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IDEAL APPROACH ON LATTICE ORDERED FUZZY SOFT GROUP AND ITS APPLICATION IN SELECTING BEST MOBILE NETWORK COVERAGE AMONG TRAVELLING PATHS

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ABSTRACT

A generic growth of the theory of lattice ordered fuzzy soft group(fsg) is furnished in this paper. Certain characterizations of lattice ordered fuzzy soft group, fuzzy soft group ideal(dual) and fuzzy soft group prime ideal(dual) are attained. The relationship between fuzzy soft group ideal and lattice ordered fuzzy soft group congruence relation has been investigated. Moreover, fuzzy soft group ideal has been applied for finding best mobile network coverage in one's long travel.

Keywords: fsg ideal, fsg prime ideal, fsg principal ideal, convex sublattice on l - fsg. mathematics subject classification (2010): 06B10, 20N25.

1. INTRODUCTION

Garrett Birkhoff [6] initiated the general development of lattice theory in the mid 1930's and exposed the emphasis of lattice theory. Gratzer [7] discussed the different types of ideals to extent the nature of ideals in distributive lattices. Zadeh [19] proposed the fuzzy set theory as obvious extension of the classical set definition and described its application in pattern recognition. In 1990, YUAN Bo *et al.* [18] presented the analogy of fuzzy ideals and fuzzy congruences on a distributive lattice. In 1994, Naseem Ajmal[13] proposed the concept of fuzzy convex sublattice and extended the unique representation theorem for fuzzy convex sublattice. Also he analyzed fuzzy ideal(dual) and fuzzy prime ideal(dual) and proved structural theorem for fuzzy sublattices. In 1995, lattice structure of various sublattices of the lattice of fuzzy subrings of a ring [14] was presented. Marcel Tonga [11] studied two aspects of fuzzy maximal filters of lattices and obtained its nice characterizations. Ivan Mezzomo *et al.* [8] proposed some new notions of fuzzy ideals and filters and provided some properties analogous to the classical theory. J.Vimala *et al.* [4] investigated the theory of fuzzy l- ideals of commutative l-group. In 1999, Molodtsov [12] offered soft set theory and constructed a measurements theory on the basis of soft set. Maji *et al.* [9, 10, 15] established fuzzy soft set theory and discussed its application in decision making problems. In 2011, Cagman *et al.* [5] defined a fuzzy soft set theory and fuzzy soft aggregation operator to formulate more effective decision processes. In 2001, Haci Aktas and Naim Cagman [2] defined the notion of soft group and derived its fundamental characterizations exerting Molodtsov's theory of soft set. In 2009, Abdulkadir Aygunoglu and Halis Aygun [1] generalized the soft group of Aktas and Cagman as fuzzy soft group and studied its unique properties. In 2016, J. Vimala *et al.* [3,16] propounded lattice ordered fuzzy soft group theory and discussed distributive and modular lattice ordered fuzzy soft group and its duality. In 2017, fuzzy soft cardinality in lattice ordered fuzzy soft group and its application [17] have been discussed.

This paper has been systematically arranged in the following way: In section 2, some basic definitions are given. In section 3, fuzzy soft group ideal(dual), fuzzy soft group principal ideal(dual) and convex sublattice of lattice ordered fuzzy soft group are defined and some pertinent properties are studied. In section 4, fuzzy soft group prime ideal(dual) is defined and relevant results are deduced. In section 5, an application of fuzzy soft group ideal has been described in decision making problem.

Throughout this paper, we use I for a unit closed interval [0,1], fsg for fuzzy soft group and l-fsg for lattice ordered fuzzy soft group.

2. PRELIMINARIES

An order $(L;\leq)$ is a lattice if sup$\{a,b\}$ and inf $\{a,b\}$ exist for all $a,b \in L$. The subset K of the lattice L is called convex if $a,b \in K$, $c \in L$, and $a \leq c \leq b$ imply that $c \in K$. The most important example of a convex sublattice is an ideal. A subset I of a lattice L is called an ideal if it is a sublattice of L and $x \in I$ and $a \in L$ imply that $x \wedge a \in I$. An ideal I of L is proper if $I \neq L$. A proper ideal I of L is prime if $a,b \in L$ and $a \wedge b \in I$ imply that $a \in I$ or $b \in I$.

Definition 2.1[19] *Let X be a non-empty set, then a fuzzy set* μ *over X is a function from X into* $I = [0,1]$.*ie.,* $\mu : X \to I$.

Definition 2.2 [12] *Let X be an initial universe set and E a set of parameters with respect to X. Let P(X) denote the power set of X and A* $\subseteq$ *E. A pair (F,A) is called a soft set over X, where F is a mapping given by F:A* $\to$ *P(X).*

A soft set over X is a parameterized family of subsets of the universe X.

Definition 2.3 [9] Let I^X denote the set of all fuzzy sets on X and A $\subset$ E. A pair (f,A) is called a fuzzy soft set over X, where f is a mapping from A into I^X. That is, for each $a \in A$, $f(a) = f_a : X \to I,$ is a fuzzy set on X.

Definition 2.4 [2] *Let X be a group and (F,A) be a soft set over X. Then(F,A) is said to be a soft group over X iff* $F(a) < X$*, for each* $a \in A$. A soft group is a parameterized family of subgroups of X.

Definition 2.5 [1] *Let X be a group and* (f,A) *be a fuzzy soft set over X. Then* (f,A) *is said to be a fsg over X iff for each* $a \in A$ *and* $x, y \in X$*,*

$(i) f_a(x.y) \geq min\{f_a(x), f_a(y)\}$

$(ii) f_a(x^{-1}) \geq f_a(x)$.

That is, for each $a \in A$, f_a is a fuzzy subgroup.

Definition 2.6 [9] *For two fuzzy soft sets* (f,A) *and* (g,B) *over a common universe X, we say that* (f,A) *is a fuzzy soft subset of* (g,B) *and write* $(f,A) \subseteq (g,B)$ *if*

(i) $A \subset B$,and

(ii)For each $a \in A, f_a \leq g_a$,

that is, f_a is fuzzy subset of g_a.

Note that for all $a \in A$, f_a and g_a are identical approximations.

Definition 2.7 [9] *Union of two fuzzy soft sets* (f,A) *and* (g,B) *over a common universe X is the fuzzy soft set* (h,C)*, where* $C = A \cup B$ *and*

$$h(c) = \begin{cases} f_c & \text{if} c \in A - B \\ g_c & \text{if} c \in B - A \\ f_c \vee g_c & \text{if} c \in A \cap B \end{cases}, \text{ for all } c \in C.$$

We write $(f,A) \mathrm{X} (g,B) = (h,C)$.

Definition 2.8 [9] *Intersection of two fuzzy soft sets* (f,A) *and* (g,B) *over a common universe X is the fuzzy soft set* (h,C)*, where* $C = A \cap B$ *and* $h_c = f_c \wedge g_c$*, for all* $c \in C$.

We write $(f,A) \prod (g,B) = (h,C)$

Definition 2.9 [3] *Let X be a group and* (f,A) *be a fuzzy soft set over X. Then* (f,A) *is said to be an* l*-fsg over X if for each* $a \in A$ *and* $x, y \in X$*,*

$(i) f_a(x.y) \geq min\{f_a(x), f_a(y)\}([1])$

$(ii) f_a(x^{-1}) \geq f_a(x)([1])$

$(iii) a \leq b$ implies $f_a \subseteq f_b$ for all $a,b \in A$

ie., for all $a,b \in A$, $f_a \vee f_b$ and $f_a \wedge f_b$ exist in (f,A).

Example 3.2 [3] Let N be the set of all natural numbers and $(\mathrm{N}, \leq)$ be a lattice. If $a,b \in N$, then $a \vee b = max\{a,b\}$ and $a \wedge b = min\{a,b\}$. Define $f : N \to I^R$ by $f(n) = f_n : R \to I$ for each $n \in \mathrm{N}$ where

$$f_n(x) = \begin{cases} 1 - 1/n & \text{if} x = k2^{-n}, k \in Z \\ 0 & \text{otherwise} \end{cases}$$

Where Z is the set of all integers. Here, for each $n_1, n_2 \in N$, $n_1 \leq n_2$ implies $f_{n_1} \subseteq f_{n_2}$. Then the pair $((f,N), \vee, \wedge, \subseteq)$ forms an l - fuzzy soft group over R.

Definition 2.10 [3] *An* l*-fsg* (f,A) *is called a distributive* l*-fsg if for all* $a,b,c \in A$*,* $f_a \vee (f_b \wedge f_c) = (f_a \vee f_b) \wedge (f_a \vee f_c)$

Definition 2.11 [16] *Let* $((f,A), \vee, \wedge, \subseteq)$ *be an* l*-fsg over X. Then* (f,A) *is called an* l*-fsg chain over X if* $f_a(x) \subseteq f_b(x)$ *or* $f_b(x) \subseteq f_a(x)$*, for all* $a,b \in A$ *and* $x \in X$.

3. IDEALS ON l-fsg

Definition 3.1 *Let X be a group and* (f,A) *be an* l *- fsg over X. Let* $\Re$ *be an* l *- fsg binary relation on* (f,A) *over X ie.,* $\Re : (f,A) \times (f,A) \to [0,1]$. *Then* $\Re$ *is said to be an* l *-fsg equivalence relation over X if for all* $a_i, a_j, a_k \in A$

$(i) \Re(f_{a_i}, f_{a_i}) = 1$ (Reflexive)

$(ii) \Re(f_{a_i}, f_{a_j}) = \Re(f_{a_j}, f_{a_i})$ (Symmetric)

$(iii) \Re(f_{a_i}, f_{a_k}) \geq \sup_{a_j \in A} min\{\Re(f_{a_i}, f_{a_j}), \Re(f_{a_j}, f_{a_k})\}$

(Transitive)

Definition 3.2 *Let X be a group and* (f,A) *be an* l *- fsg over X. Let* $\Re$ *be an* l *- fsg equivalence relation*

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on (f, A). *Then* $\Re$ *is said to be an* l*-fsg congruence relation over X if for all* $a_i, a_j, b_i, b_j \in A$,

$(i)\Re(f_{a_i} \vee f_{a_j}, f_{b_i} \vee f_{b_j}) \geq \Re(f_{a_i}, f_{b_i})$ (Join Compatible)

$(ii)\Re(f_{a_i} \wedge f_{a_j}, f_{b_i} \wedge f_{b_j}) \geq \Re(f_{a_i}, f_{b_i})$ (Meet Compatible)

Definition 3.3 *Let X be a group and* (f, A) *be an* l*-fsg over X and* (l, B) *be a non-empty fuzzy soft subset of* (f, A). (l, B) *is said to be a fsg ideal of* (f, A) *over X if*

$(i) f_x, f_y \in (\text{l}, B) \Rightarrow f_x \vee f_y \in (\text{l}, B)$

$(ii) f_x \in (\text{l}, B), f_l \in (f, A) \Rightarrow f_x \wedge f_l \in (\text{l}, B)$

Proposition 3.4 *A non- empty fuzzy soft subset* (l, B) *of an* l*-fsg* (f, A) *over X is a fsg ideal iff*

$(i) f_x, f_y \in (\text{l}, B) \Rightarrow f_x \vee f_y \in (\text{l}, B)$

$(ii) f_x \in (\text{l}, B), f_a \in (f, A), f_a \subseteq f_x \Rightarrow f_a \in (\text{l}, B)$

Proof. Let a non- empty fuzzy soft subset (l, B) of an l-fsg (f, A) over X be a fsg ideal.
From the definition 3.3, (i) is satisfied.
Let $f_x \in (\text{l}, B), f_a \in (f, A)$ and $f_a \subseteq f_x \Rightarrow f_a = f_a \wedge f_x \in (\text{l}, B)$ (by definition 3.3)
Hence $f_a \in (\text{l}, B)$, (ii) is satisfied
Conversely, Let $f_x \in (\text{l}, B), f_a \in (f, A), f_a \subseteq f_x \Rightarrow f_a \in (\text{l}, B)$
Since $f_a \wedge f_x = f_a$ and $f_a \in (\text{l}, B) \Rightarrow f_a \wedge f_x \in (\text{l}, B)$
Hence (l, B) is a fsg ideal of (f, A) over X.

Proposition 3.5 *A fsg ideal of* (f, A) *over X is also an* l*-fsg over X.*

Proposition 3.6 *Union of two fsg ideals of* (f, A) *over X is also a fsg ideal of* (f, A) *over X iff one is contained in other.*

Proof. Let (l, B), (l, C) be two fsg ideals of (f, A) over X.

Consider $(\text{l}, B)\text{X}(\text{l}, C)$ is a fsg ideal of (f, A) over X.
Then $f_x, f_y \in (\text{l}, B)\text{X}(\text{l}, C)$ implies $f_x \vee f_y \in (\text{l}, B)\text{X}(\text{l}, C)$.
$\Rightarrow f_x \vee f_y \in (\text{l}, B)$ or $f_x \vee f_y \in (\text{l}, C)$.
If $f_x \vee f_y \in (\text{l}, B)$ then as $f_y \in (\text{l}, C) \subseteq (f, A)$
$\Rightarrow (f_x \vee f_y) \wedge f_y = f_y \in (\text{l}, B) \Rightarrow (\text{l}, C) \subseteq (\text{l}, B)$.
If $f_x \vee f_y \in (\text{l}, C)$ then as $f_x \in (\text{l}, B) \subseteq (f, A)$
$\Rightarrow (f_x \vee f_y) \wedge f_x = f_x \in (\text{l}, C) \Rightarrow (\text{l}, B) \subseteq (\text{l}, C)$.
Hence either $(\text{l}, B) \subseteq (\text{l}, C)$ or $(\text{l}, C) \subseteq (\text{l}, B)$
Converse is obvious.

Proposition 3.7 *Intersection of two fsg ideals of* (f, A) *over X is also a fsg ideal of* (f, A) *over X.*

Proof. Let (l, B), (l, C) be two fsg ideals of (f, A) over X.
Since (l, B), (l, C) are non-empty, there exist some $f_b \in (\text{l}, B)$, $f_c \in (\text{l}, C)$.
Now $f_b \in (\text{l}, B)$, $f_c \in (\text{l}, C) \subseteq (f, A)$
$\Rightarrow f_b \wedge f_c \in (\text{l}, B)$, Similarly $f_c \wedge f_b \in (\text{l}, C)$. Thus $(\text{l}, B)\prod(\text{l}, C) \neq \phi$
Let $f_x, f_y \in (\text{l}, B)\prod(\text{l}, C)$
$\Rightarrow f_x, f_y \in (\text{l}, B)$ and $f_x, f_y \in (\text{l}, C)$
$\Rightarrow f_x \vee f_y \in (\text{l}, B)$ and $f_x \vee f_y \in (\text{l}, C)$
$\Rightarrow f_x \vee f_y \in (\text{l}, B)\prod(\text{l}, C)$
If $f_x \in (\text{l}, B)\prod(\text{l}, C)$, $f_l \in (f, A)$
$\Rightarrow f_x \in (\text{l}, B)$, $f_x \in (\text{l}, C)$ and $f_l \in (f, A)$
$\Rightarrow f_x \wedge f_l \in (\text{l}, B)$ and $f_x \wedge f_l \in (\text{l}, C)$
$\Rightarrow f_x \wedge f_l \in (\text{l}, B)\prod(\text{l}, C)$
Hence $(\text{l}, B)\prod(\text{l}, C)$ is a fsg ideal of (f, A) over X.

Proposition 3.8 *The set of all fsg ideals of* (f, A) *over X forms an* l *- fsg over X.*

Proposition 3.9 *Let* (f, A) *be an* l *- fsg over X and a relation defined on* (f, A) *is*

$$\Psi(f_x, f_y) = \begin{cases} Max\{f_b : f_b \wedge f_x = f_b \wedge f_y; f_b \in (f, A)\}, \forall f_x, f_y \in (f, A) & \text{if } x \neq y. \\ 1 & \text{if } x = y \end{cases}$$

if the set of all elements f_b in (f, A) forms a fsg ideal over X then the relation $\Psi(f_x, f_y)$ is congruence relation of (f, A) over X.

Proof. Let X be a group and (f, A) be an l - fsg over X. Then we have for all $a_i, a_j \in A$, $a_i \leq a_j \Rightarrow f_{a_i} \subseteq f_{a_j}$ and for all $b_i, b_j \in A$, $b_i \leq b_j \Rightarrow f_{b_i} \subseteq f_{b_j}$.

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Let $(J, D) = \{f_b : f_b \wedge f_x = f_b \wedge f_y ; f_b \in (f, A)\}$, for all $f_x, f_y \in (f, A)$

Let $f_{b_1}, f_{b_2} \in (J, D)$ then

$f_{b_1} \wedge f_x = f_{b_1} \wedge f_y \Rightarrow f_{b_1} \subseteq f_x, f_{b_1} \subseteq f_y$

and $f_{b_2} \wedge f_x = f_{b_2} \wedge f_y \Rightarrow f_{b_2} \subseteq f_x, f_{b_2} \subseteq f_y$

$\Rightarrow f_{b_1} \subseteq f_x, f_{b_1} \subseteq f_y$, and

$f_{b_2} \subseteq f_x, f_{b_2} \subseteq f_y \Rightarrow f_{b_1} \vee f_{b_2} \subseteq f_x, f_{b_1} \vee f_{b_2} \subseteq f_y$

$\Rightarrow (f_{b_1} \vee f_{b_2}) \wedge f_x = f_{b_1} \vee f_{b_2}, (f_{b_1} \vee f_{b_2}) \wedge f_y = f_{b_1} \vee f_{b_2}$

$\Rightarrow (f_{b_1} \vee f_{b_2}) \wedge f_x = (f_{b_1} \vee f_{b_2}) \wedge f_y$

$\Rightarrow f_{b_1} \vee f_{b_2} \in (J, D)$

Hence (i) (definition 3.3) is satisfied.

Let $f_b \in (J, D)$, $f_x \in (f, A) \Rightarrow f_b \wedge f_x = f_b \wedge f_y$

$\Rightarrow f_b \subseteq f_x, f_b \subseteq f_y$

$\Rightarrow f_b \wedge f_x = f_b, f_b \wedge f_y = f_b$

Since $f_b \in (J, D) \Rightarrow f_b \wedge f_x, f_b \wedge f_y \in (J, D)$, for all $f_x, f_y \in (f, A)$

Hence (ii) (definition 3.3) is satisfied.

Therefore (J, D) is a fsg ideal of (f, A) over X.

Also,

Reflexive: $\Psi(f_x, f_x) = 1$

Symmetric:

$\Psi(f_x, f_y) = Max\{f_b : f_b \wedge f_x = f_b \wedge f_y ; f_b \in (f, A)\}, \forall f_x, f_y \in (f, A)$

$= Max\{f_b : f_b \wedge f_y = f_b \wedge f_x ; f_b \in (f, A)\}$

$= \Psi(f_y, f_x)$

Transitive:

$\Psi(f_x, f_z) \wedge \Psi(f_z, f_y)$

$= Max\{f_a : f_a \wedge f_x = f_a \wedge f_z ; f_a \in (f, A)\} \wedge Max\{f_b : f_b \wedge f_z = f_b \wedge f_y ; f_b \in (f, A)\}, \forall f_x, f_y \in (f, A)$

$= Max\{f_a \wedge f_b : f_a \wedge f_x = f_a \wedge f_z, f_b \wedge f_z = f_b \wedge f_y ; f_a, f_b \in (f, A)\}$

$= Max\{f_a \wedge f_b : (f_a \wedge f_x) \wedge (f_b \wedge f_z) = (f_a \wedge f_z) \wedge (f_b \wedge f_y)\}$

$= Max\{f_a \wedge f_b : f_a \wedge (f_x \wedge f_b) \wedge f_z = f_a \wedge (f_z \wedge f_b) \wedge f_y\}$

$= Max\{f_a \wedge f_b : (f_a \wedge f_z) \wedge (f_x \wedge f_b) = (f_a \wedge f_y) \wedge (f_z \wedge f_b)\}$

$= Max\{f_a \wedge f_b : (f_a \wedge f_x) \wedge (f_x \wedge f_b) = (f_a \wedge f_y) \wedge (f_y \wedge f_b)\}$

$= Max\{f_a \wedge f_b : (f_a \wedge f_x) \wedge (f_b \wedge f_x) = (f_a \wedge f_y) \wedge (f_b \wedge f_y)\}$

$= Max\{f_a \wedge f_b : (f_a \wedge f_b) \wedge f_x = (f_a \wedge f_b) \wedge f_y ; f_a \wedge f_b \in (f, A)\}$

$\leq Max\{f_a : f_a \wedge f_x = f_a \wedge f_y ; f_a \in (f, A)\}$

$= \Psi(f_x, f_y)$

Join Compatible and Meet compatible:

$\Psi(f_{a_i} \vee f_{a_j}, f_{b_i} \vee f_{b_j})$

$= Max\{f_c : f_c \wedge (f_{a_i} \vee f_{a_j}) = f_c \wedge (f_{b_i} \vee f_{b_j}) ; f_c \in (f, A)\}$

$\geq Max\{f_c : f_c \wedge f_{a_i} = f_c \wedge f_{b_i} ; f_c \in (f, A)\}$

$= \Psi(f_{a_i}, f_{b_i})$

Similarly we get, $\Psi(f_{a_i} \wedge f_{a_j}, f_{b_i} \wedge f_{b_j}) \geq \Psi((f_{a_i}, f_{b_i})$

Hence the relation $\Psi(f_x, f_y)$ is congruence relation of (f, A) over X.

Definition 3.10 *Let (f, A) be an l-fsg over X and (J, D) be a non-empty fuzzy soft subset of (f, A). (J, C) is said to be a fsg dual ideal of (f, A) over X if*

$(i) f_x, f_y \in (J, D) \Rightarrow f_x \wedge f_y \in (J, D)$

$(ii) f_x \in (J, D), f_l \in (f, A) \Rightarrow f_x \vee f_l \in (J, D)$

Proposition 3.11 *A fsg dual ideal of (f, A) over X is also an l-fsg over X.*

Proposition 3.12 *Union of two fsg dual ideals of (f, A) over X is also a fsg dual ideal of (f, A) over X iff one is contained in other.*

Proposition 3.13 *Intersection of two fsg dual ideals of (f, A) over X is also a fsg dual ideal of (f, A) over X.*

Proposition 3.14 *The set of all fsg dual ideals of (f, A) over X forms a lattice .*

Definition 3.15 *A sublattice (f, C) of an l-fsg (f, A) over X is said to be a convex sublattice of an l-fsg (f, A) if for all $f_a, f_b \in (f, C)$, $[f_a \wedge f_b, f_a \vee f_b] \subseteq (f, C)$.*

Proposition 3.16 *Every convex sublattice of an l-fsg (f, A) over X is the intersection of a fsg ideal and a fsg dual ideal of (f, A) over X.*

Proof. Let (f, A) be an l - fsg over X and (f, C) be a convex sublattice of (f, A) over X. Consider the fsg ideal

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$(\mathsf{I},B)=\{f_x\in(f,A)\,|\,\exists\ f_c\in(f,C),f_x\subseteq f_c\}$. Then $(\mathsf{I},B)\neq\phi$ as $(f,C)\subseteq(\mathsf{I},B)$. Consider the fsg dual ideal $(\mathsf{I},B)'=\{f_x\in(f,A)\,|\,\exists\ f_c\in(f,C),f_c\subseteq f_x\}$. Then $(\mathsf{I},B)'\neq\phi$ as $(f,C)\subseteq(\mathsf{I},B)'$. Then $(f,C)\subseteq(\mathsf{I},B)$ and $(\mathsf{I},B)'\Rightarrow(f,C)\subseteq(\mathsf{I},B)\prod(\mathsf{I},B)'$

Let $f_t\in(\mathsf{I},B)\prod(\mathsf{I},B)'$. Then $f_t\in(\mathsf{I},B)$ and $f_t\in(\mathsf{I},B)'$

$\Rightarrow\exists\ f_{c_1},f_{c_2}\in(f,C)$ such that $f_t\subseteq f_{c_2},f_{c_1}\subseteq f_t$

$\Rightarrow f_{c_1}\subseteq f_t\subseteq f_{c_2}$ or that $f_t\in[f_{c_1},f_{c_2}]$

Since (f,C) is a convex sublattice of (f,A), for all $c_1,c_2\in C$ such that $[f_{c_1},f_{c_2}]\subseteq(f,C)\Rightarrow f_t\in(f,C)\Rightarrow(\mathsf{I},B)\prod(\mathsf{I},B)'\subseteq(f,C)$. Hence $(f,C)=(\mathsf{I},B)\prod(\mathsf{I},B)'$.

Definition 3.17 *Let (f,A) be an l-fsg over X and f_a be any fuzzy subgroup of (f,A) over X. Let $(f_a]=\{f_x\in(f,A)/f_x\subseteq f_a\}$, then $(f_a]$ forms a fsg ideal of (f,A) over X. It is called a fsg principal ideal of (f,A) over X generated by f_a.*

Definition 3.18 *Let (f,A) be an l-fsg over X and f_a be any fuzzy subgroup of (f,A) over X. Let $[f_a)=\{f_x\in(f,A)/f_a\subseteq f_x\}$, then $[f_a)$ forms a fsg dual ideal of (f,A) over X. It is called a fsg principal dual ideal of (f,A) over X generated by f_a.*

Proposition 3.19 *The set of all fsg principal ideals(dual) of an l-fsg over X forms an l-fsg over X.*

Proposition 3.20 *In a fsg chain (f,A) over X, every fsg ideal(dual) of (f,A) over X is a fsg principal ideal(dual) of (f,A) over X.*

Proposition 3.21 *In an l-fsg (f,A) over X, for each $a_i,a_j\in A$, $a_i\leq a_j$ implies*

$(i)(f_{a_i}]\vee(f_{a_j}]=(f_{a_i}\vee f_{a_j}]$

$(ii)(f_{a_i}]\wedge(f_{a_j}]=(f_{a_i}\wedge f_{a_j}]$

$(iii)[f_{a_i})\vee[f_{a_j})=[f_{a_i}\wedge f_{a_j})$

$(iv)[f_{a_i})\wedge[f_{a_j})=[f_{a_i}\vee f_{a_j})$

Proof. Let (f,A) be an l-fsg over X. Then for each $a_i,a_j\in A$, $a_i\leq a_j$ implies $f_{a_i}\subseteq f_{a_j}$

$(i)(f_{a_i}\vee f_{a_j}]=(f_{a_j}]=\{f_{a_i},f_{a_j}\}=\{f_{a_i}\}\vee\{f_{a_i},f_{a_j}\}=(f_{a_i}]\vee(f_{a_j}]$

(ii) Similarly we get $(f_{a_i}\wedge f_{a_j}]=(f_{a_i}]\wedge(f_{a_j}]$

$$(iii)[f_{a_i}\wedge f_{a_j})=[f_{a_i})=\{f_{a_i},f_{a_j}.......\}=\{f_{a_i},f_{a_j}.......\}\vee\{f_{a_j},f_{a_k}.........\}=[f_{a_i})\vee[f_{a_j})$$

(iv) Similarly we get $[f_{a_i}\vee f_{a_j})=[f_{a_i})\wedge[f_{a_j})$

4. PRIME IDEALS ON l-fsg

Definition 4.1 *A fsg ideal (I,B) of (f,A) over X is called a fsg prime ideal of (f,A) if (I,B) is properly contained in (f,A) and whenever $f_a\wedge f_b\in(\mathsf{I},B)$ then $f_a\in(\mathsf{I},B)$ or $f_b\in(\mathsf{I},B)$.*

Proposition 4.2 *An l- fsg (f,A) over X is a fsg chain iff all fsg ideals of (f,A) over X are prime.*

Proof. Let an l- fsg (f,A) over X be a fsg chain. Then any two elements of A are comparable implies any two fuzzy subgroups are comparable. Let (I,B) be a fsg ideal of (f,A). If $f_a\wedge f_b\in(\mathsf{I},B)$ as f_a,f_b are in chain, they are comparable. Let $f_a\subseteq f_b$. then $f_a\wedge f_b=f_a$

Thus $f_a\wedge f_b\in(\mathsf{I},B)\Rightarrow f_a\in(\mathsf{I},B)\Rightarrow(\mathsf{I},B)$ is prime

Conversely,

Let every fsg ideals (I,B) of (f,A) be prime.

Let $(\mathsf{I},B)=\{f_b:f_b\wedge f_x=f_b\wedge f_y;f_b\in(f,A)\}$, for all $f_x,f_y\in(f,A)$ be a fsg prime ideal of (f,A).

Now $f_x\wedge f_y\in(\mathsf{I},B)$, (I,B) is prime, thus $f_x\in(\mathsf{I},B)$ or $f_y\in(\mathsf{I},B)$

$\Rightarrow f_x\subseteq f_x\wedge f_y$ or $f_y\subseteq f_x\wedge f_y$

$\Rightarrow f_x\wedge f_y\subseteq f_x\subseteq f_x\wedge f_y$ or $f_x\wedge f_y\subseteq f_y\subseteq f_x\wedge f_y$

$\Rightarrow f_x=f_x\wedge f_y$ or $f_y=f_x\wedge f_y$

$\Rightarrow f_x\subseteq f_y$ or $f_y\subseteq f_x$

$\Rightarrow$ an l- fsg (f,A) is a fsg chain over X.

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Definition 4.3 *A fsg proper dual ideal* (I, B) *of* (f, A) *over X is called a fsg dual prime ideal of* (f, A) *over X if* $f_a \vee f_b \in (\mathrm{I}, B)$ *then* $f_a \in (\mathrm{I}, B)$ *or* $f_b \in (\mathrm{I}, B)$.

Proposition 4.4 *Let* (I, B) *be a fsg prime ideal of* (f, A) *over X. If* (f, A) *is a fsg chain over X then* $(f, A) - (\mathrm{I}, B)$ *is a fsg dual prime ideal of* (f, A) *over X.*

Proof. Since $(\mathrm{I}, B) \neq \phi$, $(f, A) - (\mathrm{I}, B)$ is a proper subset of an l - fsg (f, A) over X.

Let $f_x, f_y \in (f, A) - (\mathrm{I}, B)$. Then $f_x, f_y \in (f, A)$ and $f_x, f_y \notin (\mathrm{I}, B)$

$\Rightarrow f_x \wedge f_y \in (f, A) - (\mathrm{I}, B)$

Again let $f_x \in (f, A) - (\mathrm{I}, B)$, $f_l \in (f, A)$

Then $f_x \in (f, A), f_x \notin (\mathrm{I}, B)$, $f_l \in (f, A)$

$\Rightarrow f_x \vee f_l \in (f, A), f_x \notin (\mathrm{I}, B)$

$\Rightarrow f_x \vee f_l \in (f, A), f_x \vee f_l \notin (\mathrm{I}, B) \Rightarrow f_x \vee f_l \in (f, A) - (\mathrm{I}, B)$

i.e., $(f, A) - (\mathrm{I}, B)$ is a fsg dual ideal of (f, A)

Let now $f_x \vee f_y \in (f, A) - (\mathrm{I}, B)$, then $f_x \vee f_y \in (f, A)$, $f_x \vee f_y \notin (\mathrm{I}, B)$

$\Rightarrow f_x, f_y \in (f, A), f_x \notin (\mathrm{I}, B)$ or $f_y \notin (\mathrm{I}, B)$

$\Rightarrow f_x \in (f, A) - (\mathrm{I}, B)$ or $f_y \in (f, A) - (\mathrm{I}, B)$

$\Rightarrow (f, A) - (\mathrm{I}, B)$ is a fsg dual prime ideal of (f, A) over X.

Proposition 4.5 *The set of all fsg ideals of an* l *- fsg* (f, A) *is distributive iff* (f, A) *is distributive.*

Proof. Let (f, A) be distributive. Let $(\mathrm{I}, B_1), (\mathrm{I}, B_2), (\mathrm{I}, B_3) \in \mathrm{I}(f, A)$ be three fsg ideals of (f, A). Let $f_x \in (\mathrm{I}, B_1) \wedge ((\mathrm{I}, B_2) \vee (\mathrm{I}, B_3)) \Rightarrow f_x \in (\mathrm{I}, B_1)$ and $f_x \in (\mathrm{I}, B_2) \wedge (\mathrm{I}, B_3)$

$\Rightarrow \exists f_b \in (\mathrm{I}, B_2), f_c \in (\mathrm{I}, B_3)$ such that $f_x \subseteq f_b \vee f_c$

$\Rightarrow f_x \wedge (f_b \vee f_c) = f_x \Rightarrow (f_x \wedge f_b) \vee (f_x \wedge f_c) = f_x$

(as (f, A) is distributive)

Now

$f_x \in (\mathrm{I}, B_1), f_b \in (\mathrm{I}, B_2) \subseteq (f, A) \Rightarrow f_x \wedge f_b \in (\mathrm{I}, B_1)$

Again, $f_x \in (\mathrm{I}, B_1) \subseteq (f, A)$ and $f_b \in (\mathrm{I}, B_2) \Rightarrow f_x \wedge f_b \in (\mathrm{I}, B_2)$

$\Rightarrow (f_x \wedge f_b) \in (\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)$

Similarly $(f_x \wedge f_c) \in (\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3)$

Since

$f_x = (f_x \wedge f_b) \vee (f_x \wedge f_c) \Rightarrow f_x \in ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)) \vee ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3))$

i.e.,

$(\mathrm{I}, B_1) \wedge ((\mathrm{I}, B_2) \vee (\mathrm{I}, B_3)) \subseteq ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)) \vee ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3))$

Again, let $f_x \in ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)) \vee ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3))$

Then $f_x \subseteq f_{t_1} \vee f_{t_2}$ for some $f_{t_1} \in (\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2), f_{t_2} \in (\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3)$

Now $f_{t_1} \vee f_{t_2} \in (\mathrm{I}, B_1), f_x \subseteq f_{t_1} \vee f_{t_2} \Rightarrow f_x \in (\mathrm{I}, B_1)$

Also $f_{t_1} \in (\mathrm{I}, B_2), f_{t_2} \in (\mathrm{I}, B_3)$ and $f_x \subseteq f_{t_1} \vee f_{t_2} \Rightarrow f_x \in (\mathrm{I}, B_2) \vee (\mathrm{I}, B_3)$

Thus $f_x \in (\mathrm{I}, B_1) \wedge ((\mathrm{I}, B_2) \vee (\mathrm{I}, B_3))$

or that

$((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)) \vee ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3)) \subseteq (\mathrm{I}, B_1) \wedge ((\mathrm{I}, B_2) \vee (\mathrm{I}, B_3))$

i.e.

$(\mathrm{I}, B_1) \wedge ((\mathrm{I}, B_2) \vee (\mathrm{I}, B_3)) = ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_2)) \vee ((\mathrm{I}, B_1) \wedge (\mathrm{I}, B_3))$.

Hence $\mathrm{I}(f, A)$ is distributive

Conversely, $\mathrm{I}(f, A)$ be distributive. Define $\theta : (f, A) \to \mathrm{I}(f, A)$ such that $\theta(f_a) = (f_a]$, the principal ideal generated by f_a. θ is clearly well defined.

Also $\theta(f_a) = \theta(f_b) \Rightarrow (f_a] = (f_b]$ and $f_a \in (f_a] \Rightarrow f_a \in (f_b] \Rightarrow f_a \subseteq f_b$. Similarly $f_b \subseteq f_a$ and thus $f_a = f_b$. i.e. θ is one-one.

Again

$\theta(f_a \wedge f_b) = (f_a \wedge f_b] = (f_a] \wedge (f_b] = \theta(f_a) \wedge \theta(f_b)$

[by prop 3.21]

and

$\theta(f_a \vee f_b) = (f_a \vee f_b] = (f_a] \vee (f_b] = \theta(f_a) \vee \theta(f_b)$

[by prop 3.21]

Hence θ is one-one homomorphism. If (f, A) is finite, (f, A) is isomorphic to $\mathrm{I}(f, A)$. Since $\mathrm{I}(f, A)$ is distributive, (f, A) is distributive.

Proposition 4.6 *Let* (f, A) *be a distributive* l *- fsg over X, let* (I, B) *be a fsg ideal of* (f, A)*, let* (J, C) *be a fsg dual ideal of* (f, A) *and* $(\mathrm{I}, B) \prod (\mathrm{J}, C) = \phi$. *Then there exists a fsg prime ideal* (P, D) *of* (f, A) *such that* $(\mathrm{P}, D) \supseteq (\mathrm{I}, B)$ *and* $(\mathrm{P}, D) \cap (\mathrm{J}, C) = \phi$.

Proof. Let $S_p(f, A)$ be the set of all fsg prime ideals. Define $\beta(s) = \mathbf{I}\ \{(P, D) \in S_p(f, A) / (P, D) \supseteq (\mathrm{I}, B), (P, D) \prod (\mathrm{J}, C) = \phi\}$. Since $(\mathrm{I}, B) \in \beta(s)$, $\beta(s) \neq \phi$. If $\beta(s)$ is a fsg chain then $\beta(s)$ has the maximal element (P, D). Suppose

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(P,D) is not a fsg prime ideal of (f,A). Then there exist $f_a, f_b \in (f,A)$ such that $f_a, f_b \notin (P,D)$ but $f_a \wedge f_b \in (P,D)$. Maximality of (P,D) yields that $((P,D) \vee (f_a]) \prod (\mathsf{J},C) \neq \phi$ and $((P,D) \vee (f_b]) \prod (\mathsf{J},C) \neq \phi$. Thus there are $f_l, f_m \in (P,D)$ such that $f_l \vee f_a \in (P,D)$ and $f_m \vee f_b \in (P,D)$. Then $f_x = (f_l \vee f_a) \wedge (f_m \vee f_b) \in (\mathsf{J},C)$, Since (J,C) is fsg dual ideal of an l-fsg (f,A). Expanding by distributivity,
$f_x = (f_l \wedge f_m) \vee (f_l \wedge f_b) \vee (f_a \wedge f_m) \vee (f_a \wedge f_b) \in (P,D)$.
Thus $(P,D) \prod (\mathsf{J},C) \neq \phi$, which is contradiction. Hence (P,D) is a fsg prime ideal of an l-fsg (f,A).

5. TO FIND MORE OBTAINABLE MOBILE NETWORK AND THE CORRESPONDING ROUTE AMONG ALL ROUTES

In one's long journey, the following permissible routes form an lattice ordered fuzzy soft group (already proved in [17]).

R1= { P1/0, P2/0.3, P4/0.2, P7/0.5, RP/1 }

R2= { P1/0, P2/0.3, P3/0.2, P4/0.2, P7/0.5, RP/1 }

R3= { P1/0, P2/0.3, P4/0.2,P6/ 0.2, P7/0.5, RP/1 }

R4= { P1/0, P2/0.3, P3/0.2, P4/0.2,P6/ 0.2, P7/0.5, RP/1 }

R5={ P1/0, P2/0.3, P4/0.2, P5/ 0.1, P6/ 0.2, P7/0.5, RP/1 }

R6= { P1/0, P2/0.3,P3/0.2, P4/0.2, P5/ 0.1, P6/ 0.2, P7/0.5, RP/1 }

Then $(f,A) = \{R1, R2, R3, R4, R5, R6\}$.

The person wants to find the mobile network which is more obtainable among all routes and also to find the route which has maximum network coverage by comparing routes.

Step: 1

Consider a fsg prime ideal $(G,B) = \{R_1, R_2, R_4\}$ of (f,A), and fix the parameters 'n_1 network, n_2 network, n_3 network, n_4 network and n_5 network'

	n_1	n_2	n_3	n_4	n_5
R_1	0.8	0.6	0.4	0.3	0.5
R_2	0.9	0.7	0.6	0.5	0.4
R_4	0.8	0.6	0.6	0.5	0.2

Step: 2

Construct the table for finding total value of minimum obtainable network coverage by comparing routes from step1

	n_1	n_2	n_3	n_4	n_5
$R_1 \wedge R_2$	0.8	0.6	0.4	0.3	0.4
$R_1 \wedge R_4$	0.8	0.6	0.4	0.3	0.2
$R_2 \wedge R_4$	0.8	0.6	0.6	0.5	0.2
Total	2.4	1.8	1.4	1.1	0.8

Step: 3

Consider a fsg dual prime ideal $(H,C) = \{R_3, R_5, R_6\}$ of (f,A), and fix the parameters 'n_1 network, n_2 network, n_3 network, n_4 network and n_5 network'

	n_1	n_2	n_3	n_4	n_5
R_3	0.5	0.7	0.5	0.4	0.3
R_5	0.8	0.9	0.5	0.4	0.5
R_6	0.7	0.8	0.6	0.4	0.3

Step: 4

Construct the table for finding total value of minimum obtainable network coverage by comparing routes from Table-2

	n_1	n_2	n_3	n_4	n_5
$R_3 \wedge R_5$	0.5	0.7	0.5	0.4	0.3
$R_3 \wedge R_6$	0.5	0.7	0.5	0.4	0.3
$R_5 \wedge R_6$	0.7	0.8	0.5	0.4	0.3
Total	1.7	2.2	1.5	1.2	0.9

Step: 5

From step 2 and step 4, the maximum value of minimum obtainable network is n_1 network in the routes of R_1{ P1/0, P2/0.3, P4/0.2, P7/0.5, RP/1 } , R_2{ P1/0, P2/0.3, P3/0.2, P4/0.2, P7/0.5, RP/1 } and R_4{ P1/0, P2/0.3, P3/0.2, P4/0.2,P6/ 0.2, P7/0.5, RP/1 }. On these routes from step 1, the place P_3 has the heighest n_1 network coverage and the place P_6 has the lowest n_1 network coverage(since $R_1(n_1) < R_2(n_1) > R_4(n_1)$). On the basis of these analysis, The person could choose the n_1 network and the route R_2 for avoiding any network coverage problem.

6. CONCLUSIONS

In this present paper, the theory of l-fsg has been developed with lattice ideal approach. The concept of fsg Ideal, fsg prime ideal, fsg principal ideal and convex sublattices on l-fsg and its duality have been discussed and its relevant properties are attained. The idea of fsg ideal has been applied succesfully in decision making process.

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