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BIVARIATE PROBABILITY MODEL FOR WIND POWER DENSITY ANALYSIS: CASE STUDY

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ABSTRACT

The wind power density was investigated in this study to assess the wind energy potential in Kuala Terenganu, Malaysia. The monthly data were statistically analyzed to predict the best distribution that represents bivariate model of wind speed and wind direction. Subsequently, wind power density was assessed by numerical analysis. The results revealed that the estimate mean wind power densities for monthly data are significant with the monsoon seasons in that area. The northeast monsoon effects the East Coast of Peninsular Malaysia, including Kuala Terenganu. Gama distribution together with finite mixture of von Mises is best in represent the monthly bivariate model of wind speed and direction in Kuala Terenganu.

Keywords: gama distribution, finite mixture of von Mises, wind power density.

INTRODUCTION

Previous studies have established that power density analysis by using univariate probability model (Islam, Saidur, and Rahim 2011; Masseran, Razali, and Ibrahim 2012; Safari and Gasore 2010; Siti, Norizah, and Syafrudin 2011). In this study, the analysis through bivariate probability model is considered. The bivariate model consists of linear-circular data which represents the wind speed and wind direction. Studies on wind direction are limited compared to wind speed due to its nature of circular variable. Wind direction solely, is unable to generate wind power. This study combines the wind speed and wind direction in a joint probability distribution and analyze their effect on wind power generation. Previously, (José A. Carta, Ramírez, and Bueno 2008; Chen and Zhang 2009; Soukissian and Karathanasi 2017; Zhang *et al*. 2011) have studied the joint probability distribution or bivariate probability model and their effect to the wind power. But, no such studies have been done in Malaysia. The effect of wind distribution is being studied with the wind data of Kuala Terengganu. Kuala Terengganu (as shown in Figure-1) is located at East Coast of Peninsular Malaysia. It is generally fairly hot and humid all year around averaging between 28^0 to 30^0C in day time. Kuala Terengganu faces northeast monsoon season which bring heavier rainfall starts in early November and ends in March.

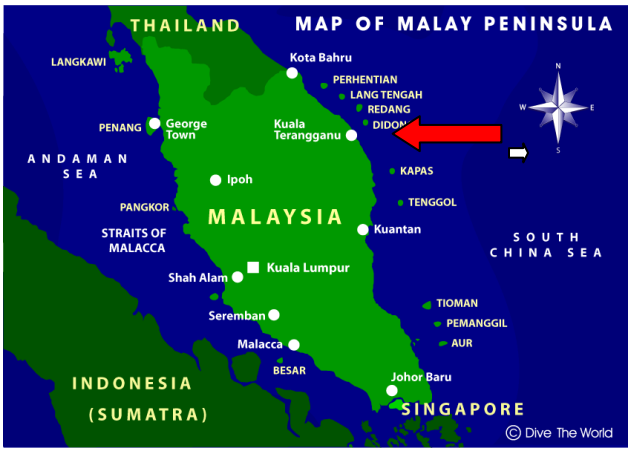

Figure-1. Location of Kuala Terengganu, Malaysia.

METHODOLOGY

Univariate probability models for wind speed and wind direction

Wind speed data consists of continuous linear variable data and can be represented by numerous distributions such as Rayleigh, Lognormal, Exponential, Inverse Gaussian and many more. This study focus on these four distribution namely Weibull, Gama, Inverse Gama and Burr. The probability density function (pdf) for each distribution is shown in Table-1. The parameters for each distribution are estimated by using the Least Square method for Weibull and Expectation Maximization method for other distribution. Nelder-Mead iteration technique is then being used for some distributions that need iteration in its estimation.

Table-1. Probability distribution for each selected distribution.

Distribution	Probability density function (pdf)
Weibull	$f(v)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}v^{\alpha-1}\exp\left(-\frac{v}{\beta}\right)$
Gama	$f(x)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right)$
Inverse Gama	$f(v)=\frac{\beta^{p}}{\Gamma(p)}v^{-p-1}\exp\left(-\frac{\beta}{v}\right)$
Burr	$f(v)=\frac{aqv^{a-1}}{b^{a-1}\left[1+(v/b)^{a}\right]^{1+q}}$

Finite mixture von Mises (FMvM) distribution is used in representing the wind direction data. The flexibility of this distribution in explaining data with more than one mode (Kamisan *et al*. 2011; Masseran *et al.* 2013; José Antonio Carta and Ramírez 2007) has made it more visible among all circular variable distribution. The probability density function (pdf) for Finite mixture von Mises distribution is shown in equation 1.

$$f_{\theta}(\theta)=\sum_{j=1}^{H}\frac{\omega_j}{2\pi I_0(\kappa_j)}\exp\left[\kappa_j\cos(\theta-\mu_j)\right] \quad (1)$$

For:

H is the number of components in the mixture; $\kappa_j \geq 0$ and $0 \leq \mu_j < 2\pi$ is the parameter; $0 \leq \omega_j \leq 1, (j=1,2,.....,H)$ and $\sum_{j=1}^{H}\omega_j = 1$ μ_j is the mean direction parameter and is the concentration parameter with $I_0(\kappa_j)$ denoting the modified Bessel function of the first kind and order zero as defined in equation (2).

$$I_0(\kappa_j)=\frac{1}{\sqrt{2\pi}}\int_0^{2\pi}\exp\left[\kappa_j\cos\theta\right]d\theta=\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{\kappa_j}{2}\right)^{2k} \quad (2)$$

Bivariate probability models for wind speed and wind direction

The bivariate probability model is in the analytic form of joint distribution of linear and angular variables which can be described and assess by Johnson and Wehrly model (Johnson R.A and Wehrly T.E 1978) which has been widely used (José A. Carta, Ramírez, and Bueno 2008; Soukissian and Karathanasi 2017). The probability distribution function defines in equation (3).

$$f_{v,\theta}(v,\theta)=2\pi g(\xi)f_v(v)f_{\theta}(\theta) \quad (3)$$

For

$0 \leq \theta < 2\pi$; $-\infty < v < \infty$ and $g(\xi)=2\pi\left[F_v(v)-F_{\theta}(\theta)\right]$

This study proposes four models which of the all four models for linear variables to be joint with the finite mixture of von Mises (FMvM) distribution for the circular variable.

(a) Weibull – FMvM
(b) Gama - FMvM
(c) Inverse Gama - FMvM
(d) Burr - FMvM

The best model for each studied station may vary to another station based on the goodness-of-fit test which is R^2 determination coefficient and Akaike Information Criterion (AIC) for the univariate model. The highest value of R^2 determination coefficient; in equation (4) and lowest value of Akaike Information Criterion (AIC) will be selected as the represented model for that particular station.

$$R^2=\frac{\sum_{i=1}^{n}\left(\hat{F}_i-\bar{F}\right)^2}{\sum_{i=1}^{n}\left(\hat{F}_i-\bar{F}\right)^2+\sum_{i=1}^{n}\left(F_i-\bar{F}\right)^2} \quad (4)$$

While for the bivariate model, the best-fit model determined by the highest R^2 determination coefficient and lowest root mean square error (RMSE).

Wind power density analysis for bivariate probability model

The wind power density, P_0 can be represent theoretically by using equation (5)

$$P_0=\frac{1}{2}\rho_k v^3 f_v(v) \quad (5)$$

As in this study, the variability of wind speed and wind direction are considered, the evaluation of wind energy in such location is considering the wind power density distribution. This is represented by the following relation

$$P=\iint\frac{1}{2}\rho_k v^3 f_{v,\theta}(v,\theta)d\theta dv \quad (6)$$

For Malaysia case, the value of air density(Sopian 1995), $\rho_k = 1.16 kg/m^3$. The Monte Carlo method which considering iteration techniques is used for obtaining numerical solutions to the double integration in equation (6). The value of wind power density is in W/m^2.

RESULTS AND DISCUSSIONS

Bivariate probability models for wind speed and wind direction

This study focus on the bivariate probability models and its power energy analysis. Therefore, this section will display and discuss the results regarding bivariate models only. Table-2 shows the value of R^2 and RMSE for each proposed model.

Table-2. Results for R^2 and RMSE.

Proposed models	R^2 coefficient value	RMSE
Weibull – FMvM	0.7125	10.788
Gama - FMvM	0.9924	10.173
Inverse Gama - FMvM	0.9817	10.246
Burr - FMvM	0.8632	10.697

Based on the highest value of R^2 and the lowest value of RMSE, result in Table-2 shows that Gama-FMvM is the best bivariate model represents Kuala Terengganu wind data. The correlation between wind speed and wind direction for Kuala Terengganu is somewhat moderate with all the values between 0.2-0.7.The circular-linear correlations of Kuala Terengganu data is shown in Figure-2.

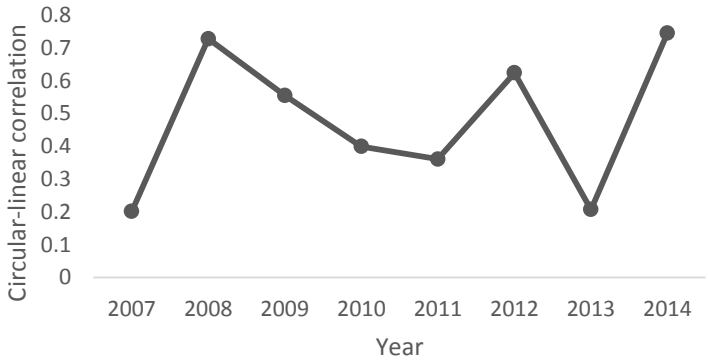

Figure-2. Circular-linear correlation between wind speed and wind direction.

Wind power density analysis for bivariate probability models

Annual wind power density (W/m^2) for Kuala Terengganu, has been analyzed and the result as in Figure-3. Based on the mean value, there is 1.72203 percent increasing on bivariate model compare to univariate probabilistic model for Kuala Terengganu annual wind power density. However, data for the year 2007 and 2013 show some decrease for about 1.527 and 0.249 respectively. The inconsistent wind power density (W/m^2) between univariate and bivariate model indicates that the higher wind speed is not concurrent with the dominant direction of wind which is parallel with (Satari *et al*. 2015) verification that there is a gradual change in the direction of wind in their studies.

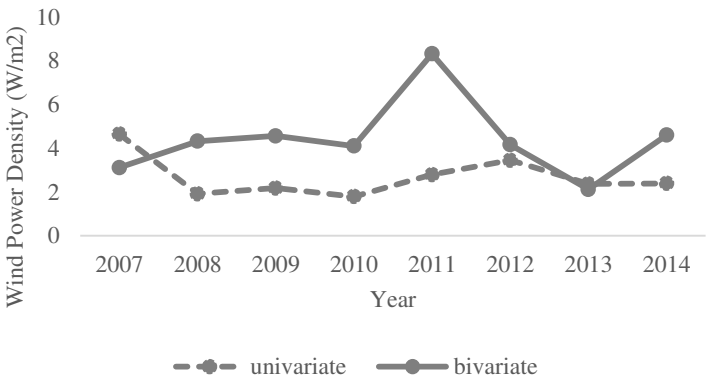

Figure-3. Wind power density (W/m^2) for Kuala Terengganu, Malaysia.

CONCLUSIONS

Gama-FMvM probability distribution is found to be the best bivariate models that represent Kuala Terengganu wind data. The estimation mean value for wind power density is higher in bivariate compare to univariate model by 1.72203 W/m^2. This increment has proven that wind direction affects the wind capture and its role cannot be denied as with the accurate position, the

sensor would capture maximum wind speed, hence will optimize the wind power generation. Although the mean value is not promising for building a wind turbine, but with wind and solar hybrid system might increase the potential of generating wind power in K. Terengganu.

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