



# NON-NEWTONIAN EFFECTS OF LOAD CARRYING CAPACITY AND FRICTIONAL FORCE USING RABINOWITSH FLUID ON THE PERFORMANCE OF INCLINED MULTI-STEPPED COMPOSITE BEARING

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## ABSTRACT

The theoretical investigations on different type of non-Newtonian fluid called as Rabinowitsh Fluid on the steady characteristics of inclined multistep composite bearings has been analysed. Here the modified Reynolds's closed form expressions are obtained using MATLAB Iterative method. The performance characteristics of different bearings such as plane inclined Slider, composite tapered land; stepped bearing and composite tapered concave bearings are established using the expressions. According to the results, the influences of Rabinowitsh fluid on the bearing characteristics provide an influence in the pressure and therefore which will lead to increase in load carrying capacity.

**Keywords:** inclined multi-stepped composite bearings, non-Newtonian lubricants, rabinowitsh fluid.

## INTRODUCTION

Moving parts are separated using the Lubricant in a system. This separation leads to less friction, Fatigue on the surface (stress limit), less heat generation, noise in an operating system and vibrations. Lubricants help to achieve this in most of the ways. The most common way is to form a physical barrier, or in general way a thin layer of lubricant which helps to separate the moving parts. We can name this process as hydroplaning or the friction loss when a car tire got separated due to moving through standing water from the surface of road. This process is called Hydrodynamic Lubrication. When the surface pressure or temperature is high, the fluid film becomes much thinner and the lubricants help the forces to transmit between the surfaces.

In 2012, Udaya P Singh and Ram S Gupta studied Homotopy Analysis of Curved Slider Bearings using Rabinowitsch Fluid as the Lubricant. He concluded that the bearing characteristics vary with the non Newtonian Pseudoplastic and Dilatant behavior of the fluid changes depending on the slider Curvature.

To study the non linear behavior of various Non-Newtonian lubricants, Rabinowitch fluid model has been taken into account which is a different type of Non-Newtonian model.

For various value of  $k$ , Rabinowitch fluid model can be applied to establish the different behaviours of fluid. For  $k=0$  the behaviour will be Newtonian, for  $k>0$  it will be the pseudo plastic fluids and for  $k<0$ , the dilatants fluids.

Stress-strain relation who holds for one dimensional fluid flow is as follows:

$$\tau_{xy} + k\tau_{xy}^3 = \mu \frac{\partial u}{\partial y} \quad (1)$$

Where  $\mu$  describes the shear rate with zero viscosity,  $k$  denotes the non linear factor responsible for the Non-

Newtonian effect of the fluid. In 2012 Aug, Jaw Ren Lin studied the Non-Newtonian squeeze film characteristic between parallel annular disks.

A theoretical analysis has been made by Naduvinamani Neminath Bhujappa and Rajashekar Mareppa in 2013 May found that the existence of a critical value for profile parameter at which the dynamic coefficient attains maximum.

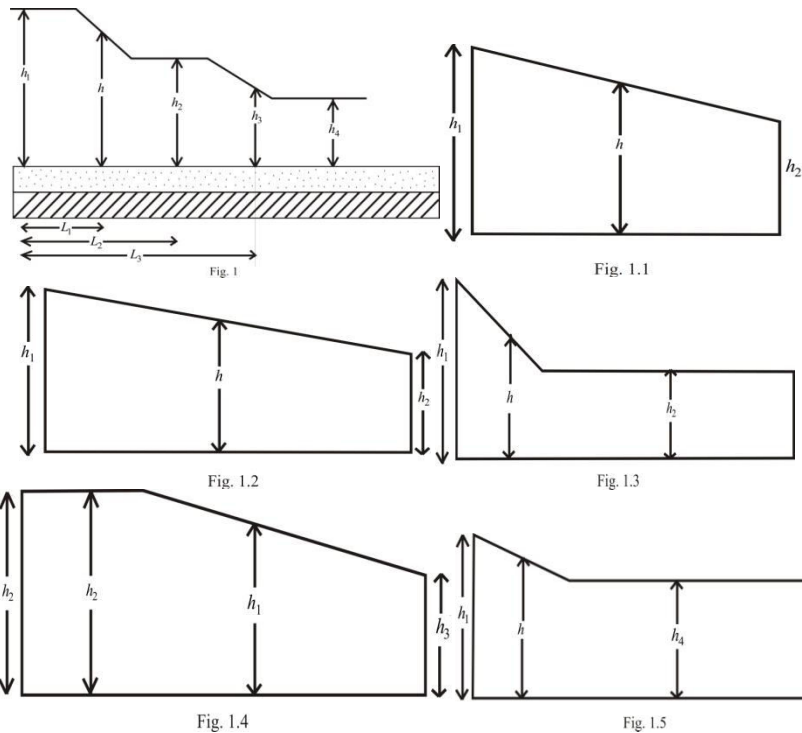
In field like agriculture, Automotive, Construction, Marine, Railroad and material handling equipment needs a long wear and low maintenance matters the most. In such field Composite Bearings are used. It has an advantage like high-load carrying capacity/high shock load capacity, self lubricating design or model, and low coefficient of friction, a resistant to temperature. The physical and mechanical properties of this composite material make it an excellent bearing material composite bearing achieve their finished dimensions using traditional machining system. This gives the designer a high degree of flexibility to produce a finished part which is perfectly aligned to the application. In 1990, N.M Bujurke, H.P Patil studied the porous slider bearing with couple stress fluid. Similarly, N. B. Naduvinamni, Siddangouda found that the load carrying capacity decrease as step height increases in Mar 2009.

## FORMULATION OF THE PROBLEM

Figure-1 shows a schematic diagram of the squeeze film geometry under consideration.

It consists of two surfaces separated by a lubricant. Along the length, the axis  $x$  has been taken while along the lubricant film,  $y$  axis is taken. Lower plane moving with a velocity along  $x$ - axis in its own plate.

Different bearing physical configuration has been shown in Figure-1. It consists of two surfaces separated by fluid called Rabinowitch fluid. The length has been taken as  $x$ -axis while  $y$ -axis is across lubricant film.



Above schematic diagram represents  
Figure-1. Inclined multi stepped composite bearing.  
Figure-1.1, 1.2. Plane slider bearing.  
Figure-1.3, 1.5. Composite tapered land bearing.  
Figure-1.4. Composite tapered concave bearing.  
With reference to Figure-1, the film thickness is defined as:

$$H(x) = h_m + h_s$$

The total film thickness over the interval is defined as:

$$\left. \begin{aligned} h_m + d_1 & \quad 0 \leq x \leq L_1 \\ h_m + d_1 \frac{(L_1 - x)}{(L_2 - L_1)} & \quad L_1 \leq x \leq L_2 \\ h_m + d_2 & \quad L_2 \leq x \leq L_3 \\ h_m + d_2 \frac{(L_3 - x)}{(L_4 - L_3)} & \quad L_3 \leq x \leq L_4 \\ h_m & \quad L_4 \leq x \leq L \end{aligned} \right\} \quad (2)$$

Where  $h_m$  is minimum film thickness and  $h_s$  is slider profile function.

The basic equation under the assumption of Hydrodynamic Lubrication governing Incompressible flow of Non-Newtonian Rabinowitsch fluid is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} \quad (4)$$

$$\frac{\partial p}{\partial y} = 0 \quad (5)$$

The velocity components having the boundary condition are as follows:

$$\left. \begin{aligned} u = U, v = 0 & \quad \text{at } y = 0 \\ u = 0, v = V & \quad \text{at } y = h \end{aligned} \right\} \quad (6)$$

On integrating equation (4) with respect to  $y$  with subject to the boundary condition (6) and using equation (1), the velocity component expression is obtained in the form:

$$u = \left[ \frac{1}{2\mu} (y^2 - yh) + k^* g^3 C_1 \right] + \left[ U - \frac{U(y + k^* g^2 C_2)}{h(1 + 0.25k^* g^2 h^2)} \right] \quad (7)$$

Where

$$C_1 = 0.25y^4 - 0.25hy^3 + 0.75h^2y - 0.125h^3y \quad \&$$

$$C_2 = y^3 - 1.5hy^2 + 0.75h^2y$$

$$g = \frac{\partial p}{\partial x}$$



Using the relevant boundary condition (6) in continuity equation (3) for  $y$ , the Non-Newtonian Rabinowitsh fluid Reynolds's type equation is obtained in the form:

$$\frac{\partial}{\partial x} \left[ 0.166 \lambda h^3 \frac{\partial p}{\partial x} + 0.025 k^* h^5 \left( \frac{\partial p}{\partial x} \right)^3 \right] = \mu U \frac{\partial h}{\partial x} + 2\mu \frac{\partial h}{\partial t} \quad (8)$$

By using the non-dimensional parameters

$$\left. \begin{aligned} h^* &= \frac{h}{h_{m_0}}, x^* = \frac{x}{l}, h_s^* = \frac{h_s}{h_{m_0}} \\ h_m^* &= \frac{h_m}{h_{m_0}}, \tau = \omega t, p^* = \frac{p h_{m_0}^2}{\mu U l} \\ \delta &= \frac{d}{h_{m_0}}, \sigma = \frac{\omega l}{U}, \alpha = \frac{k^* \mu^2 U^2}{h_{m_0}^2} \end{aligned} \right\} \quad (9)$$

The multi-stepped composite bearing dimensionless film thickness for is obtained as from (2) and (9)

$$h(x) - h_m^*(\tau) = h_s^*(x)$$

Where  $\delta$  is the dimensionless wedge parameter of the slider profile?

The Reynolds's type equation for Rabinowitsh fluid is obtained in the form:

$$\frac{1}{2} \frac{d}{dx^*} \left[ h^{*3} \frac{dp^*}{dx^*} + 0.15 h^{*5} \left( \frac{dp^*}{dx^*} \right)^3 \right] = \frac{dh_s^*}{dx^*} + 2\sigma \frac{dh_m^*}{d\tau} \quad (10)$$

$\alpha$  is the non-linear factor responsible for Non-Newtonian effects and  $\sigma$  denotes the dimensionless squeeze number.

The dimensionless minimum film thickness and film pressure can be obtained as

$$h_m^* = 1 + \varepsilon e^{i\tau} \quad (11)$$

$$p^* = p_0^* + \varepsilon p_1^* e^{i\tau} \quad (12)$$

On substituting in equation (10) and on neglecting the higher order terms of  $\varepsilon$ , the steady state characteristics respectively are obtained in the form:

$$0.166 \left[ \frac{d}{dx^*} H^{*3} \frac{dp_0^*}{dx^*} \right] = \left[ \frac{dH^*}{dx^*} - (0.025) \frac{d}{dx^*} (\alpha) H^{*5} \left( \frac{dp_0^*}{dx^*} \right)^3 \right] \quad (13)$$

Where

$$H^* = \begin{cases} H_1^* = 1 + \delta_1 & 0 \leq x^* \leq L_1^* \\ H^* = 1 + \delta_1 \left( \frac{L_2^* - x^*}{L_2^* - L_1^*} \right) & L_1^* \leq x^* \leq L_2^* \\ H_2^* = 1 + \delta_2 & L_2^* \leq x^* \leq L_3^* \\ H^* = 1 + \delta_2 \left( \frac{L_3^* - x^*}{L_3^* - L_2^*} \right) & L_3^* \leq x^* \leq L_4^* \\ H_4^* = 1 & L_4^* \leq x^* \leq 1 \end{cases} \quad (14)$$

$$p_0^* = 0 \quad \text{at } x^* = 0, 1$$

And pressure is continuous at

$$x^* = L_1^* \quad \& \quad x^* = L_4^* \quad (15)$$

$$p^* = p_{00}^* + \alpha p_{01}^* \quad (16)$$

Substituting of the above condition into equation (13) results in the following two equations containing  $p_{00}^*$  &  $p_{01}^*$

$$(0.1666) \frac{d}{dx^*} \left[ H^{*3} \frac{dp_{00}^*}{dx^*} \right] = \frac{dH^*}{dx^*} \quad (17)$$

$$\frac{d}{dx^*} \left[ H^{*5} \left( \frac{dp_{00}^*}{dx^*} \right)^3 + 6.67 H^{*3} \frac{dp_0^*}{dx^*} \right] = 0 \quad (18)$$

On integrating the equation (17) with respect to  $x^*$  gives

$$dp_{00}^* (0.1666 H^{*3}) = (H^* - H_0^*) dx^* \quad (19)$$

Where  $H_0^*$  is the film thickness at which  $\frac{dp_{00}^*}{dx^*} = 0$

Integration of equation (19) and the use of boundary equation (15) give,



$$\left. \begin{aligned}
 p_{00}^* &= \left[ \frac{H_1^* - H_0^*}{H_1^{*3}} \right] x^* & 0 \leq x^* \leq L_1^* \\
 p_{00}^* &= p_{c_1}^* + \int_{L_1^*}^{x^*} \left[ \frac{H_1^* - H_0^*}{H_1^{*3}} \right] dx^* & L_1^* \leq x^* \leq L_2^* \\
 p_{c_1}^* &= \left( \frac{H_1^* - H_0^*}{H_1^{*3}} \right) L_1^* \\
 p_{00}^* &= p_{c_2}^* + \int_{L_2^*}^{x^*} \left[ \frac{(H_1^* + H^*) - H_0^*}{H_1^{*3}} \right] dx^* & L_2^* \leq x^* \leq L_3^* \\
 p_{c_2}^* &= \left( \frac{H_2^* - H_0^*}{H_2^{*3}} \right) L_2^* \\
 p_{00}^* &= p_{c_3}^* + \int_{L_3^*}^{x^*} \left[ \frac{(H_1^* + H^* + H_2^*) - H_0^*}{H_1^{*3}} \right] dx^* & L_3^* \leq x^* \leq L_4^* \\
 p_{c_3}^* &= \left( \frac{H_3^* - H_0^*}{H_3^{*3}} \right) L_3^* \\
 p_{00}^* &= p_{c_3}^* + \int_{L_3^*}^{x^*} \left[ \frac{(1 - H_0^*)(x^* - 1)}{H_4^{*3}} \right] dx^* & L_4^* \leq x^* \leq 1
 \end{aligned} \right\} \quad (20)$$

$$H_0^* = A/B$$

Where

$$A = \frac{H_1^* L_1^*}{H_1^{*3}} - \frac{L_2^*}{H_2^*} + \int_{L_1^*}^{L_2^*} \frac{dx^*}{H^{*2}} + \int_{L_2^*}^{L_3^*} \frac{(H_1^* + H^*) dx^*}{H^{*3}} + \int_{L_3^*}^{L_4^*} \frac{(H_1^* + H^* + H_2^*) dx^*}{H^{*3}}$$

$$B = \frac{L_1^*}{H_1^{*3}} - \frac{(L_2^* - 1)}{H_2^*} + \int_{L_1^*}^{L_2^*} \frac{dx^*}{H^{*3}} + \int_{L_2^*}^{L_3^*} \frac{dx^*}{H^{*3}} + \int_{L_3^*}^{L_4^*} \frac{dx^*}{H^{*3}}$$

On integrating equation (18) and using (19) gives the following results:



$$\begin{aligned}
 p_{o1}^* &= \left( \frac{-31}{2} \right) \left[ \frac{(H_1^* - H_0^*)^3}{H_1^{*7}} \right] x^* & 0 \leq x^* \leq L_1^* \\
 p_{o1}^* &= p_{c1}^* + \left( \frac{-31}{2} \right) \int_{L_1^*}^{x^*} \left[ \frac{(H_1^* - H_0^*)^3}{H_1^{*7}} \right] dx^* & L_1^* \leq x^* \leq L_2^* \\
 p_{c1}^* &= \left( \frac{-31}{2} \right) \left[ \frac{(H_1^* - H_0^*)^3}{H_1^{*7}} \right] L_1^* \\
 p_{o1}^* &= p_{c2}^* + \int_{L_2^*}^{x^*} \left[ \frac{((H_1^* + H_2^*) - H_0^*)^3}{H_1^{*7}} \right] dx^* & L_2^* \leq x^* \leq L_3^* \\
 p_{c2}^* &= \left[ \frac{(H_2^* - H_0^*)^3}{H_2^{*7}} \right] L_2^* \\
 p_{o1}^* &= p_{c3}^* + \int_{L_3^*}^{x^*} \left[ \frac{((1 - H_0^*)^3 (x^* - 1))^3}{H_4^{*7}} \right] dx^* & L_3^* \leq x^* \leq 1 \\
 p_{c3}^* &= \left[ \frac{(H_2^* - H_0^*)^3}{H_3^{*7}} \right] L_3^*
 \end{aligned}$$

### STEADY STATE LOAD CARRYING CAPACITY

On integrating the steady film pressure over the film region, the steady load carrying capacity of the bearing is obtained as follows:

$$W = B \int_{x=0}^L p_0 dx \quad (21)$$

Dimensionless Work load expression is in the form:

$$W^* = \left\{ \frac{0.5(1-H_0^*)(L_4^*-1)}{H_0^*} + \frac{0.5(H_1^*-H_0^*)}{H_1^{*3}} + \int_{L_1^*}^{L_2^*} \left[ \frac{(H_1^*-H_0^*)x^*}{H^{*3}} \right] dx^* + \int_{L_2^*}^{L_3^*} \left[ \frac{(H_1^*+H_2^*-H_0^*)x^*}{H^{*3}} \right] dx^* + \int_{L_3^*}^1 \left[ \frac{(1-H_0^*)^3(x^*-1)^3}{H_4^{*7}} \right] dx^* \right\} \quad (22)$$

### FRICTION

The Frictional Force applied on the bearing sliding surface is given by:

$$F_f = -B \int_{x=0}^L \left[ \tau_{xy} \right]_{y=0} dx \quad (23)$$

$$\begin{aligned}
 &= \frac{H_1^*(H_1^*-H_0^*)}{H_1^{*2}} + \frac{0.5(1-H_0^*)(1-L_4^*)}{H_0^*} + 4(1-L_4^*) + \int_{L_1^*}^{L_2^*} \left[ \frac{(H_1^*-H_0^*)}{H^{*3}} \right] dx^* + \int_{L_2^*}^{L_3^*} \left[ \frac{(H_1^*+H_2^*-H_0^*)x^*}{H^{*3}} \right] dx^* \\
 &\quad + \int_{L_3^*}^1 \left[ \frac{(H_1^*+H_2^*+H^*-H_0^*)x^*}{H^{*3}} \right] dx^*
 \end{aligned}$$

### COEFFICIENT OF FRICTION

$$C^* = \frac{F_f}{W^*}$$

### RESULTS AND DISCUSSIONS

Figure-2 shows the variation of non dimensional steady film pressure  $p_0^*$  with non dimensional coordinate  $x^*$  for different bearing geometries under consideration. The highest film pressure  $p_0^*$  has been observed for inclined stepped bearing as compared to other bearing geometries.



Figure-3 shows the variation of non dimensional steady film pressure  $p_0^*$  with non dimensional coordinate  $x^*$  for various values of non linear parameter  $\alpha$ . It is found that as the parameter values increases the pressure increases gradually and reaches the maximum when  $\alpha=0.05$ .

Figure-4 shows the variation of steady film pressure  $p_0^*$  in a non dimensional form with non dimensional coordinate system  $x^*$  for different negative values of non linear parameter  $\alpha$ . It has been seen that unlike in the case as mentioned above, it shows a varying distribution, it increases and reaches a stagnated point (point of maxima) and decreases instantly with the increase in value of  $\alpha$ .

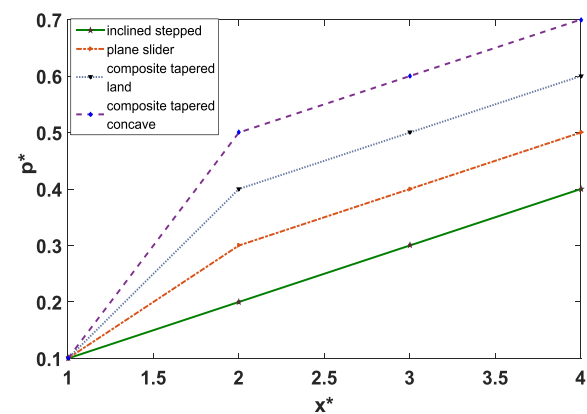
Figure-5 shows the variation of load carrying capacity  $W^*$  in a non dimensional form with dimensionless coordinate  $x^*$  for various bearing geometries. It has been seen that inclined stepped bearing shows the maximum peaks. The effect is advantageous in bearings subjected to load. It helps in inducing the fatigue loading. The danger of fatigue failure of the bearing is reduced.

Figure-6 shows the variation of non-dimensional load carrying capacity  $W^*$  with non linear parameter  $\alpha$  for different bearing geometries. It has been found that with the increase of non linear parameter, the bearing characteristics can be seen clearly, the weight age should be more in inclined stepped bearing region whereas the less weightage should be given to composite tapered concave part.

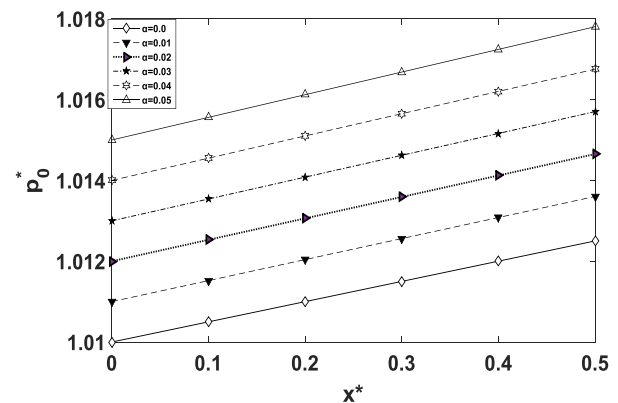
Figure-7 shows the variation of load carrying capacity  $W^*$  in a non-dimensional form with dimensionless coordinate system  $x^*$  for different non linear parameter  $\alpha$ . It can be seen that as we increase the value of non linear parameter, the load capacity increases. The higher load capacity presents possible metal to metal contact at low speeds and reduces wear of the bearing.

Figure-8 shows the variation of coefficient of Friction  $C^*$  with dimensionless coordinate  $x^*$  for different values of non linear parameter  $\alpha < 0$ ,  $\alpha = 0$  &  $\alpha > 0$  has been plotted. As the values of non linear parameter decreases, the coefficient of friction increases, this means that the direction of the force does not affect the direction the physical quantity.

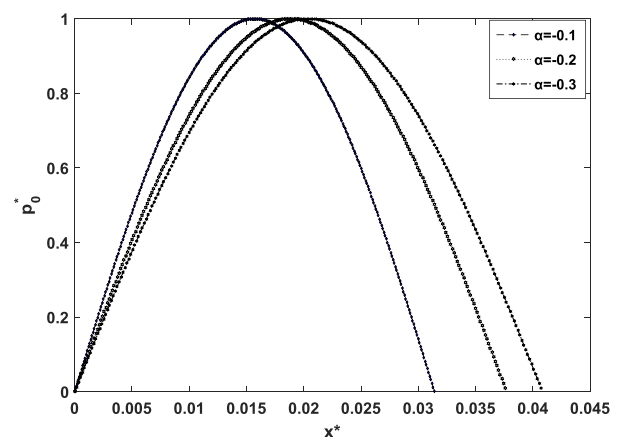
Figure-9 shows the variation of coefficient of Friction  $C^*$  with non linear parameter  $\alpha$  for different bearing geometries. The parameter which affect the performance of the bearing are the non linear parameter, the composite bearing shows the maximum characteristics.



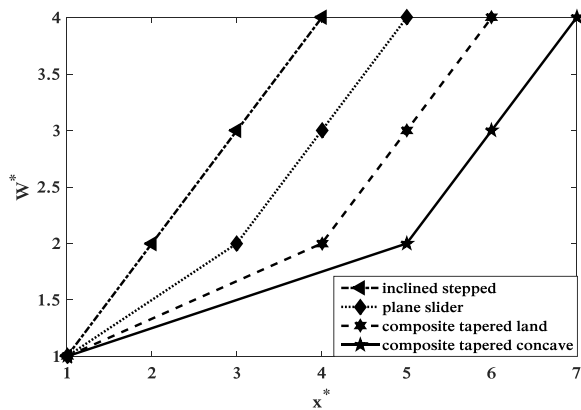
**Figure-2.** The variation of non-dimensional film pressure  $p_0^*$  with  $x^*$  for different bearing geometries with  $L_1^* = 0.25, L_2^* = 0.35, L_3^* = 0.45$  and  $L_4^* = 0.65$ .



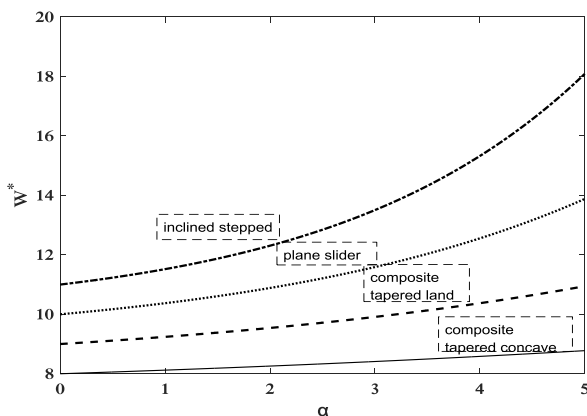
**Figure-3.** The variation of non-dimensional film pressure  $p_0^*$  with  $x^*$  for different values of  $\alpha$  with  $L_1^* = 0.25, L_2^* = 0.35, L_3^* = 0.45$  and  $L_4^* = 0.65$ .



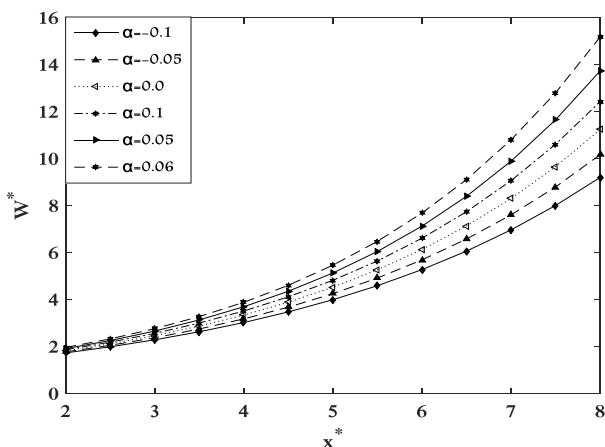
**Figure-4.** The variation of film pressure  $p_0^*$  in a non-dimensional form with  $x^*$  for different negative values of  $\alpha$  with  $L_1^* = 0.25, L_2^* = 0.35, L_3^* = 0.45$  and  $L_4^* = 0.65$ .



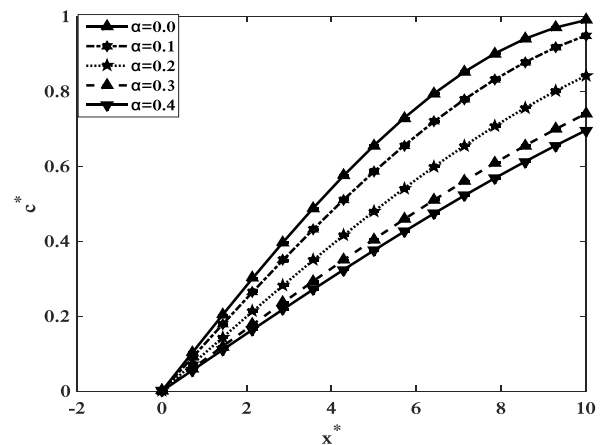
**Figure-5.** Variation of non-dimensional load carrying capacity  $W^*$  with dimensionless coordinate  $x^*$  for distinct bearing geometries.



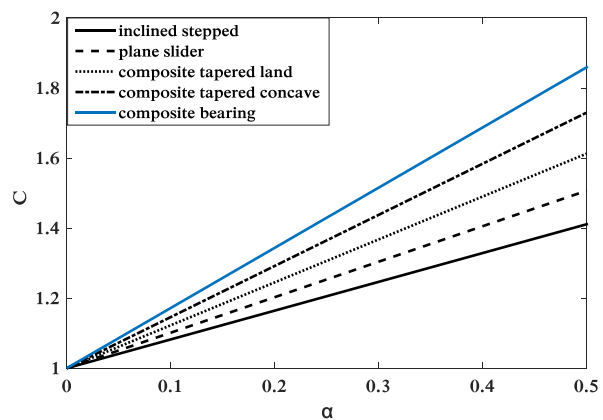
**Figure-6.** Variation of non-dimensional load carrying capacity  $W^*$  with non linear parameter  $\alpha$  for different bearing geometries.



**Figure-7.** Variation of non-dimensional load carrying capacity  $W^*$  with dimensionless coordinate  $x^*$  for different non linear parameter  $\alpha$ .



**Figure-8.** Variation of coefficient of Friction  $C^*$  with dimensionless coordinate  $x^*$  for different values of non linear parameter  $\alpha$ .



**Figure-9.** Variation of coefficient of Friction  $C^*$  with non linear parameter  $\alpha$  for different bearing geometries.

## CONCLUSIONS

Based on the study, this paper predicts the non-Newtonian effects on inclined multi-stepped Composite slider bearing for Non-Newtonian fluid model named as Rabinowitch fluid model. The following conclusions can be drawn on the basis of the results obtained.

- The highest film pressure has been found for the inclined stepped bearing than the other geometries.  $\alpha$
- As we increase the steps we have found that the multi stepped bearings seems to show the largest pressure distribution than any other geometry. We conclude that multi stepped bearing in the presence of Rabinowitch fluid control a motion by controlling the vectors of normal forces that the moving parts bears. This bearing design gives the maximum efficiency, reliability, durability and performance, a maximum outcomes and allow the demands of this application for further studies. It has a long life unless like 30 years (composite bearing).





- c) In summary, there are three observations resulting from the above numerical results. First, it is important to keep track of the work load. Without it, important bearing characteristics will be ignored. Depending upon the bearing characteristics we need to give the load.

The second thing is the consequence of friction i.e. wear, which is responsible for poor performance or performance degradation and/or damage to components of the bearing systems. In any of the system, the flow of material shows an increase in energy density, because initial phase of the transformation and displacement of the material demand acceleration of material and high pressure. Or in general way we can say we can't deform a solid material using direct contact without applying a high pressure. Somewhere along the process must acceleration and deceleration take place, i.e., the high pressure must be applied on all the sides of the deformed material for more effect. Due to phase transformation, the flowing material will immediately exhibit energy loss and will have reduced ability to flow if ejected from high pressure into low pressure. Coefficient of friction helps to holds the high pressure and energy density in the contact zone and decreases the amount of energy or friction force.

And the last one is effect of coefficient of friction on the bearing characteristics. The material combinations of sliding surfaces, along with the lubricant should provide a low friction coefficient for reducing damage and lower running cost. The characteristic of the bearing should be like it must not fatigue, flatten, or split under these loads. If the bearing load resistance is too low, the bearing can flatten which ultimately results in abnormal function of the bearing system.

## REFERENCES

- [1] Stokes V.K. 1966. Couple stresses in fluids. The physics of fluids. 9, Vol. 1966, 1709-1720.
- [2] Hamrock B.J. 1994. Fundamentals of fluid film lubrication. 1<sup>st</sup> edition, New York, Mc Graw-Hill.
- [3] Murti P.R.K. 1974. Analysis of Porous slider bearing. Wear. 28: 131-135.
- [4] Hughes W.F. 1963. The magneto hydrodynamic finite step slider bearing. Journal of Basic Engineering. 85: 129-136.
- [5] Pinkus O. and Sterlicht B. 1961. Theory of hydrodynamic lubrication. McGraw Hill, New York.
- [6] Ramanaiah.G. 1979. Slider bearings lubricated by fluids with couple stress. Wear. 52: 27-36.