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A CALCULATION METHOD FOR ESTIMATION OF THE MEAN TIME TO FIRST FAILURE OF THE TECHNICAL SYSTEMS ON BASIS OF THE TOPOLOGICAL CONVERSION OF THE MARKOV RELIABILITY MODEL

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ABSTRACT

This scientific paper deals with the reliability models of technical systems on basis of continuous-time Markov chains (CTMC).An existing operator method for calculating the mean time to first failure (MTTFF), based on the reduction of Markov chain and solving the system of Kolmogorov-Chapman differential equations, is also discussed. The work also highlights the topological method offered by the author for calculating the mean time to first failure on basis of the special conversion of Markov chain. The calculation examples of MTTFF for the asymmetric computing system with triple modular redundancy by the existing operator method and the offered topological methods are also presented. The experimental research of calculation time by the operator and the topological methods are also discussed. According to the research results, the topological method offered by the author is significantly faster than existing operator method.

Keywords: technical systems, reliability model, continuous-time Markov chain (CTMC), mean time to first failure (MTTFF), kolmogorov-chapman differential equations system, laplace transform, topological method, linear algebraic equations system.

INTRODUCTION

In present days the reliability models [1, 2] on the basis of well-known continuous-time Markov chains [3, 4] and a set of reliability indices like stationary availability factor, mean time to failure and mean to repair, are widely used for the reliability analysis of technical systems. For the calculation of such reliability indices an efficient topological method is applied [5, 6].

However, for both non-repairable and repairable systems in most cases the mean time to first failure (MTTFF) also represents an important reliability index, which allows for predicting the mean time to first failure state of the system after its startup. To calculate this reliability index the modern reliability theory offers a reduction of the Markov chain and a calculation method based on time-consuming solution of a system of differential equations for the given initial operable state of the technical system. The operator method based on the Laplace transform [7, 8] simplifies the solution of differential equations; however, the computational complexity is still high.

Within the research work in the field of reliability models of different technical systems on the basis of Markov chains, the author derived the formulas for calculating various reliability indices, including the mean time to first failure, for different data transmission, processing and storage systems [9-12]. As a result, by summarizing the research results and integrating the existing methods for the calculation of the reliability indices, the author developed an effective topological method for calculating the mean time to first failure on basis of conversion of Markov chain.

Existing operator method for calculation of the mean time to first failure

In general case, the reliability models (Figure-1) based on continuous-time Markov chains contain a finite set of states E, including a subset of operable states E_+ and a subset of failure states E_-.

The subset E_+, in its turn, contains the subset of border operable states H_+, which has the direct transition links to the failure states. The subset E_-, in its turn, contains the subset of border failure states H_-, which has direct transition links to the operable states.

Each state $i \in E$ (Figure-2) can have either inbound links with the given transition rates γ_{ri} from the subset of states $\{r \in R_i\}$, which have the transition to the i-th state, and outbound links with the given transition rates γ_{is} to the subset of states $\{s \in S_i\}$, which have the transition from the i-th state.

For calculation of the mean time to first failure in the reliability model, based on a continuous-time Markov chain, modern reliability theory suggests reducing the base reliability model and solving the Kolmogorov-Chapman differential equations system for the reduced chain for the given initial operable state of the technical system.

The initial operable state of the system should belong to the subset of operable states E_+. The initial state in Markov chain is typically designated as «0».

The reduction of the base Markov chain includes the following steps:

- removing all outbound links from all failure states within subset E_- to any other states;
- removing all failure states that have no direct transitions to the operable states.

As a result, only «dead-end» border failure states within subset H_- remain in the reduced Markov chain (Figure-3), and they have direct inbound transition links from the operable states and no outbound transition links.

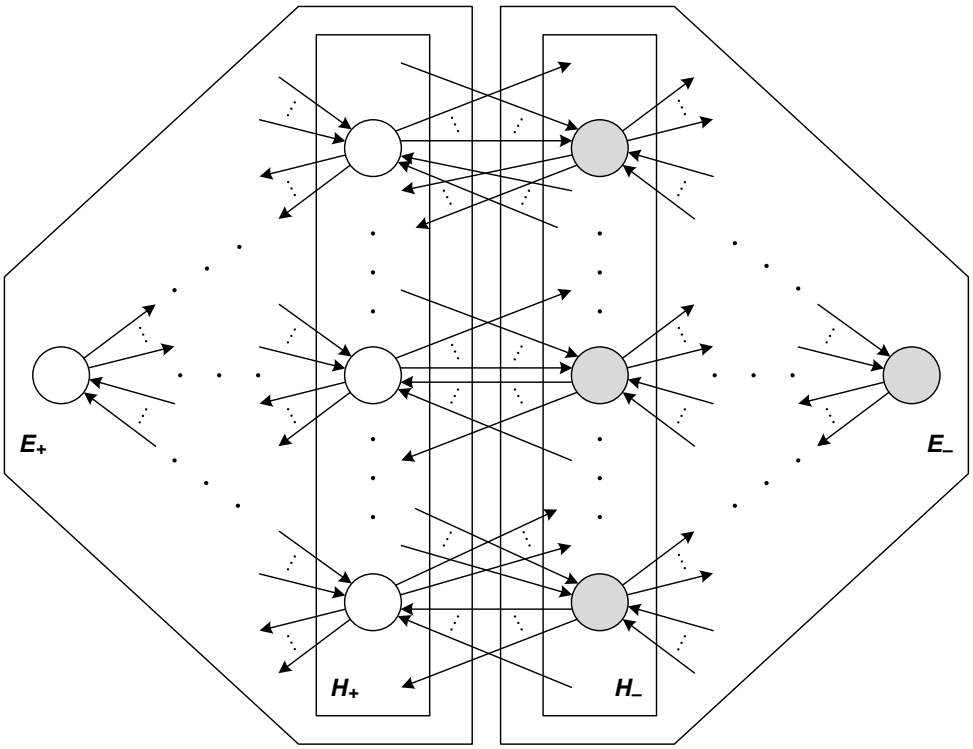

Figure-1. Base reliability model of the system on basis of Markov chain.

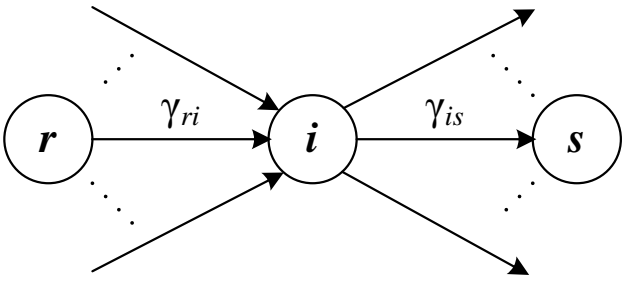

Figure-2. Inbound and outbound transitions links for *i*-th state.

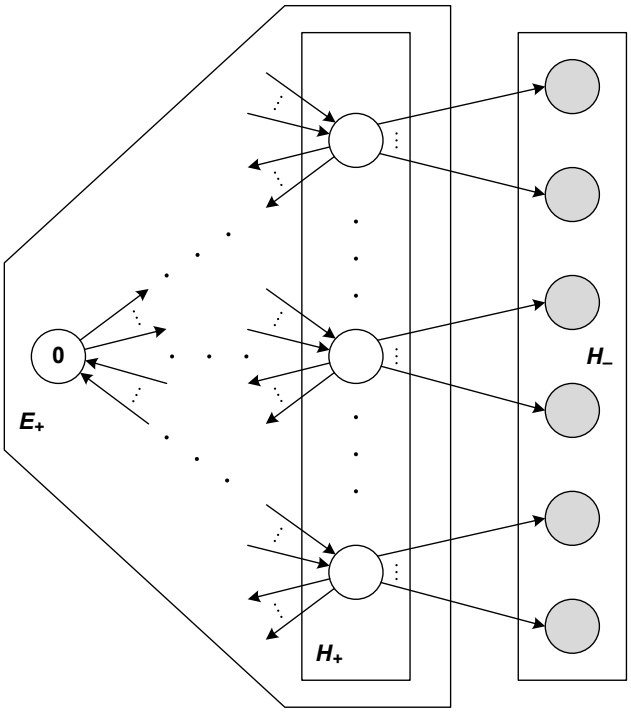

Figure-3. Reduced Markov chain for the operator method.

Further, taking into consideration that the operable states in the reduced Markov chain have no inbound transitions from the failure states and in the

differential equations for the operable states the probability functions of the failure states are not used in equations, only the equations for the operable states within subset E_+ are included in the Kolmogorov-Chapman system of differential equations. As for the initial conditions, the initial probability of the initial system state «0» is considered 1, and for the all other states the initial probabilities are considered 0:

$$\begin{cases} P_0(0) = 1; \\ \forall i \in E_+ \setminus \{0\} : P_i(0) = 0; \\ \dfrac{dP_0(t)}{dt} = \sum\limits_{r \in R_0} (\gamma_{r0} P_r(t)) - P_0(t) \sum\limits_{s \in S_0} \gamma_{0s}; \\ \forall i \in E_+ \setminus \{0\} : \dfrac{dP_i(t)}{dt} = \sum\limits_{r \in R_i} (\gamma_{ri} P_r(t)) - P_i(t) \sum\limits_{s \in S_i} \gamma_{is}. \end{cases} \quad (1)$$

Finally, the mean time to first failure of the system with the given initial state 0 can be calculated as the mean time spent by the system in the operable states before the first failure:

$$T_{\text{FF}} = \int_0^\infty \sum_{i \in E_+} P_i(t) dt. \quad (2)$$

It must be mentioned that, solving the system of differential equations, obtaining unknown probability functions and calculating the definite integral are quite time-consuming tasks. Therefore, in modern reliability theory, the operator method based on the Laplace transform is used for calculation of the mean time to first failure.

In accordance with the operator method, the differential equations system is converted by the Laplace transform, taking into consideration initial condition $P_0(0) = 1$:

$$\begin{cases} pY_0(p) - 1 = \sum\limits_{r \in R_0} (\gamma_{r0} Y_r(p)) - Y_0(p) \sum\limits_{s \in S_0} \gamma_{0s}; \\ \forall i \in E_+ \setminus \{0\} : pY_i(p) = \sum\limits_{r \in R_i} (\gamma_{ri} Y_r(p)) - Y_i(p) \sum\limits_{s \in S_i} \gamma_{is}. \end{cases} \quad (3)$$

Further, the mathematical expressions for the transformed probability functions $Y_i(p)$ for all operable states are algebraically derived from the converted equations system. Finally, the mean time to first failure is calculated as a limit of the sum of all these probability functions at $p \to 0$:

$$T_{\text{FF}} = \lim_{p \to 0} \sum_{i \in E_+} Y_i(p). \quad (4)$$

EXAMPLE OF CALCULATION OF THE MEAN TIME TO FIRST FAILURE BY THE OPERATOR METHOD FOR THE ASYMMETRIC COMPUTING SYSTEM WITH TRIPLE MODULAR REDUNDANCY

Let us assume that an asymmetric computing system with three functional identical computer nodes and different reliability parameters for each node are given. The computing nodes provide the same calculations, and there is a special voting circuit, which selects final result by using the majority principle. The voting circuit is considered to be highly reliable, and the failures of the voter are not taken into consideration. The computing system is considered as operable when at least two of the three nodes are operable and provide correct calculations.
The computing nodes are considered as simple repairable elements with exponential distribution of failure and repair times. Moreover, the nodes are independent from the viewpoint of failure and repair events.

The node failure rates are: $\lambda_1 = 1/8760$ hour^{-1}, $\lambda_2 = 2/8760$ hour^{-1} and $\lambda_3 = 3/8760$ hour^{-1}.
The node repair rates are: $\mu_1 = 1/24$ hour^{-1}, $\mu_2 = 3/24$ hour^{-1} and $\mu_3 = 2/24$ hour^{-1}.

Accordingly, the base Markov chain (figure 4), which represents the discussed above reliability model of the asymmetric computing system with triple modular redundancy, is as follows:

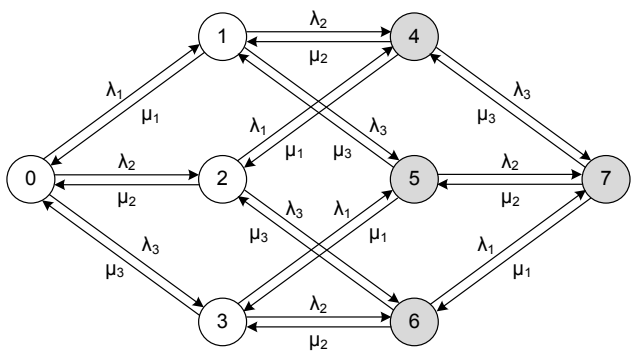

Figure-4. Base Markov chain for the asymmetric computing system with triple modular redundancy.

State 0 - all nodes are operable. State 1 - only node 1 is failed, state 2 - only node 2 is failed, state 3 - only node 3 is failed. State 4 - nodes 1 and 2 are failed, state 5 -nodes 1 and 3 are failed, state 6 - nodes 2 and 3 are failed. State 7 - all three nodes are failed.

States 0, 1, 2 and 3 are considered as operable, because in these states at least two of three nodes (majority quantity) are operable and provide correct results.
State 0 is the initial state of the system.

For calculation of the mean time to first failure by existing operator method for the discussed above asymmetric computing system with triple modular redundancy, we delete all outbound transition links from

the failure states 4-7 in the base Markov Chain (Figure-4). Next, we also delete state 7, because it has no direct transition links to any of operable states. As a result, we obtain the following reduced Markov chain (Figure-5) for a computing system.

The algorithm for calculation of the mean time to first failure includes the following steps:

- Forming the Kolmogorov-Chapman differential equations system for the reduced Markov chain with due consideration of initial state of the system.
- Applying the Laplace transform to the differential equations system and obtaining the system of transformed functional algebraic equations.
- Solving the transformed algebraic equations system and obtaining the algebraic expression for the transformed probability functions for all operable states.
- Calculating the mean time to first failure as a limit of sum of the transformed probability functions for all operable states at argument $p \to 0$.

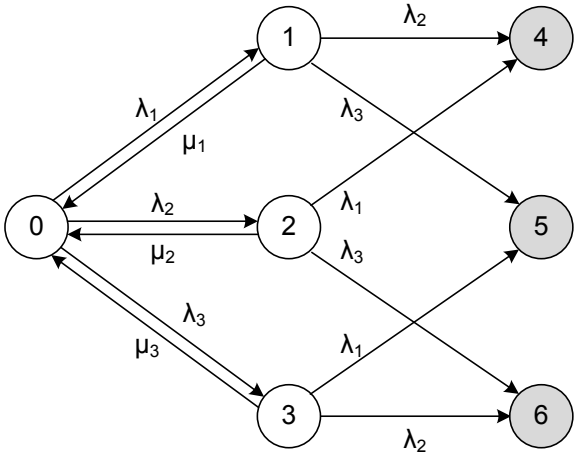

Figure-5. Reduced Markov chain for the asymmetric computing system with triple modular redundancy for calculation of the mean time to first failure by the existing operator method.

For the above-discussed reduced Markov chain of the computing system with triple modular redundancy, the Kolmogorov-Chapman differential equations system, considering the state 0 as the initial one, is as follows:

$$\begin{cases} P_0(0)=1;\quad P_1(0)=0;\quad P_2(0)=0;\quad P_3(0)=0; \\ \dfrac{dP_0(t)}{dt}=-(\lambda_1+\lambda_2+\lambda_3)P_0(t)+\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_3(t); \\ \dfrac{dP_1(t)}{dt}=\lambda_1P_0(t)-(\mu_1+\lambda_2+\lambda_3)P_1(t); \\ \dfrac{dP_2(t)}{dt}=\lambda_2P_0(t)-(\mu_2+\lambda_1+\lambda_3)P_2(t); \\ \dfrac{dP_3(t)}{dt}=\lambda_3P_0(t)-(\mu_3+\lambda_1+\lambda_2)P_3(t). \end{cases} \tag{5}$$

In accordance with the reliability theory, the mean time to first failure of the system with the given initial state 0 can be calculated as the mean time spent by the system in the operable states 0-3 before the first failure:

$$T_{\text{FF}}=\int_0^\infty (P_0(t)+P_1(t)+P_2(t)+P_3(t))dt. \tag{6}$$

By applying the Laplace transform to the differential equation systems (1), we obtain the following system of the transformed functional algebraic equations:

$$\begin{cases} pY_0(p)-1=-(\lambda_1+\lambda_2+\lambda_3)Y_0(p)+\mu_1Y_1(p)+\mu_2Y_2(p)+\mu_3Y_3(p); \\ pY_1(p)=\lambda_1Y_0(p)-(\mu_1+\lambda_2+\lambda_3)Y_1(p); \\ pY_2(p)=\lambda_2Y_0(p)-(\mu_2+\lambda_1+\lambda_3)Y_2(p); \\ pY_3(p)=\lambda_3Y_0(p)-(\mu_3+\lambda_1+\lambda_2)Y_3(p). \end{cases} \tag{7}$$

Accordingly, the mean time to first failure can be calculated as the limit of the sum of transformed probability functions for the operable states 0-3 for argument $p \to 0$:

$$T_{\text{FF}}=\lim_{p\to 0}(Y_0(p)+Y_1(p)+Y_2(p)+Y_3(p)). \tag{8}$$

Next, we obtain algebraic expression for the transformed probability functions for all operable states 0-3 by solving the system of the transformed functional algebraic equations (7):

$$\begin{cases} Y_0(p)=\dfrac{1}{p+\dfrac{\lambda_1(p+\lambda_2+\lambda_3)}{p+\mu_1+\lambda_2+\lambda_3}+\dfrac{\lambda_2(p+\lambda_1+\lambda_3)}{p+\mu_2+\lambda_1+\lambda_3}+\dfrac{\lambda_3(p+\lambda_1+\lambda_2)}{p+\mu_3+\lambda_1+\lambda_2}}; \\ Y_1(p)=\dfrac{\lambda_1}{p+\mu_1+\lambda_2+\lambda_3}Y_0(p); \\ Y_2(p)=\dfrac{\lambda_2}{p+\mu_2+\lambda_1+\lambda_3}Y_0(p); \\ Y_3(p)=\dfrac{\lambda_3}{p+\mu_3+\lambda_1+\lambda_2}Y_0(p). \end{cases} \tag{9}$$

Finally, we obtain the formula for calculating the mean time to first failure by substituting the obtained functions into the equation (8):

$$T_{\text{FF}}=\frac{1+\dfrac{\lambda_1}{\mu_1+\lambda_2+\lambda_3}+\dfrac{\lambda_2}{\mu_2+\lambda_1+\lambda_3}+\dfrac{\lambda_3}{\mu_3+\lambda_1+\lambda_2}}{\dfrac{\lambda_1(\lambda_2+\lambda_3)}{\mu_1+\lambda_2+\lambda_3}+\dfrac{\lambda_2(\lambda_1+\lambda_3)}{\mu_2+\lambda_1+\lambda_3}+\dfrac{\lambda_3(\lambda_1+\lambda_2)}{\mu_3+\lambda_1+\lambda_2}}. \tag{10}$$

By using the given values for the failure and repair rates for the computing nodes, we obtain value $T_{\text{FF}} \approx$ 267165 hours for MTTFF of the asymmetric computing system with triple modular redundancy.

OFFERED TOPOLOGICAL METHOD FOR CALCULATION OF THE MEAN TIME TO FIRST FAILURE

Now, it should be noted that if we offer some kind of additional conversion of the reduced Markov chain, which converts the reduced chain to some another «repairable» chain, for which the mean time to first failure of the reduced chain will be equal to the mean time to failure of the converted chain, then we will be able to apply topological approach. The topological approach is based on solving the stationary Kolmogorov-Chapman system of linear algebraic equations, and it calculates the mean time to first failure. The calculated mean time to failure of the converted chain in our case will be equal to the mean time to first failure of the reduced chain.

The author has obtained such additional conversion, when the mean time to first failure of the reduced chain is equal to the mean time to failure of the converted chain. The conversion is based on the union of all border failure states within subset H_- to the single aggregate failure state F, and usage of the additional fictitious transition link from the aggregate failure state F to the initial state 0 with some transition rate $\delta > 0$.

Finally, by uniting the described above reduction of the base Markov chain and the offered by the author additional conversion of reduced Markov chain, we obtain next algorithm of topological conversion of the base Markov chain, which includes:

- removing all outbound links from all failure states within subset E_- to any other states;
- removing all failure states, which have no direct transitions to the operable states;
- uniting all border failure states within subset H_- to one aggregate failure state F. For each border operable state within subset H_+ all outbound transition links to the failure states should be also united and rerouted to the aggregate failure state F, and the appropriate transition rates should be summed;
- addinga fictitious transition link from the aggregate failure state F to the initial state 0 with some transition rate $\delta > 0$.

As a result, the converted Markov chain (Figure-6) consists of a subset of operable states E_+ of the base Markov chain and one aggregated failure state F.

$$E^* = E_+ \cup \{\mathrm{F}\}. \tag{11}$$

The transition links between the operable states and their rates remain unchanged. However, for each border operable state within subset H_+ all outbound transition links to the failure states are united and rerouted to the aggregate failure state F, and the appropriate transition rates are summed:

$$\forall i \in H_+ : \gamma_{i\mathrm{F}} = \sum_{j \in E_-} \gamma_{ij}. \tag{12}$$

Moreover, a fictitious transition link from aggregate failure rate F to the initial state 0 with some rate $\delta > 0$ is added:

$$\gamma_{\mathrm{F}0} = \delta. \tag{13}$$

Now, taking into consideration that the converted Markov chain is «repairable» due to the fictitious transition link from aggregate failure rate F to the initial state 0, we can write a stationary Kolmogorov-Chapman system of linear algebraic equations, which allows us to obtain the stationary probabilities for all operable states within subset E_+ and aggregate failure state F:

$$\begin{cases} \sum_{i \in E_+} P_i + P_{\mathrm{F}} = 1; \\ \forall i \in E_+ \cup \{\mathrm{F}\}: \sum_{r \in R_i} (\gamma_{ri} P_r) - P_i \sum_{s \in S_i} \gamma_{is} = 0. \end{cases} \tag{14}$$

Finally, using the obtained stationary probabilities P_i for all operable states within subset E_+, we can easily calculate the mean to the failure of the converted chain by the topological formula, and this value is also equal to mean time to first failure of the base and reduced Markov chains:

$$T_{\mathrm{FF}} = \sum_{i \in E_+} P_i \Big/ \sum_{i \in H_+} P_i \gamma_{i\mathrm{F}}. \tag{15}$$

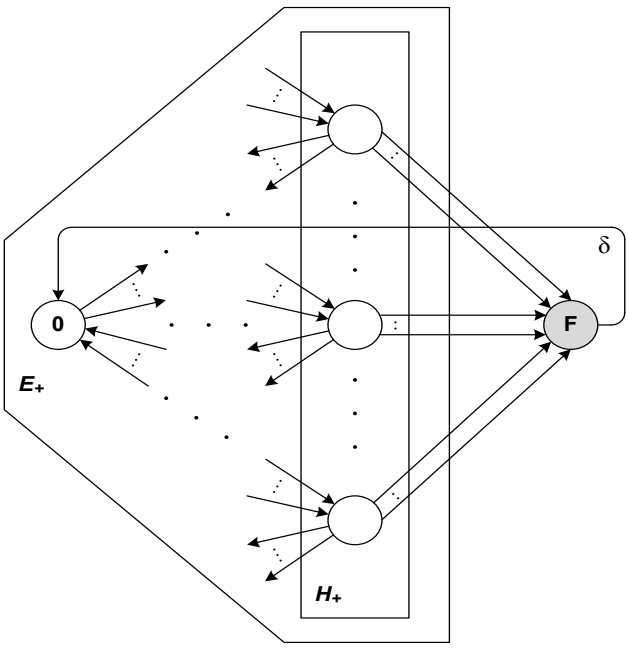

Figure-6. Converted Markov chain for the topological method.

The mean time to first failure is calculated as a ratio of sum of the stationary probabilities of all operable states to a weighted sum of the stationary probabilities of all border operable states. The weight for each border operable state is calculated as a sum of the rates of outbound transitions to the aggregate failure state F.

Equality between the mean time to first failure and the mean time to failure in the converted Markov chain is guaranteed by the usage of fictitious transition link from aggregate failure rate F to the initial state 0. Due to the fictitious transition link in reliability model, the system after the first failure always passes to the initial state 0, and mean time to next failures will be same as the mean time to first failure. Thus, the special conversion of base Markov chain offered by the author allows for the application of a faster topological method for calculation of mean time to first failure, based on solving the stationary Kolmogorov-Chapman system of linear algebraic equations.

EXAMPLE OF CALCULATION OF THE MEAN TIME TO FIRST FAILURE BY THE TOPOLOGICAL METHOD FOR THE ASYMMETRIC COMPUTING SYSTEM WITH TRIPLE MODULAR REDUNDANCY

For calculation of the mean time to first failure by topological method for the discussed above asymmetric computing system with triple modular redundancy, we remove all transition links from failure states 4-7 and delete the failure state 7, which has no direct transitions links to the operable states, in the base Markov chain (Figure-4).

Next, we unite all failure states to the aggregate failure state F and for each operable state 1-3 we unite all outbound transitions to the failure states 4-6 and reroute them to the aggregate failure state F with summarization of the appropriate transition rates.

Finally, we add a fictitious transition link from the aggregate failure state F to the initial state 0 with some rate $\delta > 0$.As a result, we obtain the following converted Markov chain (Figure-7) for the asymmetric computing system with triple modular redundancy:

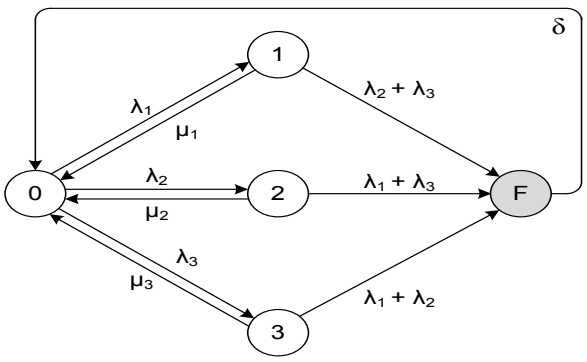

Figure-7. Converted Markov chain for the asymmetric computing system with triple modular redundancy for calculation of the mean time to first failure by the offered topological method.

Accordingly, the stationary Kolmogorov-Chapman equations system for the converted Markov chain is as follows:

$$\begin{cases} P_0 + P_1 + P_2 + P_3 + P_F = 1; \\ 0 = -(\lambda_1 + \lambda_2 + \lambda_3)P_0 + \mu_1 P_1 + \mu_2 P_2 + \mu_3 P_3 + \delta P_F; \\ 0 = \lambda_1 P_0 - (\mu_1 + \lambda_2 + \lambda_3)P_1; \\ 0 = \lambda_2 P_0 - (\mu_2 + \lambda_1 + \lambda_3)P_2; \\ 0 = \lambda_3 P_0 - (\mu_3 + \lambda_1 + \lambda_2)P_3; \\ 0 = (\lambda_2 + \lambda_3)P_1 + (\lambda_1 + \lambda_3)P_2 + (\lambda_1 + \lambda_2)P_3 - \delta P_F. \end{cases} \quad (16)$$

The mean time to first failure, which is equal to the mean time to failure, for the converted chain can be calculated as a ratio of sum of the stationary probabilities of operable states 0-3 to the weighted sum of the stationary probabilities of all border operable states 1-3, where the weight for each border operable state is calculated as a sum of the rates of outbound transitions to the state F:

$$T_{FF} = \frac{P_0 + P_1 + P_2 + P_3}{(\lambda_2 + \lambda_3)P_1 + (\lambda_1 + \lambda_3)P_2 + (\lambda_1 + \lambda_2)P_3}. \quad (17)$$

Next, by solving the stationary equations system (16), we obtain stationary probabilities for all operable states 0-3 of the converted Markov Chain:

$$\begin{cases} P_0 = \dfrac{\delta}{\delta + \dfrac{\lambda_1(\delta + \lambda_2 + \lambda_3)}{\mu_1 + \lambda_2 + \lambda_3} + \dfrac{\lambda_2(\delta + \lambda_1 + \lambda_3)}{\mu_2 + \lambda_1 + \lambda_3} + \dfrac{\lambda_3(\delta + \lambda_1 + \lambda_2)}{\mu_3 + \lambda_1 + \lambda_2}}; \\ P_1 = \dfrac{\lambda_1}{\mu_1 + \lambda_2 + \lambda_3} P_0; \quad P_2 = \dfrac{\lambda_2}{\mu_2 + \lambda_1 + \lambda_3} P_0; \quad P_3 = \dfrac{\lambda_3}{\mu_3 + \lambda_1 + \lambda_2} P_0. \end{cases} \quad (18)$$

Finally, we obtain formula for calculation of the mean time to first failure by substituting the obtained expression into the formula (17):

$$T_{FF} = \frac{1 + \dfrac{\lambda_1}{\mu_1 + \lambda_2 + \lambda_3} + \dfrac{\lambda_2}{\mu_2 + \lambda_1 + \lambda_3} + \dfrac{\lambda_3}{\mu_3 + \lambda_1 + \lambda_2}}{\dfrac{\lambda_1(\lambda_2 + \lambda_3)}{\mu_1 + \lambda_2 + \lambda_3} + \dfrac{\lambda_2(\lambda_1 + \lambda_3)}{\mu_2 + \lambda_1 + \lambda_3} + \dfrac{\lambda_3(\lambda_1 + \lambda_2)}{\mu_3 + \lambda_1 + \lambda_2}}. \quad (19)$$

It is easy to see, that the equation (19), obtained by the offered topological method, is similar to equation (10), obtained by the existing operator method.

By using the above-given values for the failure and repair rates and equation (19), we obtain value $T_{FF} \approx$ 267165 hours, which is equal to the calculated above value.

EXPERIMENTAL RESEARCH OF MTTFF CALCULATION TIMES

For comparison of the existing operator method and the topological method offered by the author from the viewpoint of calculation time, both methods were

programmed in Maplesoft® Maple™ 15, and an experimental research of the calculation time was carried out on the desktop computer based on processor Intel® Pentium™ IV 3.0 GHz.

As an example of the reliability model for calculation of the mean time to first failure, a well-known generalized Markov birth-death chain (Figure-8) was used with a subset of operable states 0…s–1 and one failure state s with initial operable state 0.For such Markov chain it is quite easy to program the equations system and carry out the research of calculation times by both methods for different number of states s + 1.

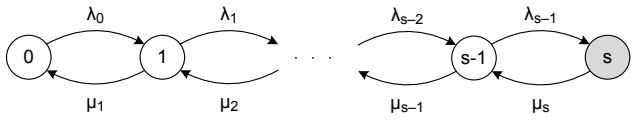

Figure-8. Generalized Markov Birth-Death Chain.

To control the correctness of the calculated by the both methods MTTFF values the author also obtained and used next analytical formula for estimation of the MTTFF of the Markov Birth-Death Chain:

$$T_{\mathrm{FF}} = \sum_{k=0}^{s-1} \frac{1}{\lambda_k} \left(\sum_{q=0}^{s-1-k} \prod_{j=1}^{q} \frac{\mu_{k+j}}{\lambda_{k+j}} \right). \tag{20}$$

Within the experimental research, a series of calculations of mean time to first failure for the reliability model based on Markov birth-death chain was carried out using the operator and topological methods. For each s = 5, 10, 15, 20, 25, 30 the series of 10000 calculations by both methods was carried out for the different transition rates λ_i and μ_j with the measurement of calculation times. Further, the measured times were averaged within each series for the operator and topological methods. The results of the time measurements for both methods and different s are shown in Table-1.

Table-1. Average times of MTTFF calculations by the operator and topological methods.

s	**Operator method** (sec)	**Topological method** (sec)
5	0.0195	0.0031
10	0.0597	0.0061
15	0.1239	0.0094
20	0.2159	0.0132
25	0.3462	0.0176
30	0.5097	0.0229

It is easy to see, that the topological method offered by the author is faster than the existing operator method.

CONCLUSIONS

This article covers such issues as the reliability models of technical systems on the basis of Markov chains, the existing operator method for calculating the mean time to first failure, based on the reduction of Markov chain and solution of the differential equation systems. The topological method offered by the author for calculating the mean time to first failure, based ona special conversion of Markov chain, is also highlighted.

The calculation examples of MTTFF for the asymmetric computing system with triple modular redundancy by the existing operator method and the offered topological methods are also presented.

The experimental research of calculation time by the operator and the topological methods are also discussed. According to the research results, the topological method offered by the author is faster than the existing operator method.

The topological method was applied for calculating MTTFF for various technical systems. In all cases it gave values similar to the values obtained by the existing operator method.

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