VOL. 13, NO. 2, JANUARY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

HYBRID ALGORITHM APPLICATION FOR PREDICTION OF NONRENEWABLE ENERGY PRICE

Wahab Musa and Wrastawa Ridwan
Department of Electrical Engineering Universitas Negeri Gorontalo Kota Gorontalo, Indonesia
E-Mail: wmusa@ung.ac.id

ABSTRACT

In the last decade, energy consumption in Indonesia has seen an average increase of 7-8% per year as population and economic growth continue to improve. This condition requires the availability of good energy to support economic activities and social dynamics of the community. Nevertheless, there are various challenges and obstacles to meet the energy needs such as petroleum production, which tends to decline, while the acceleration of new renewable energy development is expected to become the new backbone of national energy is still not maximized. Under these conditions, all efforts to realize energy security must be a priority agenda for Indonesia. The importance of realizing energy security is due to the dynamics of the global energy sector in the coming years not only influenced by supply, demand and price, but also other factors such as geopolitical issues and stability of areas where world energy sources are located. In this research, we will develop a hybrid algorithm application to predict the non renewable energy price in Indonesia. Hybrid algorithm in this study is a combination of genetic algorithm with Nelder Mead and named rvGA-eNM. The development model of computational intelligence conducted in this research is utilizing the advantages of the Nelder Mead algorithm in exploiting the optimal solution through local search and Genetic algorithm capability in conducting optimal solution exploration in the global search area. Data on non-renewable energy prices will be used to measure the performance of proposed hybrid models in the form of historical data of non-renewable energy prices several months earlier. The average prediction error will be the reference in choosing the right model for the non-renewable energy price prediction the next few months. The purpose of this research is to improve the accuracy of non-renewable energy prediction pricing model based on computational intelligence. Non-renewable energy prices are predicted using hybrid algorithm optimization. Predicted non-renewable energy prices during 2005-2014 are shown in figure visualizes the comparison between the actual value and the non-renewable energy price prediction. The values shown shows that in most test points, the prediction value approximates the actual (adjacent) values. This explains that the accuracy of the rvGA-eNM model used in the prediction of non-renewable energy prices has high robustness properties.

Keywords: non renewable energy, energy price, prediction accuracy, hybrid algorithm.

1. INTRODUCTION

In general, frequently used prediction methods include: Time series, statistical approach, econometric, End-use, Neural Network based model and Hybrid Algorithm based method.

Time series forecasting methods are an important aspect of the field of research, which includes energy demand and computer science. Traditional procedures such as the combination of auto-regression (AR) and Moving Average (MA) are included in the Time series' method popularized by Box and Jenkins in the 1970s. However, predictive problems arise for nonlinear models because they require much data availability, complex patterns that often cannot be extracted linearly.

A statistical approach was developed to simplify the medium-term energy price forecast model, make them more accurate, and to avoid the use of large amounts of data. A statistical model for load parameters was developed in a study by Feinberg *et al*. (2003). For energy price prediction, regression method is usually used to model the relation of energy price and the factors that influence it. The regression model incorporates deterministic, stochastic and exogenous influences.

Econometrics is a set of quantitative tools for analyzing economic data. Economists need to use economic data, among others, to predict the impact of policy changes and predict what might happen in the future (Contos *et al*, 2009). The econometric approach incorporates economic theory and statistical techniques for prediction. The approach estimates the relationship between energy prices (dependent variable) and the factors that influence it.

The End-Use method is widely used for estimating long-term electrical energy. The End-use model focuses on a wide range of electrical energy usage in industrial, commercial, and household sectors based on the principle that electricity demand comes from customer demand for heating, cooling, light, etc. In this method, the age distribution of equipment is important for certain types of equipment. The End-use model describes the energy demand as a function of the amount of equipment on the market (Feinberg *et al.,* 2003).

However, the simulation model based on the End-use approach requires a description of the equipment used by the customer, customer behavior, house size, population dynamics, equipment age and technological changes.

Neural Network has been widely used to solve prediction problems. One of the most promising approaches is the combination of NN with other techniques such as genetic algorithms, evolutionary strategies, etc. This technique, if used correctly, can be very high efficiency (Reyhani & Moghadam, 2011). However, in terms of fitness functions, there are still some

shortcomings in finding better predictive results. In applying Neural Network for energy price prediction, it is necessary to choose one of a number of architectures.

Hybrid algorithms perform optimizations that are divided into global search methods and local searches. Local search methods usually converge on local optima. The Genetic Algorithm (GA) has recently attracted many researchers' attention as a reliable stochastic search algorithm for solving problems. GA has been used by researchers in Ozturk & Ceylan (2005) and Azadeh *et al.* (2006) as an optimization tool for complex problems involving many variables and combinations of linear and nonlinear equations. However, these models have not found the best solution yet, still have a high predictive error rate, and long iterations, causing high commuting costs and long operating time.

2. NONRENEWABLE ENERGY PRICE PREDICTION

Some nonrenewable energy price prediction techniques have been used, among others, Linear Regression. Khazem, H. A (2008) has used the Artificial Neural Network (ANN) in his research to predict future nonrenewable energy prices. The variables affecting nonrenewable energy prices when using the ANN model are Federal Reserve Interest Rate, the consumer price index (CPI), the world crisis and events (EVENT), the natural gas (NG) futures contracts, the Heating Oil futures contracts (HO), the West Texas Intermediate (WTI) Cushing, Oklahoma, crude oil Spot Prices (SPOT), and the futures contracts of crude oil.

3. RESEARCH STEPS

The flow diagram used in the study as shown in Figure-1.

- The first step in the proposed research step is to select an appropriate prediction model. Selection depends on the approximate time horizon, available data, available time, and operating costs. Preferred prediction models use available data for energy prices collected from energy agencies, model variable data from valid sources.

The second step is the collection of data. In developing a non-renewable energy price prediction model, the simulation and processing of each model with available data is needed to obtain an estimated distribution of energy prices. Each prediction method is then tested in a special way with respect to the above non-renewable energy price variable. Data for non-renewable energy prices collected can be categorized as time series data and may have an integer sequence, so it needs to be normalized. After the prediction process, the data is returned to its original value by denormalization.

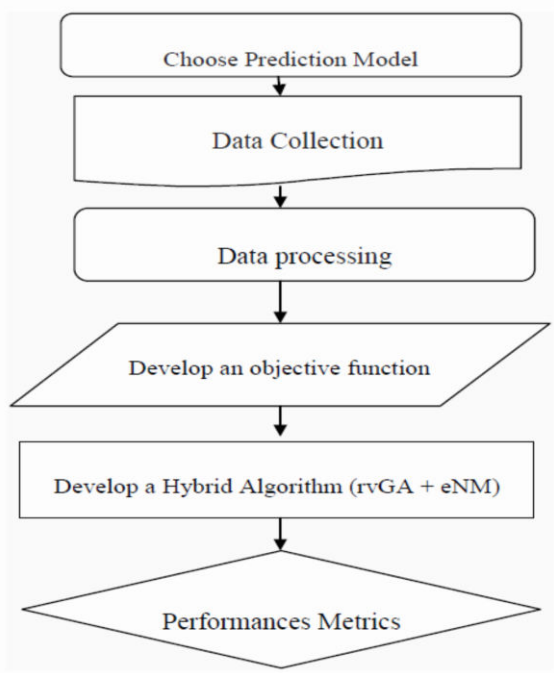

Figure-1. Research steps.

The third step is data processing. Representation of the objective function of each model is a fitness function that represents the relationship between non-renewable energy prices in the form of dependent variable with the independent variable. In this method, objective functions are linear and nonlinear models. Models that have been developed can then be applied to predictions according to the time period.

The fourth step is to develop an objective function in the form of a formula that states the relationship between non-renewable energy prices and the variables that influence them. This relationship is expressed by mathematical formulas and the influence of each variable on the magnitude of non-renewable energy prices, associated with the algorithm used to calculate the price.

The fifth step is developing algorithms. The proposed approach is known as the real-valued genetic algorithm and the extended-Nelder-Mead (rvGA-eNM). Nelder Mead's improved local search algorithm is used to exploit solutions around individuals in the local environment, while genetic algorithms make exploration in the global population. Individual solutions will experience a good evolution of rvGA and the exploitation of local environmental solutions from eNM in each iteration. This can be done, among other things, taking the result of a solution from rvGA as the initial solution of eNM.

The sixth step is to measure the performance of the hybrid algorithm model. The performance of the non-renewable energy price prediction model is measured by the proposed hybrid algorithm, the actual value of non-renewable energy prices and the estimated results from each model are compared. Once the approximate results of the proposed model are obtained, it needs to be validated

for accuracy. The model selection for this study is based on several criteria. In this methodology, models with sufficient degree of accuracy will be the nominee chosen for predicting future non-renewable energy prices. An important consideration that the purpose of any predictive activity is to provide estimates with sufficient accuracy at the lowest possible cost.

4. RESULTS AND DISCUSSIONS

4.1 Model development

This section describes the development of a non-renewable energy price forecasting model that includes a description of the hybrid algorithm between rvGA and eNM and the formulation of the mathematical model used to calculate the energy price.

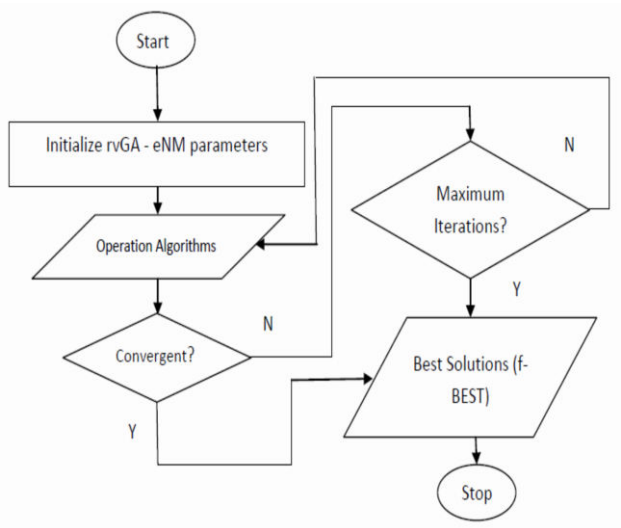

Figure-2. rvGA-eNM flowchart.

4.1.1 Description of rvGA-eNM

The steps in the hybrid algorithm (rvGA-eNM) follow the flowchart in Figure-2. The Hybrid algorithm will calculate the fitness value of the objective function obtained at each iteration. This value is the minimum individual solution (global minima) that can be achieved when convergent. If the targeted search is an error value that states the deviation between the actual data and the predicted result, then under ideal conditions this value is zero. But this ideal condition is difficult to achieve, or in other words, it is difficult to achieve 100% accuracy.

The steps in the rvGA algorithm can be divided into six stages: (i) initialization, (ii) encoding the individual, (iii) crossover and mutation, (iv) decoding the individual, (v) fitness evaluation, (vi) stops checking criteria.

For initialization, the initial population range should cover the overall solution space possible. Each individual variable will be generated randomly within the specified range.

The 'derivative' solution will be first produced by a crossover process in which all the variables of the individual solution will be grouped and converted to binary form. In this study, a uniform crossover was used for 40 bits of each individual. One-point, two-point, and uniform crossovers, have different rules on how derived solutions inherit the characteristics of the 'parent'.

The mutation process will prevent premature convergence in poor solutions. The mutation operator involves an absolute bit (arbitrary) in a genetic sequence that has a probability to change from its original state. This is done by reversing some random sections of the genetic sequence '0' to '1' or from '1' to '0'.

The solutions obtained from rvGA after crossover and mutation will be converted back to real form. This solution is used as a starting point (x0) for Nelder Mead's local search algorithm (eNM).

Evaluation step. In this study, MSE, RMSE, MAD, and MAPE were used as a fitness evaluation function to measure the smallest error (f-BEST) between actual nonrenewable energy prices and forecasting values. The reproduction process will be repeated until one of the stop conditions is met. Typically, the final criterion will be one of the following conditions: (i). Solutions that meet the minimum criterion found (convergent). (ii). Maximum iteration is reached.

4.1.2 Formulation of mathematical models

The objective function for the non-renewable energy price model is expressed by the following mathematical equations:

$$Yt = f[\gamma\ (Ytn)] \quad (1)$$

Where Yt is the actual value, and f [γ (Ytn)] is the predicted value in the nth month, γ is the parameter to be calculated by the hybrid algorithm.

Mathematically the relationship between nonrenewable energy prices and operational variables can be expressed by Equations 4.2 to 4.5 as follows:

$$HE1 = \gamma 0 + \gamma 1 \log x1 + \gamma 2 \log x2 + \gamma 3 \log x3 + \gamma 4 \log x4 \quad (2)$$

Where HE1 is Energy Price model 1, x1 is the energy price of B-1 (month-1), x2 is the energy price of B-2 (month-2), x3 is the energy price B-3, x4 is the energy price B-4; γ0, γ1, γ2, γ3, γ4 are model parameters when the algorithm reaches the global optimum value. This algorithm will determine how important this parameter is in influencing the non renewable energy price in Indonesia every month.

Non-renewable energy prices can also be expressed by non-linear relationships, such as mixes between linear and nonlinear (HE2), exponential (HE3), and quadratic (HE4) (Musa & Tansa, 2016).

Mixed (HE2):

$$HE2 = \gamma 1 + \gamma 2 * \exp(\gamma 3 + \gamma 4\ x1 + \gamma 5\ x2 + \gamma 6\ x3 + \gamma 7\ x4) \quad (3)$$

Exponential (HE3):

$$HE3 = \gamma 1 + \gamma 2\ x1\ \gamma 3 + \gamma 4\ x2\ \gamma 5 + \gamma 6\ x3\ \gamma 7 + \gamma 8\ x4\ \gamma 9 \quad (4)$$

Quadratic (HE4):

$$HE4 = \gamma1 + \gamma2\ x1\ \gamma3 + \gamma4\ x2\ \gamma5 + \gamma6\ x3\ \gamma7 + \gamma8\ x4\ \gamma9 + \gamma10\ x1\ x2 + \gamma11\ x1\ x3 + \gamma12\ x1\ x4 + \gamma13\ x2\ x3 + \gamma14\ x2\ x4 + \gamma15\ x3\ x4 \quad (5)$$

The respective parameters of the four mathematical models will be calculated using rvGA-eNM to find out which model is more precise or more accurate in predicting nonrenewable energy prices.

4.2 Experimental data

4.2.1 Input data for rvGA-eNM algorithm

The data for the rvGA-eNM algorithm is very important because it is necessary to test the performance of the model in the calculation of parameter values. The parameters to be calculated consist of independent variable data and its effect on the dependent variable.

Independent variable include energy price data of previous months, for this research takes variable data until 5 months before the base month of calculation. Data are obtained from legitimate sources that provide data for use in research (EMR, 2017). Table 4.1 shows non-renewable energy price data within 120 months ie from January 2005 to December 2014.

Table-1. Indonesia crude oil price (ICP) US$/barrels.

Year	Jan	Feb	Mar	Apr	May	Jun
2014	105.80	106.08	106.90	106.44	106.20	108.95
2013	111.07	114.86	107.42	100.19	99.01	99.97
2012	115.91	122.17	128.14	124.63	113.76	99.08
2011	97.09	103.31	113.07	123.36	115.18	113.82
2010	77.29	74.01	78.67	85.48	76.96	75.22
2009	41.89	43.10	46.95	50.62	57.86	68.91
2008	92.09	94.64	103.11	109.30	124.67	132.36
2007	52.81	57.62	61.49	67.91	68.60	69.14
2006	62.26	61.19	61.72	68.92	70.01	67.85
2005	42.39	44.74	53.00	54.88	48.72	52.92
Year	Jul	Ags	Sep	Oct	Nov	Des
2014	104.63	99.51	94.97	83.72	75.39	59.56
2013	103.12	106.50	109.69	106.39	104.69	107.20
2012	102.88	111.72	111.02	109.85	106.68	106.90
2011	117.15	111.67	111.00	109.25	112.94	110.70
2010	73.74	75.94	76.76	82.26	85.07	91.37
2009	64.85	72.47	67.07	72.53	77.08	75.58
2008	134.96	115.56	99.06	70.66	49.32	38.45
2007	75.50	72.32	76.10	82.55	92.10	91.54
2006	71.95	72.82	62.49	55.98	55.90	60.15
2005	55.42	61.09	61.36	58.11	53.96	54.64

http://kip.esdm.go.id/pusdatin/index.php/data-informasi/data-energi/minyak-dan-gas-bumi/harga-minyak-mentah-icp, Accessed on 11 Jan 2017 10.19 PM

4.2.2 Output data rvGA-eNM algorithm

Experiment using the normalization data is done to estimate parameter value which can minimize rvGA-eNM error, so that f_BEST value is obtained as shown in Table-4.2. All parameters (γ1, γ2, γ3, γ4, γ5, γ6, γ7 and γ8) are used to measure the prediction accuracy.

4.2.3 Output data rvGA-eNM algorithm

Experiment using the normalization data is done to estimate parameter value which can minimize rvGA-eNM error, so that f_BEST value is obtained as shown in Table-4.3. All parameters (γ1, γ2, γ3, γ4, γ5, γ6, and γ7) are used to measure the prediction accuracy.

Table-2. HE2 parameters.

Iter	γ1	γ2	γ3	γ4	γ5	γ6	γ7	γ8	f-BEST
100	0.3	0.01	1.56	1.79	0.4	0.34	0.11	0.06	0.62
101	0.3	0.01	1.56	1.79	0.4	0.34	0.11	0.06	0.23
102	-1.07	0.01	5.03	0.82	-0.2	0.02	-0.17	0.11	0.23

Table-3. Best value (f-BEST) by rvGA-eNM algorithm.

Algorithm	rvGA				
Iteration	**80**	**85**	**90**	**95**	**100**
f-BEST	0.83	0.83	0.83	0.80	0.62
Algorithm		eNM			
Iteration			100	101	102
f-BEST			0.62	0.23	0.23

The parameter values for obtaining f-BEST in Table-4.3 are as shown in Table-4.2 for computing HE2. HE2 (Mixed Model) is better than other models as indicated by a smaller 'maximum absolute percentage error' (MAPE=3.0131).

The visual result of the convergence rate of the global optimum solution by rvGA-eNM obtained from the simulation is given in Figure-3. From the image, it can be seen that rvGA-eNM experience, strength, in terms of convergence.

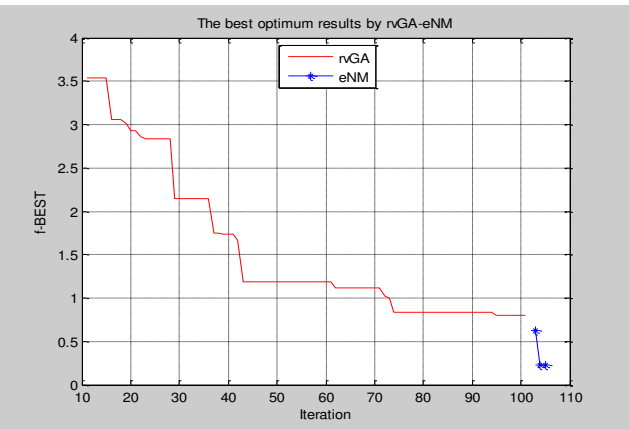

Figure-3. rvGA-eNM convergence rate.

4.2.4 Performance metrics

Predicted non-renewable energy prices during 2005-2014 using these parameters is shown in Figure-4.

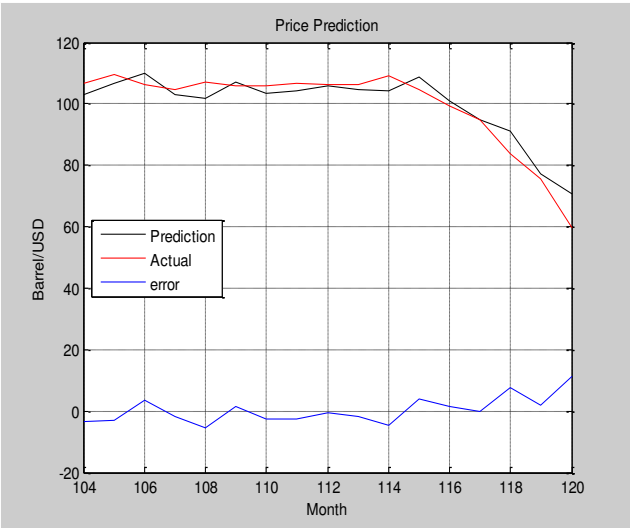

Figure-4. rvGA-eNM prediction.

Model performance is measured in terms of accuracy using MAPE, MSE, RMSE and MAD. The input data are normalized to the value between '0' and '1' so as to facilitate the prediction system to respond well, so that the prediction goes according to the specifications required by the algorithm. Observation time from month 1 to 120 or 10 years (from January 2005 to December 2014). To test the prediction system, we used some non-renewable energy price data for 22 months from month 104 to 120.

Table-4. HE2 predictions.

Month	Actual (USD/Barrel)	Prediction (USD/Barrel)	Error
104	106.5	103.3227	3.1773
105	109.69	106.5312	3.1588
106	106.39	109.5567	3.1667
107	104.69	102.8368	1.8532
108	107.2	101.694	5.506
109	105.8	105.9598	0.1598
110	106.08	103.3419	2.7381
111	106.9	104.4493	2.4507
112	106.44	105.0558	1.3842
113	106.2	104.3575	1.8425
114	108.95	104.0953	4.8547
115	104.63	108.2108	3.5808
116	99.51	100.6305	1.1205
117	94.97	95.2179	0.2479
118	83.72	90.5933	6.8733
119	75.39	77.5894	2.1994
120	59.56	71.4316	11.8716
MAPE (%)	3.0131		
MAD (%)	3.305		
MSE (%)	1.5744		
RMSE (%)	1.2547		

As can be seen, Figure-5 visualizes the comparison between the actual value and the non-renewable energy price prediction. The values shown show that in most test points, the prediction value approximates the actual (adjacent) values. This explains that the accuracy of the rvGA-eNM model used in the prediction of non-renewable energy prices has high robustness properties.

4.3 Projections

The projection is to predict energy prices in the coming months, here taken the price of non-renewable energy in the 120th month to project energy prices in the 121st month until the 122nd month.

Table-5. Price projection (USD/Barrel).

Projection		
120th month	**121st month**	**122nd month**
71.4316	69.7132	70.0202

The price of non-renewable Energy projection for the 121st month is 69.7132 (USD / Barrel) and the 122nd month is 70.0202 (USD / Barrel). As a basis of projection, the price of non-renewable energy is predicted in the 120th month, which is 71.4316 (USD / Barrel).

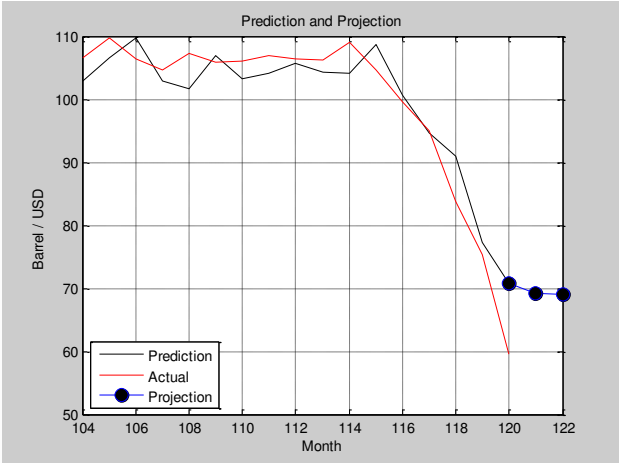

Figure-5. Prediction and projection.

If desired, the long-term projections can be done using the last month as the base value to predict. However, the long-term projection accuracy is less than the actual price. However, this is deviated from the goal of predicting the most recent (urgently needed) short-term energy prices.

In comparison, the following predictions of non-renewable energy prices for the next four months use the base month of the 120th calculation with the actual value.

Table-6. Projection based on the predicted value of the 120th month projection (USD/Barrel).

120th month	121st month	122nd month	123th month	124th month
71.4316	69.7132	70.0202	71.5590	71.7896

Table-7. Projection based on the actual value of the 120th month projection (USD/Barrel).

120th month	121st month	122nd month	123th month	124th month
59.5600	54.3815	55.4896	58.5111	64.3957

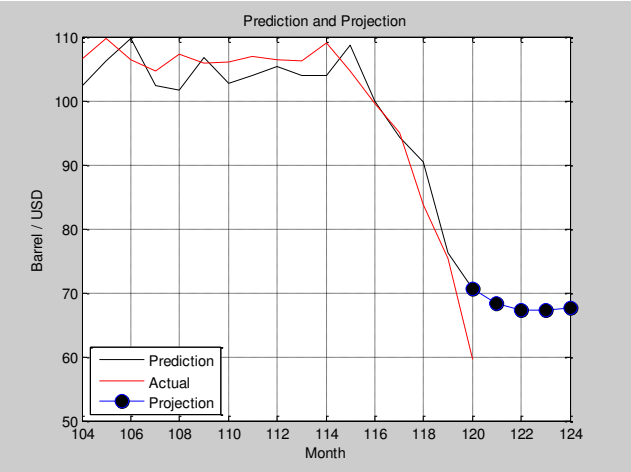

Figure-6. Projection of non-renewable energy price based on prediction value.

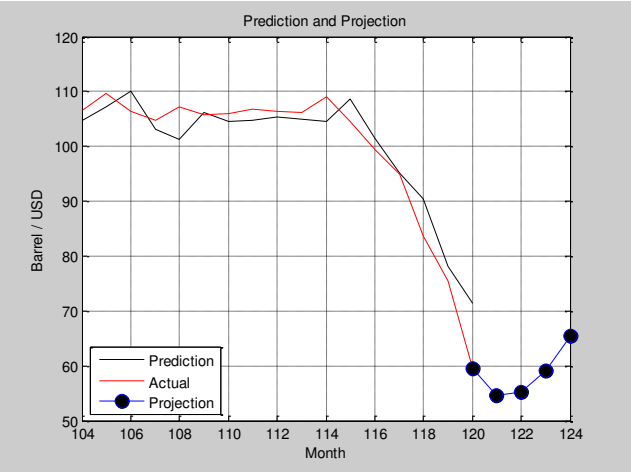

Figure-7. Projection of non-renewable energy prices based on actual values.

5. CONCLUSIONS

One type of hybrid algorithm that proves to be effective for this type of non-renewable energy price prediction application is the real value Genetic Algorithm - extended Nelder Mead (rvGA-eNM). The motivation underlying the use of rvGA-eNM is the ability of this hybrid method to exploit and explore possible solutions so as to accurately estimate non-linear energy price data.

The process of exploration and exploitation of algorithms in search of optimum global solutions is one of the challenges of using hybrids between Nelder Mead and Genetic Algorithm. The problem with using Nelder Mead is the determination of the initial point (IP) in exploring the search for solutions in the local area, whereas Genetic Algorithm is difficult to find the best solution in the global search area.

Sometimes a search by GA is stuck in a state of stagnation even though the search for a solution is repeated. However, GA is easy to find the area where the best solution is located, the results of this discovery being the starting point of the search by NM.

Experiments result visualizes the comparison between the actual value and the non-renewable energy price prediction. The figure shown that in most test points, the prediction value approximates the actual values. This explains that the accuracy of the hybrid rvGA-eNM used in the prediction of non-renewable energy prices has high robustness properties. The accuracy of the prediction based rvGA-eNM can reach 97%.

REFERENCES

Aljanabi A. I. 2010. Interacted Multiple Ant Colonies for Search Stagnation Problem. PhD Thesis, Universiti Utara Malaysia.

Azadeh A., Ghaderi S. F., Tarverdian S. & Saberi M. 2006. Integration of Artificial Neural Networks and Genetic Algorithm to Predict Electrical Energy consumption. 32nd Annual Conference on Industrial Electronics. 2552 -2557.

Contos G., Eftekharzadeh A., Guyton J., Erard B. & Stilmar S. 2009. Econometric simulation of the income tax compliance process for small businesses. Proceedings of the 2009 Winter Simulation Conference. 2902-2914.

El-Mihoub T.A., Hopgood A.A., Nolle L. & Battersby A. 2006. Hybrid Genetic Algorithm: A Review. Engineering Letters. 13, 2-11.

Fan S., Methaprayoon K. & Lee W.-J. 2010. Multi-region load forecasting considering alternative meteorological predictions. IEEE Power and Energy Society General Meeting. 1-7.

Feinberg E.A., Hajagos J.T. & Genethliou D. 2003. Statistical Load Modeling. Proceedings of the 7th IASTED International Multi-Conference: Power and Energy Systems. 88-91.

Franco M., Blanco D., Blequett W., Guglia M. & Alvarado E. 2006. Cointegration Methodology and Error Correction Model used to forecast the Electricity Demand of The Venezuelan Electric System - Period 2004-2024. IEEE/PES Transmission Distribution Conference and Exposition. 1-8.

Huang Y. 2009. Hybrid genetic algorithms and neural network based function approximators for water quality modeling. Dalhousie University (Canada). ProQuest Dissertations and Theses, 276. Retrieved from http: //eserv.uum.edu.my/docview/ 305060608?accountid=42599. (305060608).

Khazem H.A. 2008. Using artificial neural networks to forecast the futures prices of crude oil, ProQuest Dissertations and Theses.

Lian, K., Zhang, C., Li, X., & Gao, L. 2009. An Effective Hybrid Genetic Simulated Annealing Algorithm for

Process Planning Problem. Fifth International Conference on Natural Computation. 5, 367-373.

Mamta M. & Sushila M. 2010. Convalesce Optimisation for Input Allocation Problem Using Hybrid Genetic Algorithm. Journal of Computer Science. 4, 413 -416.

Musa W. & Tansa S. 2016. HAM: A Hybrid Algorithm Prediction Based Model For Electricity Demand. Journal of Theoretical and Applied Information Technology. 92(2): 303-310.

Musa W. 2016. A Hybrid Modern and Classical Algorithm for Indonesian Electricity Demand Forecasting. ARPN Journal of Engineering and Applied Sciences. 11(3): 1782-1786.

Ozturk H. K. & Ceylan H. 2005. Forecasting total and industrial sector electricity demand based on genetic algorithm approach: Turkey case study. International Journal of Energy Research. 29, 829-840.

Pham N., Malinowski A. & Bartczak T. 2011. Comparative Study of Derivative Free Optimization Algorithms. IEEE Transactions on Industrial Informatic. 7(4): 592-600.

Piltan M., Shiri H. & Ghaderi. S.F. 2012. Energy Demand Forecasting in Iranian Metal Industry using Linear and Nonlinear Models Based on Evolutionary Algorithms. Energy Conversion and Management. 58, 1-9.\

Pusdatin. 2011. Kajian Analisis Isu-isu Sektor ESDM, Pusat Data dan Informasi Energi dan Sumber Daya Mineral, Kementerian Energi dan Sumber Daya Mineral.

Reyhani R. & Moghadam A.-M. E. 2011. A heuristic method for forecasting chaotic time series based on economic variables. Sixth International Conference on Digital Information Management. 300-304.

Tan T.-H., Huang Y.-F., Hsu L.-C. & Wu C.-H. 2010. Joint channel estimation and multi-user detection for MC-CDMA system using genetic algorithm and simulated annealing. IEEE International Conference on Systems Man and Cybernetics, 249-256.

Wu L., Wang Y., Yuan X. & Zhou S. 2010. A hybrid simplex differential evolution algorithm. Chinese Control and Decision Conference. 3039-3045.

Yen J., Randolph D., Lee B. & Liao J.C. 1995. A Hybrid Approach to Modelling Metabolic System Using Genetic Algorithm and Simplex Method. IEEE Transaction on System, Man and Cybernetics. 1205-1210.

Zhang X., Pu Y., Ishida K., Ryu Y., Okuma Y., Chen P.-H., Watanabe K., *et al.* 2010. A 1-V input, 0.2-V to 0.47-V output switched-capacitor DC-DC converter with pulse density and width modulation (PDWM) for 57% ripple reduction. IEEE Asian Solid State Circuits Conference. 1-4.