



CONSTRUCTING MINIMAL ADJACENT DOMINATING SETS IN SEMIGRAPHS FOR CLUSTERING IN WIRELESS NETWORKS

S. Saravanan¹, R. Poovazhaki² and N. R. Shanker³

¹Department of Mathematics, R.M.D. Engineering College, Kavaraipettai, Tamil Nadu, India

²E.M.G. Yadava Women's College, Madurai, Tamil Nadu, India

³Department of Electronics and Communication Engineering, Aalim Muhammed Salegh College of Engineering, Avadi, Chennai, Tamil Nadu, India

E-Mail: mathssaravanan77@gmail.com

ABSTRACT

Researchers propose Connected Dominating Set (CDS) of graphs in which each node in the cluster wireless network cover via dominating neighbors, define many dominating sets such as strongly connected dominating sets and weakly connected dominating sets etc. In this paper, we extend the dominative capacity of nodes such that each node dominates not only itself and all its adjacent neighbors completely called Adjacent Dominating Set (ADS) in semigraphs. Furthermore, an ADS construction algorithm to find minimal ADS in wireless networks is proposed for cluster head selection. The efficiency and performance of the ADS construction algorithm confirm through theoretical analysis and simulations. This paper addresses the behavior of the protocols in different network model in ADS based cluster network. Simulation result shows that DSR and DSDV perform better in graph and semigraph structure, whereas AODV is more adaptable in the randomly chosen network.

Keywords: connected dominating set (CDS), semigraph, adjacent dominating set (ADS), wireless sensor network (WSN), energy efficiency.

1. INTRODUCTION

Wireless sensor network (WSN) is an active research area, gain the attention of many researcher in monitoring Environmental parameter, such as Health care monitoring, Area monitoring, Industrial monitoring etc. Furthermore, the major problem in WSN is energy conservation, reliability, mobility, scalability, and ease of use. In addition, clustering the node in WSN provide benefit in terms of energy-efficiency, reducing routing delay and scalability. However, wireless sensor network (WSN) consist of many number of sensor node. In addition, clustering node is a challenging task in WSN for researcher. Among the node in the cluster, a set of nodes act as cluster-head (CH). In last decade, clustering in WSN plays a vital role in the energy conservation for individual node. However, cluster formation, CH selection, and data transmission are three phases of operation in clustering protocol. The major work of clustering starts with the CH selection algorithm that gives the energy efficiency of any network. In addition to the energy efficiency, CH covers major node and fault-tolerance.

2. RELATED WORK

The clustering algorithm with convergence time perform and energy efficient. The convergence time algorithm executes with constant and variable convergent time [1]. However, probability theory plays an important role in clustering. The most common clustering algorithms are LEECH, HEED, and EEHC. The probability cluster classifies the node either in probability or non-probability method. In non-probability scheme, the algorithm divides into node proximity, graph, weight, biological inspired protocol, and clustering protocol. A survey in cluster routing protocol of 16 different methods [2] show the QoS in WSN. However, the challenges in cluster formation

become difficult when the size of network increase. A study on clustering and their challenges such as selecting the optimal frequency of CH rotation, ensuring connectivity, computing the optimal cluster sizes, and clustering the network in the presence of a node duty cycle, presented in Younis *et al.*[2], also classifies the clustering based on the parameters of the CH selection and the execution nature of a clustering algorithm (probabilistic or iterative). Furthermore, the various routing protocols studied by Heinzelman *et al.* [3], Younis *et al.* [2], Basagni, Xu *et al.* [4], and Chen *et al.* [5], respectively. In [6], describes PEGASIS protocol is near optimal for a data gathering problem in sensor network. However, a comparative study on hierarchical routing protocol performed by Xu and Gao. The important hierarchical routing protocol, such as LEACH, TEEN, APTEEN (Manjeshwar *et al.*[7,8]) and PEGASIS (Lindsey *et al.*[6]), etc. examined.

In our daily life, the usage of sensor network increases considerably. Wireless sensor network (WSN) transfer the data between node in appropriate time and efficient routing. The efficient routing and time can obtain from various routing algorithm such as hierarchical, clustering, graph theory, color theory, and hypergraph theory. Hierarchical routing protocol (HRP) [9,10,11] show considerable conservation in total energy consumption of WSN. The HRP create the cluster with cluster head for each cluster. Moreover, the selection for best Cluster heads and Clusters make difficult in any cluster routing of the network. During the selection of cluster heads in any cluster consume more energy and time. The Color-theory-based Dynamic Localization (CDL) algorithm [12] identifies each sensor node location for efficient energy routing.

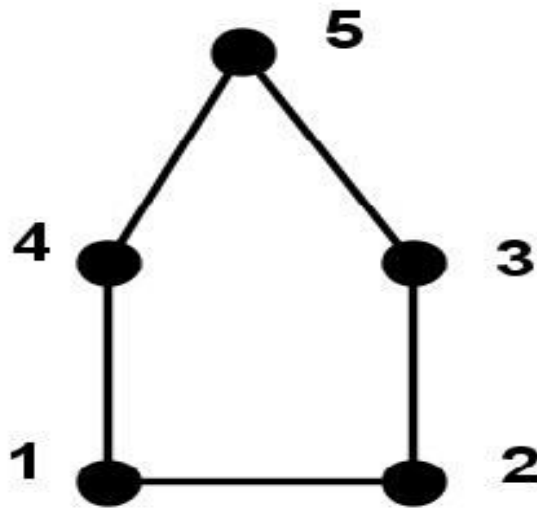


Figure-1.

The method to connect dominating set (CDS) as a virtual backbone for routing was first proposed by Ephermides *et al.*[13] in 1987. Furthermore, Clark *et al.*[14] extended to the relative complexity of problems under the restriction from Unit Disk Graphs (UDG) to grid graphs. The vertices distance in the circle must below or equal to unit value. In addition, Guha and Kuller[15] introduce the approximation algorithms to calculate minimum CDS as virtual backbone in UDG in graph theory. In graph model, as shown in Figure-1 each edge is connected with two vertices. However, the virtual backbone construction plays an important role in clustering the network. The Figure-1, need at least 3 cluster heads (nodes) to form a connected dominating set (CDS) say, 1, 2, and 4. Furthermore, in hypergraph [16] instead of connecting two vertices, a set of vertices connects with an edge. However in semigraph theory, the number of vertices in an edge is arranged in an order. Hence we consider semigraph [18,19] model in our work.

3. SEMIGRAPH MODEL

In WSN, a semigraph $S = (U, X)$ consider as a network where U is the set of sensor nodes, and X is the set of links between the nodes. A semigraph $S = (U, X)$ is a natural extension of a graph obtained by removing the constraint on the cardinality of an edge: Any nonempty subset of U can be an element of the edge set X .

Definition 1 (Dominating set). [17]

A dominating set (DS) for a graph $G(V, E)$ is a subset $D \subseteq V$ such that each node in $V - D$ adjacent to at least one node in D .

Definition 2.(Connected dominating set). [17]

A connected dominating set (CDS) in a connected graph G is a dominating set in G whose vertices induce a connected subgraph.

Definition 3.[18].

A semigraph is a pair $S(V, E)$ and V is a nonempty set of vertices of S and E is a set of n -tuples, called edges of S of distinct vertices for $n \geq 2$ satisfying the following conditions:

(SG1) Any two edges have at most one vertex in common.

(SG2) Two edges (u_1, u_2, \dots, u_n) and (v_1, v_2, \dots, v_m) are considered to be equal if and only if (i) $m = n$ and (ii) $u_i = v_i$ for $1 \leq i \leq n$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$. Thus the edge (u_1, u_2, \dots, u_n) is the same as $(u_n, u_{n-1}, \dots, u_1)$.

For the edge $e_1 = (u_1, u_2, \dots, u_n)$, u_1 and u_n are to be the end vertices of e_1 and u_2, u_3, \dots, u_{n-1} are the middle vertices of edge e_1 .

Definition 4 ([19]). Let $S(U, E)$ be a semigraph. For any vertex $u \in U$, the adjacent neighborhood of u is $N_a(u) = \{x \in U \mid x, \text{ adjacent to } u\}$ and $N_a[u] = N_a(u) \cup \{u\}$.

Definition 5 [18]. The highest adjacent degree of all the nodes of semigraph (network) S is denoted by $\Delta_a(S)$.

Definition 6. Adjacent Dominating Set[18] A set $D \subseteq U$ is called adjacent dominating set [7] (ad-set) if for every $u \in U - D$ there exists a $v \in D$ such that u is adjacent to v in S . The adjacency domination number $\gamma_a = \gamma_a(S)$ is the minimum cardinality of an ad-set of S .

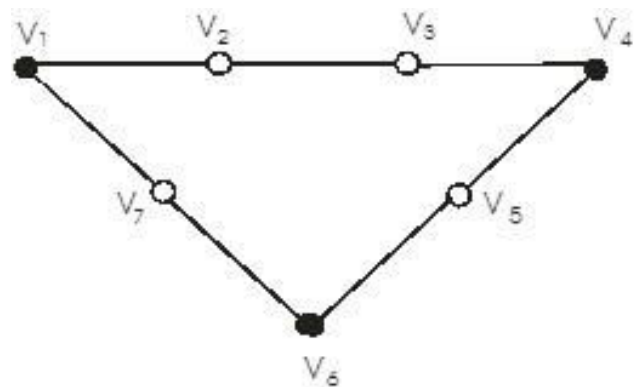


Figure-2.

In the semigraph model shown in Fig 2, the vertex set $D = \{v_1, v_4\}$ is an ADS. Since every node not in D has at least one adjacent neighbor in D . A survey of well-known results related to domination parameters contained in the book [17].

4. MOTIVATION AND CONTRIBUTION

Researchers consider UDG as a network model. However, Mohanty *et al.* [20] proposed a Connected Pseudo Dominating Set using 2 Hop Information



(CPDS2HI). The algorithm constructs Pseudo Dominating Set (PDS) in the first phase, and then improves the Steiner tree construction method, the PDS nodes are connected and finally, the redundant dominators are omitted from the CDS. The performance ratio of CPDS2HI is $(4.8 + \ln 5)|\text{opt}| + 1.2$, whereas $|\text{opt}|$ is the size of any optimal CDS. Also, the time and message complexities of $O(n\Delta)$ and $O(D)$, whereas D is the network diameter and Δ is the maximum degree of nodes of the network. Furthermore, the major contribution of the proposed algorithm improves the energy efficiency and performance of the network. However, the ADS construction method has very low time complexity $O(p^2)$. Also the method has the best performance ratio of $(1 + \ln \Delta_a)|\text{opt}|$, where $|\text{opt}|$ to the optimum size of the network. Finally, we analyze the three different network performances in three routing protocols AODV, DSR, and DSDV in the backbone of ADS using network simulator (NS2).

5. ADS CONSTRUCTION-ALGORITHM

In the first phase of this section, we obtain adjacent neighborhood set. In the last phase, we introduce the proposed algorithm to build ADS of semigraph S .

Algorithm 1. Determination of adjacent neighborhood sets of a semigraph.

Input: A connected semigraph $S(U, E)$ with $|U| = p$ and $|E| = q$.

Output: Adjacent neighborhood sets $N_a(u_i)$ of each node in $u_i \in U$ and $N_a(u_i) \subseteq U$.

```

1:  $N_a(u_i) \leftarrow \emptyset, U \leftarrow \{u_1, u_2, \dots, u_p\}, E \leftarrow \{E_1, E_2, \dots, E_q\}$ 
2: Let  $E_j = \{(u_1, u_2, \dots, u_i, u_k) | u_i \in U, k \geq 2\}$ 
3: for  $i = 1$  to  $p$  do
4:   for all positive integer  $k < p$  and  $k \neq i$  do
5:   if  $u_i$  satisfies at least one of the following conditions:
   (i)  $u_k$  belongs to an edge containing  $u_i$ 
   (ii)  $u_k$  adjacent to the node  $u_i$  then
6:   Add  $u_k$  to  $N_a(u_i)$ 
7: end if
8: end for
9: end for
10: return  $N_a(u_i)$ 

```

The next algorithm starts with sorting the nodes into the ascending order of its adjacent degrees and end with ADS of a connected semigraph.

Algorithm 2. Construction of minimal adjacent dominating set (ADS) of a semigraph.

Input: A connected semigraph $S(U, E)$ with $|U| = p$ and $|E| = q$.

Output: Minimal ADS Dof the semigraph $S(U, E)$, $D \subseteq U$.

```

1: ADS  $D \leftarrow \emptyset, U \leftarrow \{u_1, u_2, \dots, u_p\}, E \leftarrow \{E_1, E_2, \dots, E_q\}$ 
2: for  $i = 1$  to  $p$  do
3:   Sort the nodes in the ascending order of  $|N_a(u_i)|$ 
4: end for
5: Add last  $\left\lceil \frac{p}{\Delta_a + 1} \right\rceil$  nodes to  $D$ 
6: for  $i = 1$  to  $p$  do
7:   for every node  $u_i \in U - D$ 
8:   if  $|N_a(u_i) \cap D| = 0$  then
9:   add  $u_i$  to  $D$ 
10:  end if
11: end for
12: end for
13: return  $D$ 

```

In algorithm 2, while constructing ADS in the highest degree of the last $\left\lceil \frac{p}{\Delta_a + 1} \right\rceil$ node added to D .

Since the above number is a lower bound for γ_a [18]. Hence the adjacent dominating set D obtained from algorithm 2 is the minimal ADS.

6. OPTIMALITY ANALYSIS OF ADS CONSTRUCTION ALGORITHM

Theorem 1. D is an ADS.

Proof. Let D be the set of end node obtained from line 13 of Algorithm 2, and there is no middle node in D . By the construction of D , every middle node $u \in D - A$ should be adjacent to at least one end node in D . Hence D is an adjacent dominating set (ADS).

In this section, the performance ratio of the proposed algorithm analyze with message and time complexity. For algorithm, consider $|ADS|$, the size of ADS as the parameter.

Theorem 2. The proposed ADS algorithm determines the corresponding ADS in finite time.

Proof. Algorithm 1 shown in section 5 construct adjacent neighborhood set $N_a(u_i)$ for each $u_i \in U$. Since there are p node in semigraph S , the number of iteration is

bounded. In Algorithm 2, we add the last $\left\lceil \frac{p}{\Delta_a + 1} \right\rceil$ node,

after arranging the node into the ascending order of adjacent degree. Then a set of node added to ADS D , in such a way that each node not in D adjacent to at least one node in D . So the number of round is finite. Hence the finite number of node in ADS, will take finite time.

Theorem 3. The size of ADS obtained by ADS construction algorithm is at most $(1 + \ln \Delta_a)|\text{opt}|$,



whereas $|opt|$ the size of any optimal ADS, and Δ_a the maximum adjacent degree of the network.

Proof. Let $|ADS|$ be the size of ADS of the network (Semigraph) S . The first phase of construction algorithm determines adjacent neighborhood set for each node of S . In the second phase, minimal adjacent dominating set (ADS) is determined.

Then $|ADS| = (\ln \Delta_a + 1) |opt|$ [21].

Hence the performance ratio of ADS construction algorithm is $(\ln \Delta_a + 1) |opt|$.

Theorem 4. The message and time the complexities of ADS algorithm are $O(p^2)$ and $O[p(p + \log p)]$ respectively.

Proof. The time and message complexities in phase 1 are $O(p^2)$ and $O(p^2)$ respectively.

In phase 2, each node $u \in U - D$ has at least one adjacent node in D [22]. The algorithm need $O(p^2)$ time and $O(p \log p + p^2)$ message [24]. Therefore, the overall time complexity of the proposed algorithm is $O(p^2)$ and total message complexity is

$$O(p^2 + p \log p) = O[p(p + \log p)].$$

Since the smaller virtual backbone gives the better performance [25]. As shown in Figure-1 and Figure-2, size of ADS in semigraph is smaller than the size of CDS in graph. Since each edge of semigraph not only connected with end nodes but also connected with middle nodes. Furthermore, simulation of proposed ADS construction algorithm applies with three routing protocols AODV, DSDV, and DSR and compare with performance metrics such as throughput, packet loss ratio, residual energy, and end-to-end delay. Furthermore, all the above metric is discussed in further section.

7. PERFORMANCE METRICS OF ADS

The performance metrics in the experiments are throughput, the packet loss ratio, the residual energy, and end-to-end delay. Furthermore, Simulator (NS-2) software used to simulate the proposed novel ADS construction algorithm. In addition, in virtual backbone simulation, the deployment area for each protocol is of dimension $2000m \times 1000m$ square units. Also, in the simulation work, we consider the following models connected graph, connected semigraph and connected random network. However, the result of the simulation is reported in the subsequent Figures.

Table-1. The table shows the parameters of simulation in NS2.

S.No.	Parameters	Details
1	Node placement	semigraph
2	Number of nodes	20
3	Number of sink(destination)	One(Node 0)
4	Number of sources	19

5	Area of simulation	2000m×1000m
6	Traffic interval	0.55 sec
7	Packets generated by each source	500
8	Total packets generated in N/W	500×19=9500
9	Size of each packet	100 bytes
10	Model	Energy model
11	Initial energy	1000J
12	Transmitting power	36.00mw
13	Receiving power	14.4mw
14	Protocol	AODV/DSR/DSDV
15	MAC layer	802.11

Throughput is the measure of number of packets or data successfully transmitted to their final destination via a communication link per unit time [24]. Furthermore, it is measured in bits per second (bit/s or bps).

End to End Delay defines as the average time between packets sent and received. It is given by

$$A = \frac{1}{N} \sum_{i=1}^p (r_i - s_i), \quad N \text{ is the number of successfully}$$

received packets, i is unique packet identifier, r_i is time in which a packet with unique id i is received, s_i is time at which a packet with unique id i is sent and A measure in ms. [24]. Packet Loss ratio is calculated by subtract to the number of data packets sent to source and number of data packets received at destination through the number of packet originated by the application layer of the source.

8. SIMULATION RESULTS

Throughput

In the first set of simulation, we compare throughput for the DSR, DSDV and AODV protocols as shown in Figure-3, Figure-4, and Figure-5. However, the semigraph model obtains the best throughput of 12kb/sec over time in DSR. Still it achieves similar results for AODV and DSDV with all three network models as shown in Figure-3, Figure-4, and Figure-5.

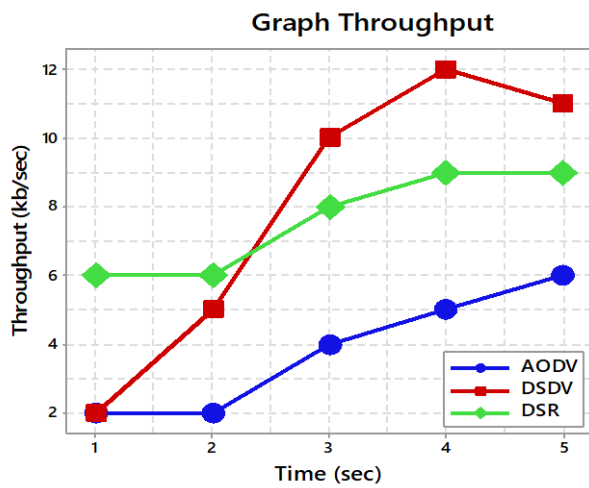


Figure-3.

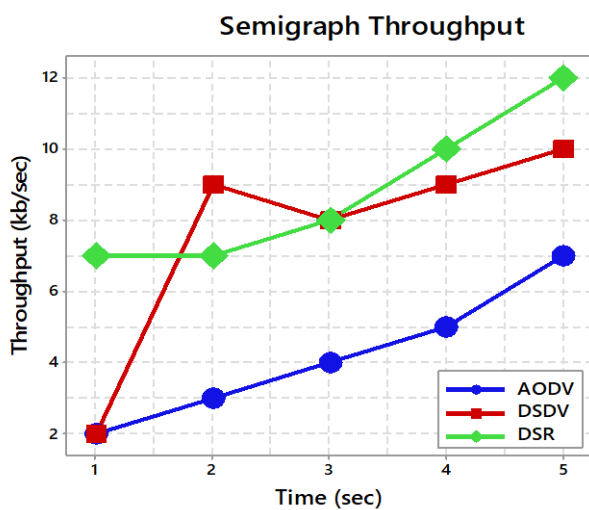


Figure-4.

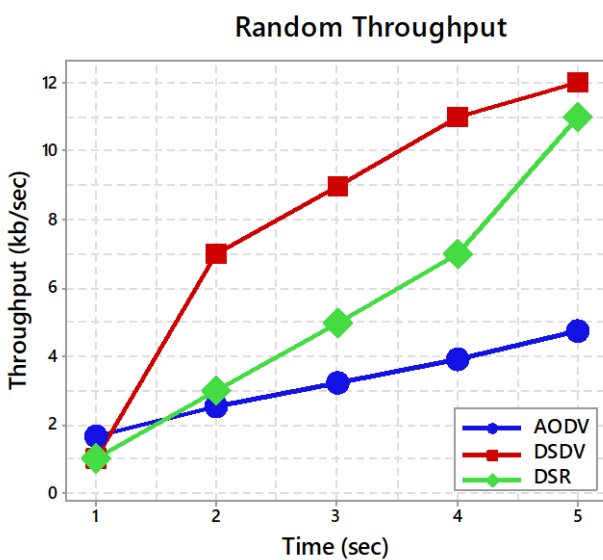


Figure-5.

Residual energy

Semigraph model (ADS) consume low energy compared to graph model and random network models as shown in Figure-6, Figure-7, and Figure-8.

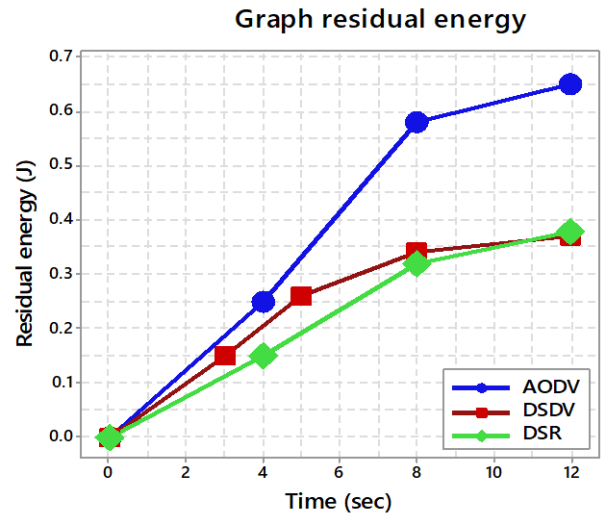


Figure-6.

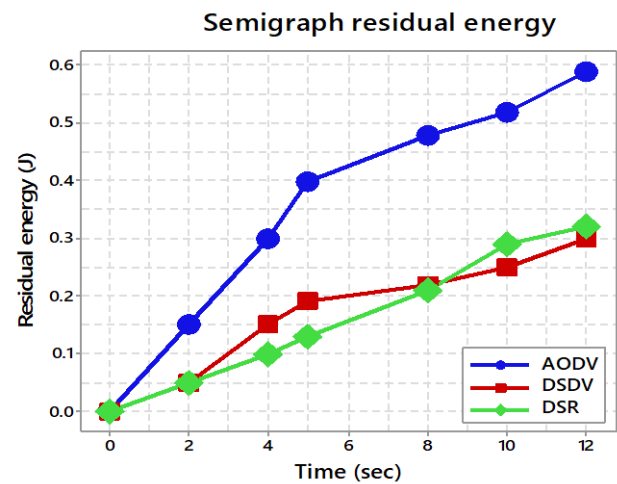


Figure-7.

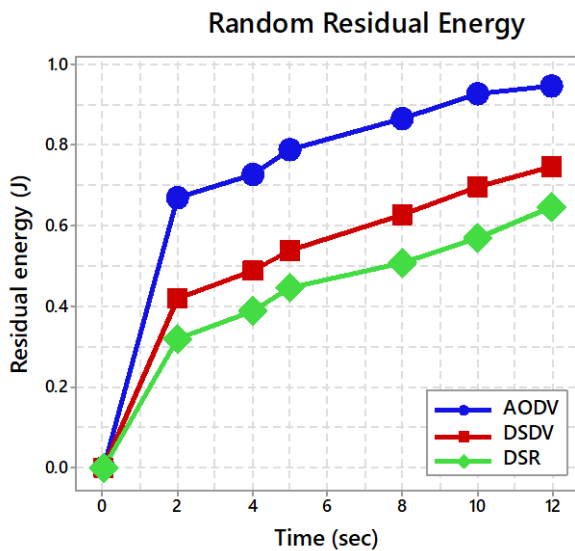


Figure-8.

Packet loss ratio

Refer to the graph of Figure-9, Figure-10, and Figure-11 the packet loss ratio of ADS is lower than the other two networks in all the three protocol. Furthermore, for AODV, the packet loss is higher than DSDV and DSR for all the three network models.

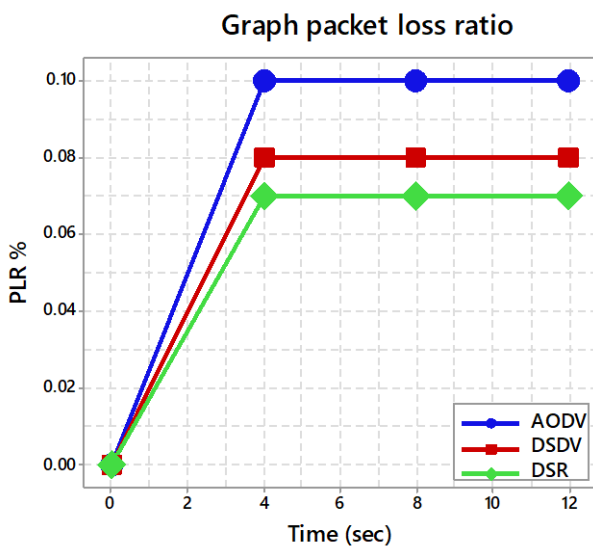


Figure-9.

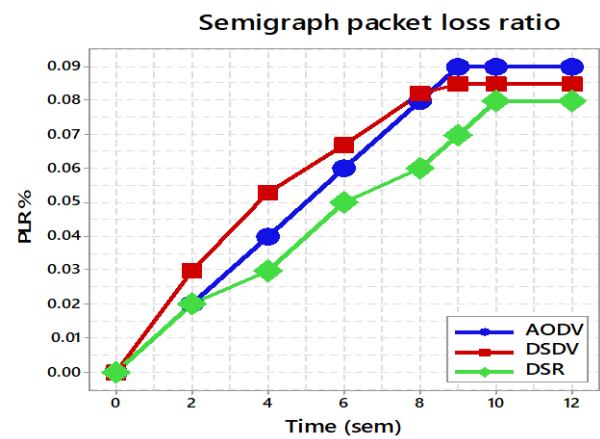


Figure-10.

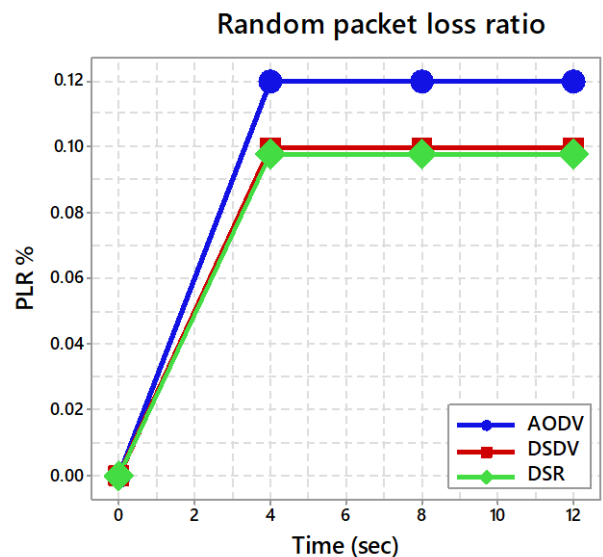


Figure-11.

End-to-end delay ratio

From the graphs of Figure-12, Figure-13, and Figure-14, the AODV has high delay compare to DSDV and DSR in all the three different network models. In addition, the semigraph model (ADS) shows a lower end-to-end delay while comparing the other two network models.



Graph end to end delay

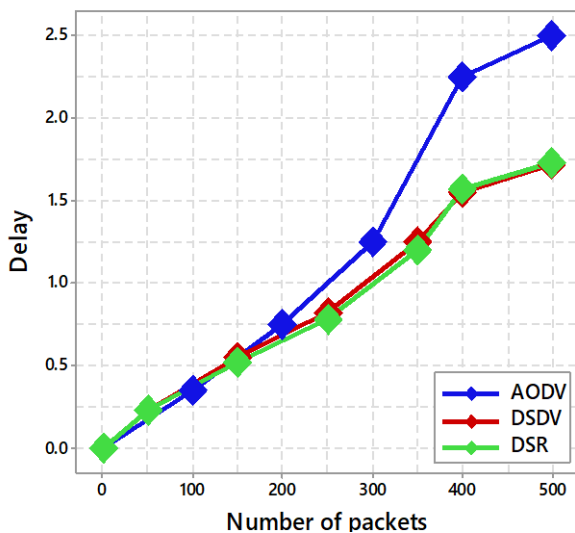


Figure-12.

Semigraph end to end delay

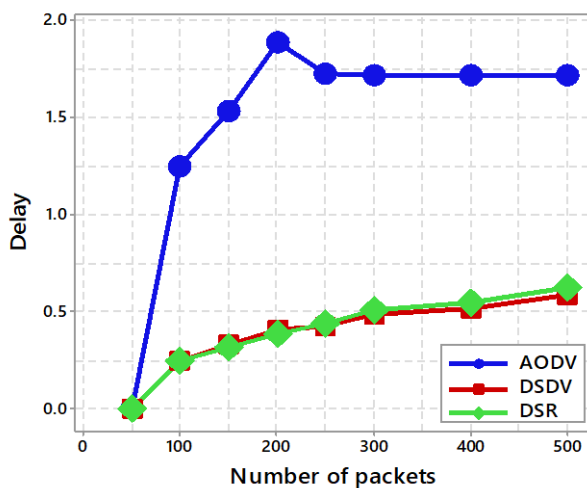


Figure-13.

Random End to end delay

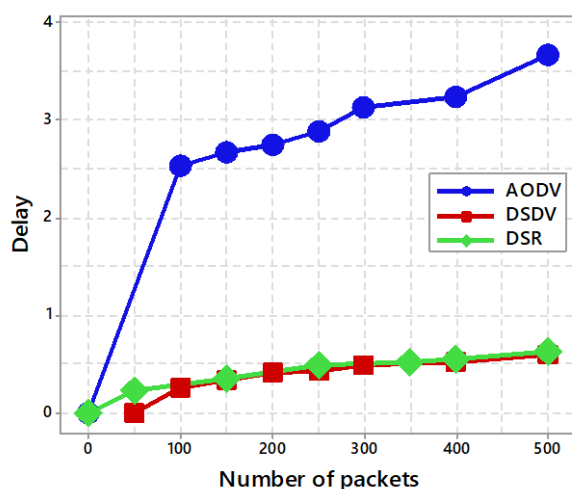


Figure-14.

9. CONCLUSION

In this paper, we have investigated the problem of clustering and cluster head selection by constructing minimal ADS for a wireless network. The performance ratio of our ADS construction algorithm is $(1 + \ln \Delta_a) |opt|$, whereas $|opt|$ the size of any optimal ADS, and Δ_a the maximum adjacent degree of the network. Furthermore, from the simulation result, we conclude the semigraph model (ADS) is the energy efficient model in all the metrics of three protocols AODV, DSDV, and DSR. As a future work, we can build new ADS construction schemes considering other issues such as bandwidth, and fault tolerance.

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