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SURFACE CRACK GROWTH IN A SOLID CYLINDER UNDER COMBINED CYCLIC BENDING-TORSION LOADING

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ABSTRACT

Here, we present fatigue crack growth (FCG) of a surface crack in a solid cylinder under combined cyclic torsion-bending load. The effective stress intensity factors were found to be fluctuated during the crack growth. When the loading ratio of the maximum shearing stress over the maximum bending stress was being unity, for a given crack length, the crack aspect ratios between 0.5 and 1 resulted in insignificant differences on the estimated fatigue lives. The effect of the crack depth on the fatigue life was found to be insignificant when the crack position was away from the maximum bending stress location.

Keywords: solid cylinder, surface crack, fatigue crack growth, combined torsion-bending load, boundary element method.

1. INTRODUCTION

The cylindrical bars are widely used in engineering applications for machine components and structures. Surface cracks or flaws are frequently initiated in these components due to cyclic applied loads, material defects and improper manufacturing processes. During service loading, the crack or flaw grows into a critical stage, which can then result in an undesirable catastrophic failure. To ensure the safety of the component to be fit in service, scheduled inspections shall be performed. Thus, understanding of the fatigue crack behavior in components is essential. Linear elastic fracture mechanics (LEFM) approach has been widely used in analyzing the fatigue crack behavior where elastic stress-strain field in the vicinity of crack tip are usually evaluated by calculating the stress intensity factors.

The stress intensity factor (SIF) solutions for a surface crack in a smooth round bar have been reported by many researchers. Most of the available solutions are limited to simple loading cases such as under either of tension, bending or torsion (Rajuand Newman [20], Shih and Chen [21], Shin and Cai [22], Carpinteri [4], Fonte and Freitas [12]). Under such loading, fatigue crack growth and crack shape evolution of a surface crack in a smooth round bar have also been reported by Carpinteri [5], Carpinteri and Brighenti [6], Carpinteri and Vantadori [8], Couroneau and Royer [9] [10], Lin and Smith [14] [15], Thompson and Sheppard [24] and Toribio *et al.* [25] [26] [27]. In practices, engineering components or structures are often subjected to combined loading which in turn causes mixed mode fractures (Modes I, II and III). Thus, fatigue crack growth analyses become more complex. Research on fatigue crack growths under combined loads has been of interest worldwide. Carpinteri and Brighenti [7]evaluated propagation of a surface crack in round bars under combined axial and bending load using finite element method. For a given value of the total combined stress, the propagation path is independent to the loading ratio.

Predan *et al.* [19] investigated the fatigue crack growth in a hollow cylinder subjected to torsion load through experimentations. Yang *et a*l. [28] studied crack growth of a straight-front crack in solid cylinder under cyclic tension and steady torsion loads. They reported that steady torsion loading superimposed on the cyclic tension load led to a significant reduction in the crack growth rates. Abreu *et al.* [1] predicted fatigue life of a notched tubular specimen subjected to combined bending-torsion load by the use of stress-strain intensity and energy-base approaches. Branco *et al.* [3] predicted fatigue life of lateral notched round bar under combined bending-torsion loads. The crack initiation site, surface crack path and surface crack angle were predicted by principle stress field criterion. Firat [11] simulated crack initiation and propagation of notched shaft subjected to in phase and out phase of combined bending-torsion loads using ANSYS finite element with a critical plane criteria approach. The relatively longer fatigue life were found in the case under out-of-phase load in comparison to that under in-phase load. Tanaka[23] assessed fatigue crack growth rate and fatigue life of circumferential notched round bar of steel under combined cyclic torsion and static axial load. Fonte *et al.*[13]evaluated crack growth and crack evolution of a surface crack in a solid shaft under combined cyclic bending and steady torsion loads. Similar to that reported by Yang *et al.* [28], a significant reduction of the crack growth rate was shown when a steady torsion was superimposed to cyclic bending. Maligno *et al.* [16] simulated fatigue crack growth of a surface crack in a solid cylinder under combined cyclic bending and static torsion loads using Zencrack FE code to verify the results reported by Fonte *et al*. [13]. They also simulated the fatigue growth of a surface crack in solid and hollow shafts under combined cyclic torsion and axial loads. Marciniak *et al.* [17] experimentally evaluated fatigue life of circular smooth specimen under proportional and non-proportional bending- torsion loadings. It was found that, for the same levels of normal and shear stresses, fatigue

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lives under non-proportional loading were greater than those under proportional loadings.

The findings related to the fatigue crack growth analyses under combined loadings are always actively sought for updating the knowledge. In this study, fatigue crack growths of a surface crack in a smooth round bar subjected to a few scenarios of proportional cyclic bending-torsion loading are simulated by using a BEM software package of BEASY (BEASY [2]). The fatigue growthsand crack shape evolutionsfor different crack aspect ratio and different loading ratio of bending stress to torsional stress are presented.

2. MATERIALS AND METHOD

The meshed model of a solid cylinder containing a surface crack, its notations and the crack locations are illustrated in Figure-1. Smooth round bar has diameter D= 8 mm and length L= 80 mm. Displacement constraints in x, y and z directions are applied on either end of round bar, and a combination of torsional (anti-clock wise direction) and bending stresses are applied on another end. A semi-elliptical surface crack was introduced at the mid-length of the round bar. The crack plane is set to be normal to the axis of the round bar.

Material used in this study is magnesium alloy AZ-6A-T5. This material is widely used in aerospace and automotive components. Table-1 lists the mechanical and fracture properties of the AZ-6A-T5 alloy for the NASGRO constants/parameters taken from BEASY material database (BEASY [2]).

Table-1. Mechanical and fracture properties of AZ-6A-T5 Magnesium alloy used in simulations (BEASY [2]).

Modulus of elasticity (GPa)	45
Poisson's ratio	0.35
Yield stress (0.2%) (MPa)	268.89
Ultimate Tensile strength (MPa)	344.74
Plane stress fracture toughness, K_c (MPa√mm)	972.96
Plane strain fracture toughness, K_{Ic} (MPa√mm)	694.97
Crack growth rate coefficient, C	7.96E-10
Crack growth exponent coefficient, N	2.58
NASGRO coefficient, P	0.25
NASGRO coefficient, Q	0.25
Threshold SIF at R=0, ΔK_{th}(MPa√mm)	48.65
Plane stress/strain constraint factor	1.5
The ratio of the peak stress to the material flow stress in a stress cycle	0.3

The cases for the combined torsion-bending stress acting on the round bar are specified and presented in Table-2. All the surfaces of round bar are defined to be in outward normal direction and discretized by two-dimensional quadratic elements. Three different crack aspect ratios a/c = 0.4/0.8, 0.8/0.4and 0.8/0.8are specified. The NASGRO crack propagation law with a stress ratio R = 0 is chosen to simulate fatigue crack growths. The J-integral method is used to quantify the stress intensity factors during the crack growth.

Table-2. Combinations of loadings acting on round bar.

Case	**Torsion** (MPa)	**Bending** (MPa)
A	100	150
B	150	150
C	200	150
D	200	200

3. NUMERICAL RESULTS AND DISCUSSIONS

A comparison of the normalized SIFs for a surface crack on smooth round bar subjected to bending stress (100 MPa) obtained by using BEASY(BEASY, 2013) and from the Raju-Newman solution (Newman and Raju [18])is plotted in Figure-2a. The round bar has a diameter and length of 10 mm and 50 mm, respectively and a surface crack with the aspect ratio a/c of 1 and the crack depth a of 2 mm. It can be seen that the results are shown to be in good agreement. The stress intensity factor K_o shown in Figure-2a is defined as:

$$K_0 = \sigma_b \sqrt{\pi \frac{a}{Q}}$$

where σ_b is a bending stress, a is crack depth and Q is shape factor for elliptical crack.

The empirical expression Q for $a/c \leq 1$ is defined as (Newman and Raju [18]):

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

Next, to validate the fatigue growth result by BEASY (BEASY [2]) (based on the NASGRO crack propagation law for Al 2024-T351), the estimation for a surface crack (a = 0.5 mm and c = 0.5 mm) in a 15.9 mm-diameter smooth cylinder subjected to a maximum cyclic torsion loading of 134.28 MPa with a constant stress ratio R = 0.1 is compared with the experimental result reported by Thompson and Sheppard (1992) as presented in Figure-2b. At the stable stage, even though the slope of the curves is shown to be similar, the estimation by BEASY (BEASY [2]) shows more conservative than that by Thompson and Sheppard [24]. However, both results are shown to be in very good agreement during the unstable stage. These features are desirable in the preliminary engineering design stage.

Fatigue crack growths for different loading cases and crack locations are presented in Figure-3. It can be obviously seen from Figure-3a, for given crack aspect ratio a/c and bending loading, a larger torsional loading

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expectedly leads to a shorter fatigue life. The results show similar phenomenon with those in shaft under combined cyclic bending and steady torsion loads as reported by Yang *et al*. [28], Fonte *et al.* [13] and Maligno *et al.* [16].When the loading ratio is being in unity (maximum bending stress = maximum shear stress) with larger torsion and bending, for a given crack length c with $a/c \geq 0.5$, the fatigue lives are shown to be insignificant (see Case D in comparison to Case B in Figure-3a).It may be understood that larger maximum bending and shear stresses would lead to shorter fatigue lives, resulting in the effect of a/c on the fatigue life becoming less significant. However, for cases of $0.5 \leq a/c \leq 1$ and the loading ratio being in unity, the differences of the estimated fatigue lives are observed to be insignificant. The corresponding effective SIFs for all cases are presented in Figure-3b. The effective SIFs are shown to be fluctuated during the crack growth. This is as a result of the change of the crack path orientation to be inclined and twisted during the crack growth. It can be observed from Figures-3a and 3c, as expected, the fatigue crack growths under Case A for Position 1 have shorter fatigue lives than those for Positions 2 and 3. For a given crack length c, the effect of the crack depth a on the fatigue life becomes insignificant as the crack position is away from the location of the maximum bending stress.

Figure-4 presents the y-z plane view (see Figure-1) of the crack shape evolutions for different loading cases and crack aspect ratios. The initial cracks are shown to be in a planar shape. Then, the cracks are growing to form U-shapes in the following increments as a result of the inclined and twisted crack path (see Figure-5).It can be observed from Figure-5, for a given bending loading, a larger torsional loading produces a larger inclining crack path α. In turn, the y-z plane views of the crack shape evolution for Case C are shown to be wider that those for the other cases (Figure-4). A shorter line in the final crack shape (Figure-4) indicates more twisted crack plane which would depend on the loading ratio and the crack aspect ratio a/c. When the loading ratio of torsion to bending is being in unity, the crack paths produce similar inclining angles (see Cases B and D in Figure-5).

The x-y plane (cross section) view of the crack shape evolutions for different crack locations under Case A is presented in Figure-6. As indicated in Figure-1, positions 2 and 3 make the inclination angles of 30^o and 60^o, respectively. At those positions, larger crack extensions are shown on the crack front which have larger principal stresses resulting from the distribution of bending and torsional loadings. Next, when the cracks are further growing towards unstable stage, the crack shapes evolve to be similar to those for Position 1.

4. CONCLUSIONS

The fatigue crack growths of a surface crack in a solid cylinder under combined cyclic torsion-bending loads were evaluated. A few statements may be fairly drawn as follows:

- The effective stress intensity factors showed fluctuated values during the crack growth.
- For a given crack length c and $a/c \geq 0.5$, the effect of the crack aspect ratio on the fatigue life became less significant as the ratio of the larger maximum shear stress to the bending stress was being in unity.
- For a given crack length c and the loading ratio of the maximum shearing stress over the maximum bending stress being in unity, insignificant differences on the estimated fatigue lives were observed in the cases with $0.5 \leq a/c \leq 1$.
- For given crack aspect ratio and bending loading, a larger torsional loading led to a larger inclination angle of the crack path and a shorter fatigue life.
- When the crack position was away from the maximum bending stress location, the effect of the crack depth a on the fatigue life became insignificant.
- Combinations of the crack aspect ratio and the torsion-bending loading gave different crack path and crack shape during the crack growth.

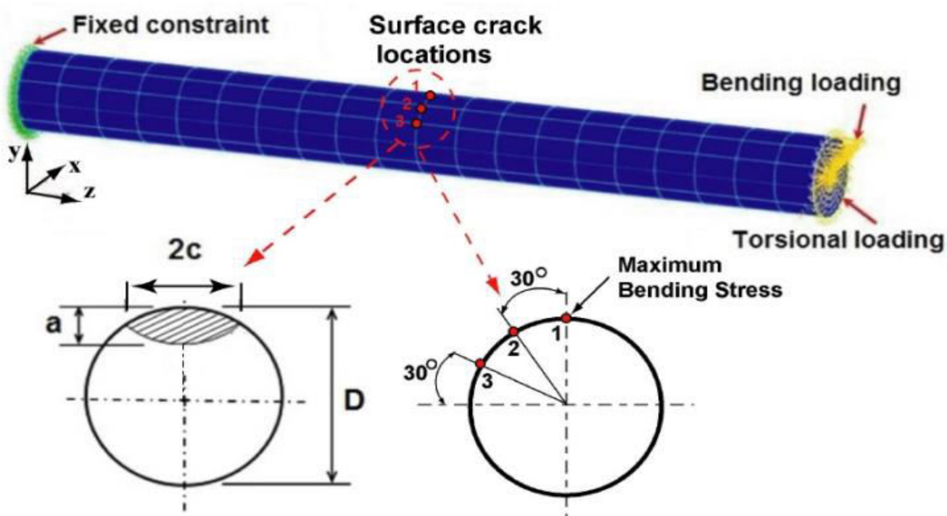

Figure-1. Schematic diagram of a surface crack in a solid cylinder and the notations used in the analysis.

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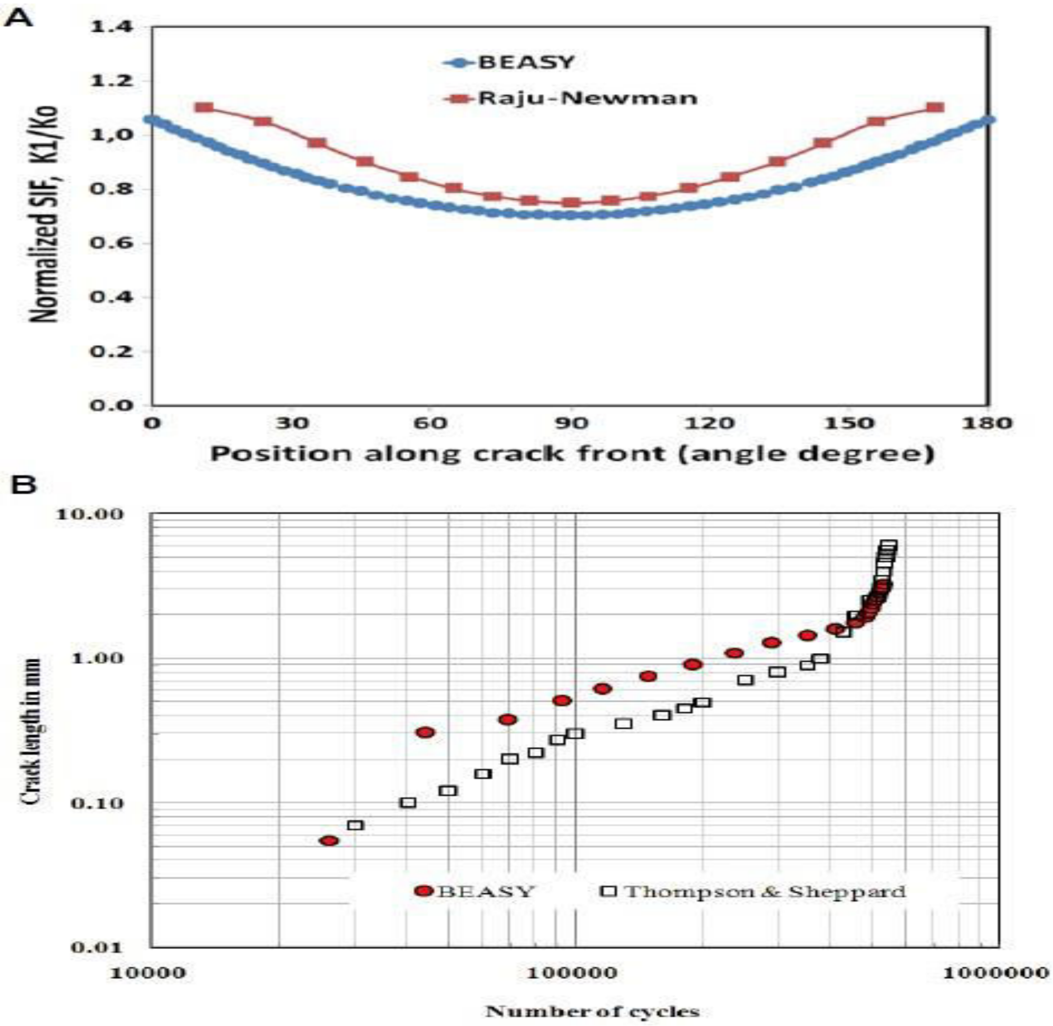

Figure-2. Verification results for a surface crack in a solid cylinder. a) SIFs under bending loading by Raju and Newman (Raju and Newman [20]) and BEASY (BEASY [2]).b) Fatigue crack growths by Thompson and Sheppard(Thompson and Sheppard [24]) and BEASY (BEASY [2]).

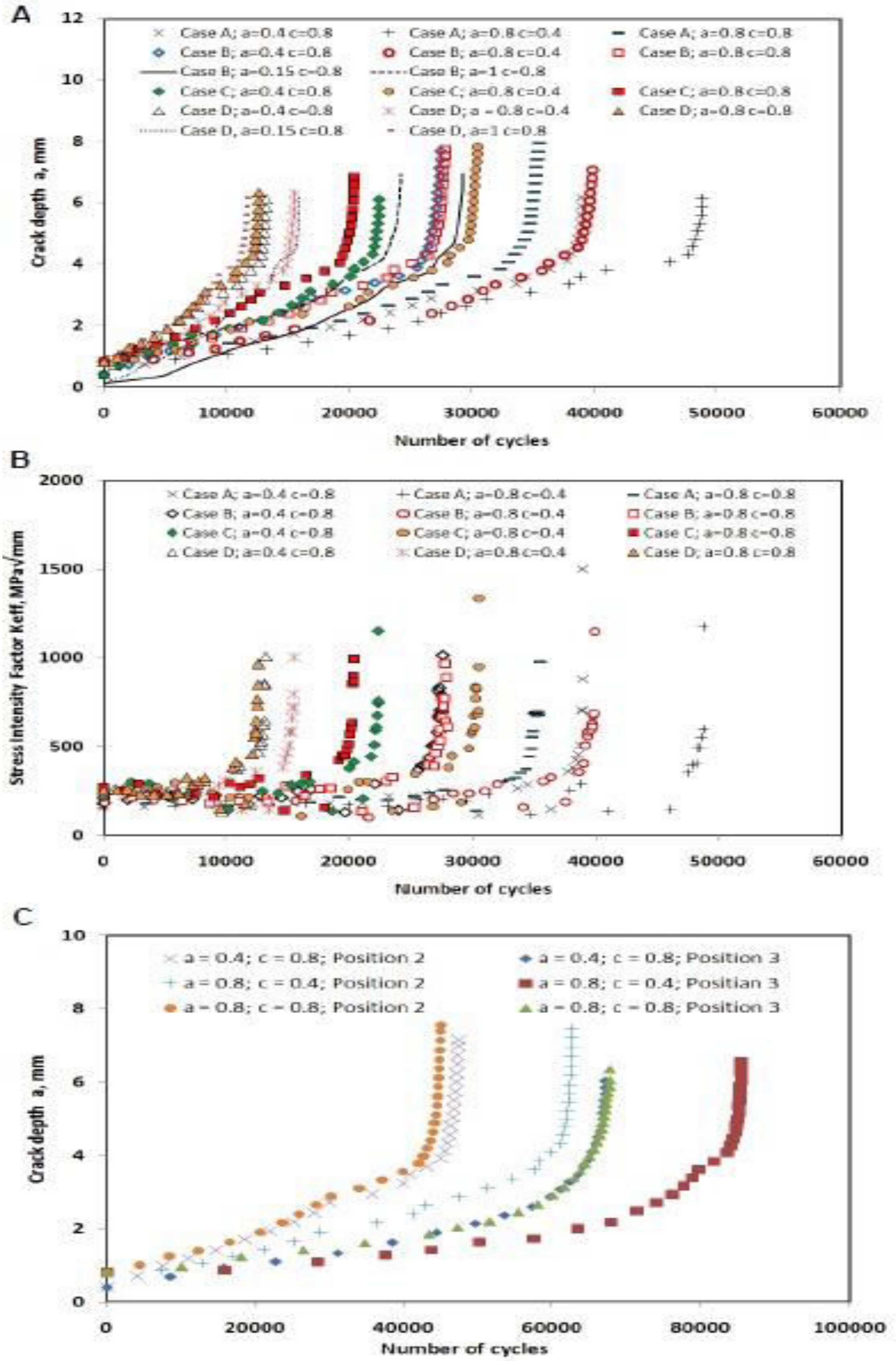

Figure-3.a) Fatigue crack growths for different loading cases and crack aspect ratios. b) The effective SIFs for different loading cases and crack aspect ratios and c) Fatigue crack growths for different initial crack locations (Positions 2 and 3).

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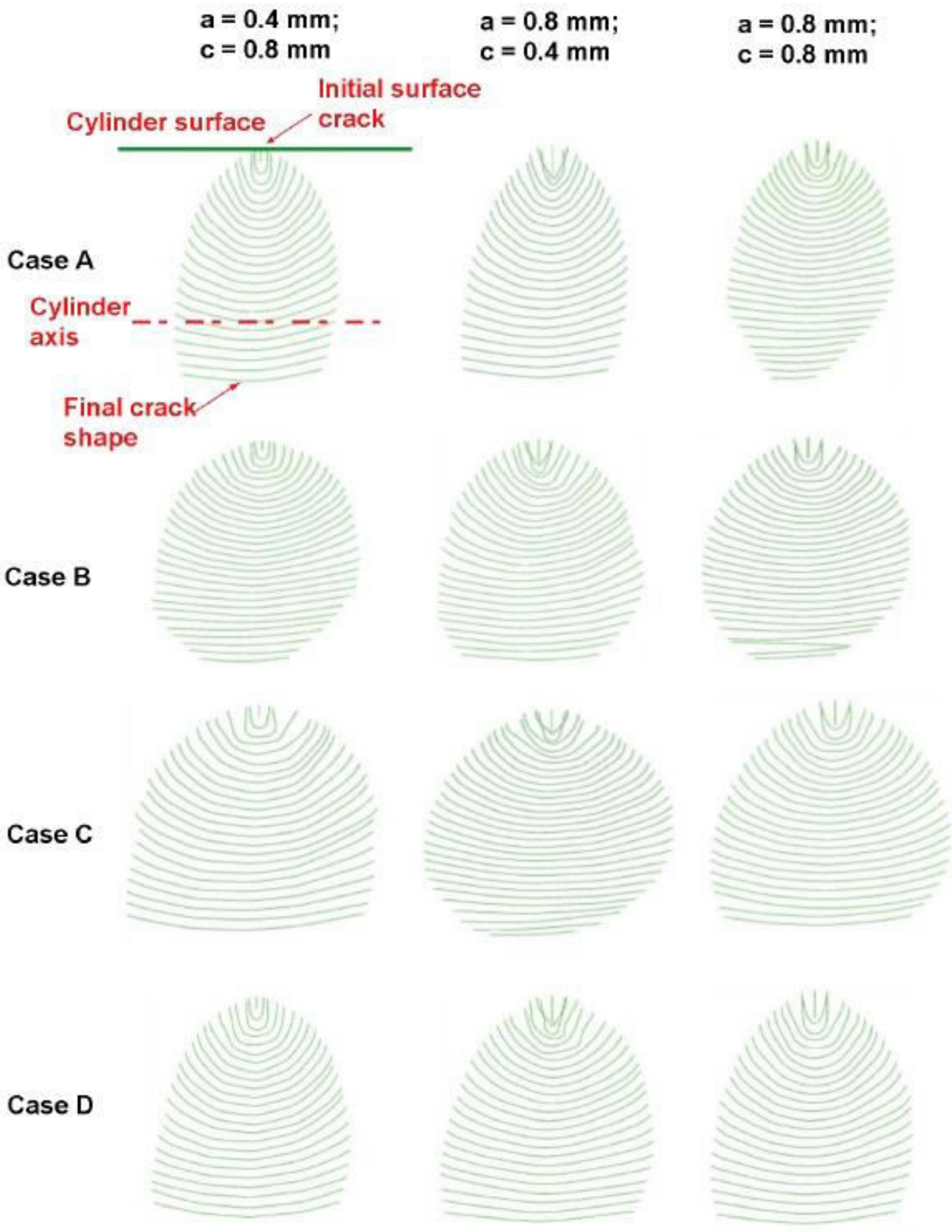

Figure-4. The *y-z* plane view (see Figure-1) of the crack shape evolutions for different loading cases and crack aspect ratios.

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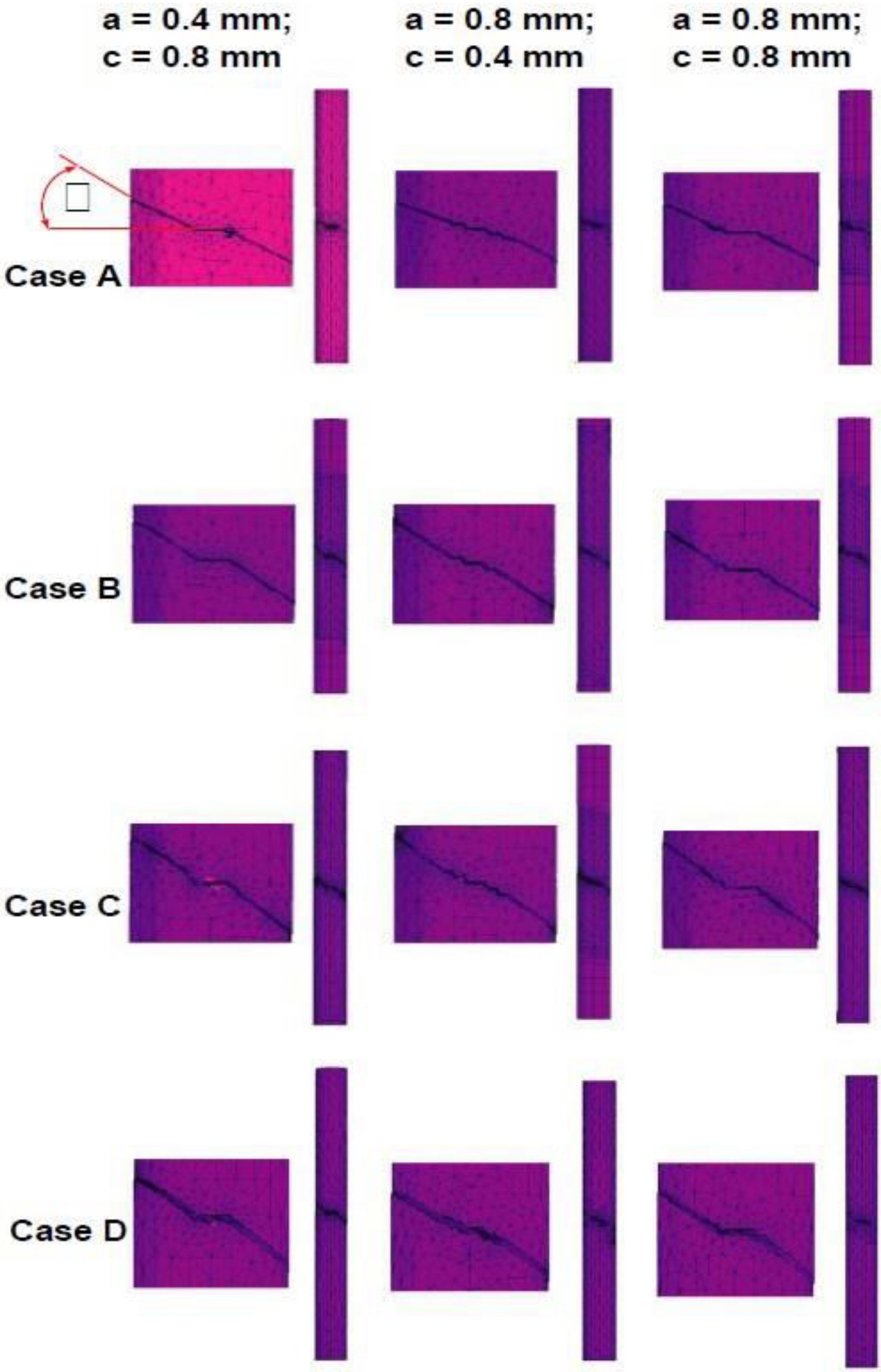

Figure-5. Crack paths for different loading cases and crack aspect ratios.

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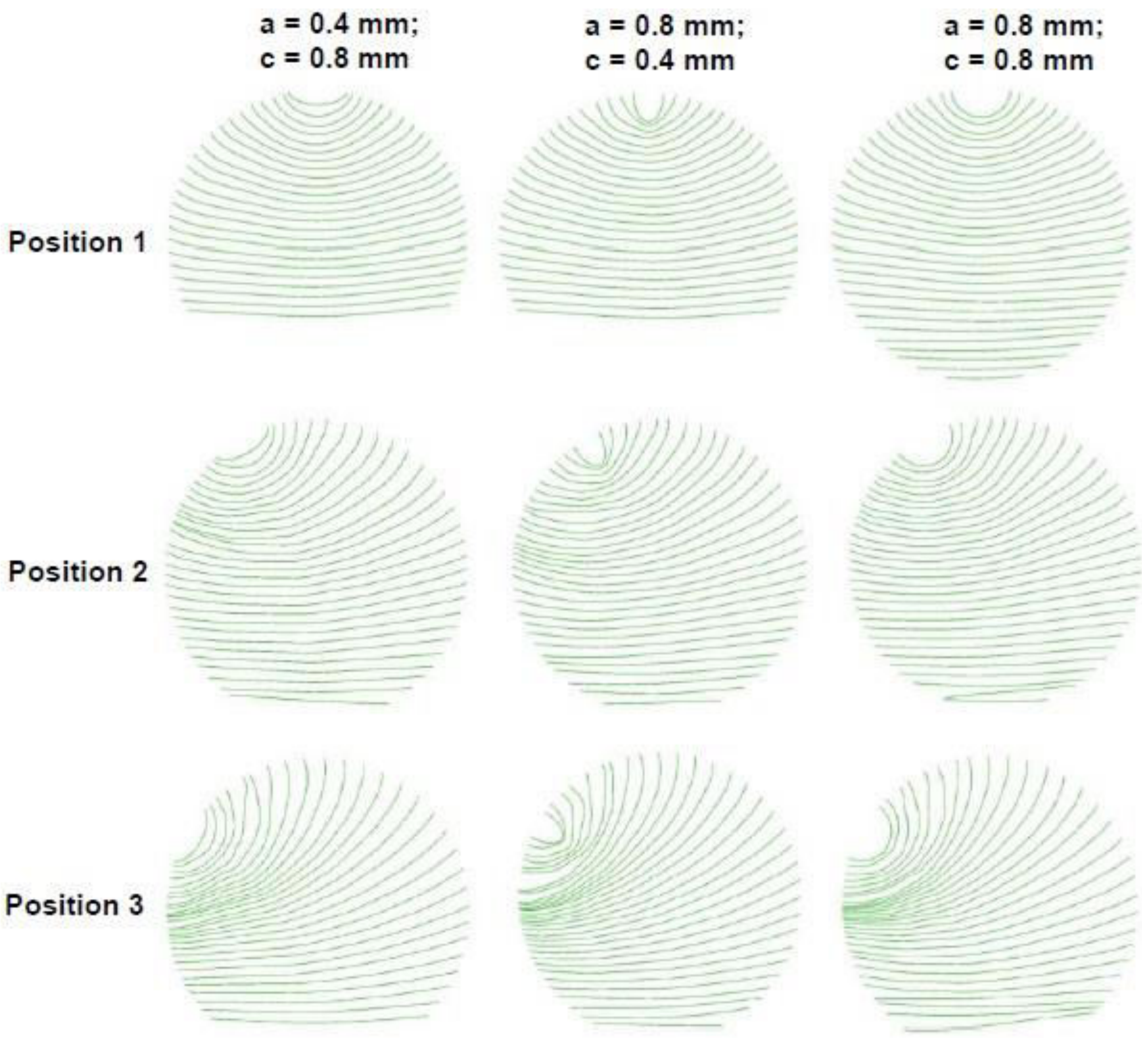

Figure-6. The x-y plane (cross section) view of the crack shape evolutions for different crack locations under Case A.

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