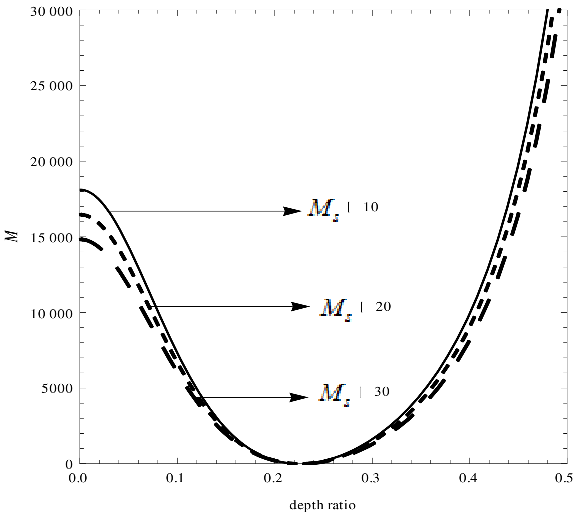
(5c)

Figure-5a, 5b & 5c. The effects of ε on the thermal Marangoni number M.

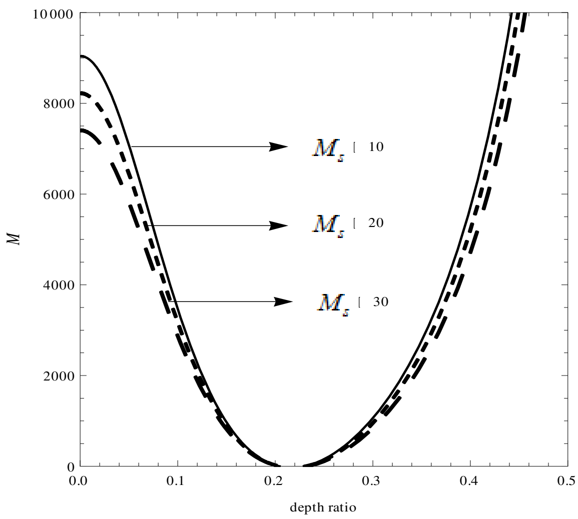
Figures 5a, 5b and 5c depict the effects of porosity ε, on the Thermal Marangoni numbers $M_1, M_2 \& M_3$ for the Linear, parabolic and inverted parabolic profiles respectively. For the both the profiles, upto the value of depth ratio $\hat{d} = 0.2$ there is no effect of porosity on the thermal Marangoni number. For the values of the depth ratio $\hat{d} \ge 0.2$ the curves are diverging and for a fixed value of depth ratio, increase in the value of porosity decreases the thermal Marangoni number that is to destabilize the system. In other words the increase in the void volume of the porous layer decreases the thermal Marangoni number and hence destabilizes the system.



(6a)



(6b)



(6c)

Figure-6a, 6b & 6c. The effects of M_s on the thermal Marangoni number M.

Figures 6a, 6b and 6c displays the effects of the solute Marangoni number M_s on the thermal Marangoni number M. The graph has three converging curves. This number has dual effect on the thermal Marangoni number. For values of $M_s \leq 0.2$ the curves are converging and here for a fixed depth ratio the increase in value of M_s increases the thermal Marangoni number where as, for the values of depth ratio $M_s \geq 0.2$ the curves are diverging, and here for a fixed depth ratio the increase in value of M_s decreases the thermal Marangoni number.

9. CONCLUSIONS

The increase in the values of Horizontal wave number a, the Chandrasekhar number Q, and the porosity ε increases the thermal Marangoni number for both the parabolic and inverted parabolic temperature profiles, hence their effect is to delay the surface tension driven convection i.e., to stabilize the system. Whereas the increase in the values of the Porous parameter β, the viscosity ratio $\hat{\mu}$ and solute Marangoni number M_s, the thermal Marangoni number decreases so the effect of these parameters is to destabilize the system for both the parabolic and inverted parabolic temperature profiles.

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