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EOQ INVENTORY MODEL FOR TIME DEPENDENT DETERIORATING PRODUCTS WITH QUADRATIC TIME VARYING DEMAND, VARIABLE DETERIORATION AND PARTIALBACKLOGGING

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ABSTRACT

In this article we present a suitable EOQ Inventory model for deteriorating items having time dependent demand, allowing shortages. We have considered a variable deterioration rate which is sensitive to changes in time. The demand rate is assumed to be a quadratic function of time.The permitted shortages are considered to be partly backlogged. The backlogging rate is changing and reliant on the length of the following renewal. The mathematical formulation of the model is first developed and solved analytically to minimize the average inventory cost. Finally we have discussed the proposed model with a numerical example and analysed its sensitivity to change in parameters.

Keywords: inventory, varying deterioration, time varying quadratic demand, partial backlogging.

INTRODUCTION

In the study of inventory models, deterioration of items and demand assumes an essential part. There is a common misconception that inventory cannot go to waste. Inventory can be damaged in many ways and at some point every business that sells products has to deal with damaged goods. Deterioration in inventory is a practical feature which needs consideration. Deterioration refers to the process of becoming second rate in quality, character, value or usefulness. Some organizations offer goods such as food products that have the tendency to deteriorate over time, which results in the possibility of not being able to sell some of the inventory in time, before it deteriorates. Similarly, certain types of products, such as computers and other electronic devices tend to become obsolete quickly. Consumers might not be willing to buy old renditions of products at a value that is gainful when new or updated renditions become accessible. The loss due to deterioration of inventory cannot be overlooked. Due to deterioration, inventory system confronts the issue of deficiencies and loss of good-will or benefit. Lost deals and lost good-will can discolour a brand's notoriety in specified time.

Inaccurate forecasting of consumer demand can also lead to a loss incurred by the retailer, since at the end of a product's life cycle, it would have lost its value. Due to this, we can make the proper planning to maintain the inventory economically to increase the revenue of industry. Certain goods will not have any offer due to changes in market demand. For example, a clothing store that stocks too many woollen clothes during the winter may find itself unable to get rid of them before summer, leaving the company with a large quantity of goods on hand that simply take up space. Such commodities whose demand rate undergoes seasonal variations may follow quadratic function of time. Many researchers have argued that the demand rate need not follow either linear or exponential pattern. So it is reasonable to expect that the demand rate in a couple of merchandise may follow quadratic function of time.

In most researches, the deteriorating rate in the inventory models of deteriorating items has been considered to be fixed, which is not possible in real life situations. The research on inventory began with Whitin [19] who studied the deterioration of fashion items after a specific time of storage. Later Ghare and Schrader [7] analysed the inventory model of deteriorating items considering the decay at an exponential rate. Covert and Philip [5] in their article have considered the deterioration rate to follow time dependent Weibull distribution. Anchal Agarwal and Singh [1] considered the deterioration rate to be Weibull distributed with two parameters along with time dependent demand and shortages. Rangarajan and Karthikeyan [14] formulated an EOQ model in which the rates of deterioration and demand are piecewise continuous cubic function of time. Many items like fruits, blood, alcohol, etc. do not start to deteriorate instantly. They have a period of time before the start of deterioration. Jeyaraman and Sugapriya [8] found a common time for a production cycle of an inventory model with non - instantaneous deteriorating items permitting price discount, utilizing reasonable deferral in payments. Palani and Maragatham [10] built an inventory model in which the deterioration rate could be controlled using preservation technology. Chang and Dye [4] developed an inventory model with demand varying with time and partially backordered, considering stock out cost. They discussed that the backlogging rate would decrease if the waiting time increases. So the number of the clients who would like to accept backlogging also decreases with the waiting time for the next renewal. Research in this direction was continued by Ouyang *et al.* [5].

Donaldson [6] was the first to examine about the inventory replenishment strategy for a linear pattern in demand. The same was continued by Singh and Pattanayak [15] who established an EOQ inventory model for deteriorating items that deteriorates according to a variable deterioration rate with the demand function being linear with partial backlogging. Peter *et al* [12] have proposed an expanded stock model for items which have an increasing demand until a particular period after which demand becomes consistent, the demand rate being ramp type. Anil Kumar Yadav and Meenakshi Yadav [2] built a stock model with time dependent quadratic demand and partial backlogging in which deficiencies are permitted, while rate of deterioration and holding cost are considered to be consistent. Many researchers have assumed the deterioration of goods as a complete loss to the inventory system. Venkateswarlu and Mohan [17] formulated the inventory model to study the impact of salvage value related with time dependent deteriorated items during cycle duration, when demand rate follows time dependent quadratic demand. The justifications for considering the demand rate to be quadratic are discussed. Begum et. al. [3] gave a new approach to the inventory model considering the time dependent demand trends. They have proposed that the quadratic demand pattern depicts the rise or fall in demand in the best way. In some practical situations customers go in search of new suppliers or they may wait for the consignment. Hence there is a loss in the sales which is a partial loss. Researchers like Ramen Patel [13], Mishra and Singh [18], Manna and Chaudhuri [16] and PJ Mishra *et al.*[11] have considered the case of partial backlogging rates in their models. Yang [20] proposed another model in which partial backlogging of deteriorating items was done in two warehouses under inflation.

In the present paper we have developed the model by taking the demand rate to be quadratic function of time. Quadratic function of time expresses the steady growth or decline of the demand in the best way. Variable rate of deterioration is considered since constant deterioration rate is not possible in reality and deterioration is a serious issue which cannot be ignored. Shortages are allowed with partial backlogging. The backlogging rate is inversely proportional to the time taken for the next replenishment. We have solved the model to optimize the total profit which is maximum. A numerical example is given to illustrate the developed model and the sensitivity analysis of the solution is carried out.

ASSUMPTIONS

The following assumptions are made in developing the model.

- The Demand $D(t)$ at time t is assumed to be

$$D(t) = \begin{cases} a + bt + ct^2, I(t) > 0 \\ D_0 \ , \qquad I(t) \le 0 \end{cases}$$

where $a > 0, b > 0, c > 0$. Here a is the initial rate of demand, b is the rate at which the demand rate increases and c is the rate at which the change in the demand rate itself increases

- Replenishment rate is infinite.
- Shortages are allowed and partially backlogged.
- The lead time is zero.
- The planning horizon is infinite.
- The deteriorating rate $\theta(t) = \theta t, 0 < \theta < 1$ is variable and there is no replacement of deteriorated items during the period of consideration.
- During shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. So the backlogging rate for negative inventory is $\frac{1}{1+\delta(T-t)}$ where δ is the backlogging parameter and $(T - t)$ is the waiting time $(t_1 \le t \le T)$.

NOTATIONS

The following notations have been used in developing the model.

- C_1 : holding cost per unit per unit time.
- C_2 : cost of the inventory per unit.
- C_3 : ordering cost of inventory per order.
- C_4 : Shortage cost per unit per unit time.
- C_5 : Opportunity cost due to lost sales per unit.
- t_1 : The time at which the inventory level reaches zero.
- T : Length of each ordering cycle.
- W : The maximum inventory level for each ordering cycle.
- S : The maximum amount of demand backlogged for each ordering cycle.
- Q : The order quantity for each ordering cycle.
- $I(t)$: The inventory level at time t .
- t_1^* : The optimal solution of t_1.
- T^* : The optimal solution of$T$.
- TC^* :The minimum total cost per unit time.

FORMULATION AND SOLUTION OF THE MODEL

The length of the cycle is$T$. At the time t_1 the inventory level becomes zero and shortages occurring in the period (t_1, T) is partially backlogged.

Let $I(t)$ be the inventory level at time t , $(0 < t < t_1)$.

The differential equations for the instantaneous state over $(0, T)$ are given by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a + bt + ct^2),\ 0 \le t \le t_1 \qquad (1)$$

With boundary conditions$I(t_1) = 0$.

The solution of equation (1) is

$$I(t) = \left[a\left(t_1 + \frac{\theta t_1^3}{6}\right) + b\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right) + c\left(\frac{t_1^3}{2} + \frac{\theta t_1^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}} - \left[a\left(t + \frac{\theta t_1^3}{6}\right) + b\left(\frac{t^2}{2} + \frac{\theta t_1^4}{8}\right) + c\left(\frac{t^3}{2} + \frac{\theta t_1^5}{10}\right)\right] e^{-\frac{\theta t^2}{2}} \qquad (2)$$

Maximum inventory level for each cycle is obtained by putting boundary condition $I(0) = W$ in equation (2).

Therefore,

$$I(0) = W = a\left(t_1 + \frac{\theta t_1^3}{6}\right) + b\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right) + c\left(\frac{t_1^3}{2} + \frac{\theta t_1^5}{10}\right) \qquad (3)$$

VOL. 13, NO. 7, APRIL 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

During the shortage interval $[t_1, T]$, the demand at time t is partially backlogged at the fraction $\frac{1}{1+\delta(T-t)}$.

Therefore the differential equation governing the amount of demand backlogged is

$$\frac{dI(t)}{dt} = -\frac{D_0}{1+\delta(T-t)}, t_1 < t \le T \quad (4)$$

With boundary conditions $I(t_1) = 0$.
The solution of equation (4) is

$$I(t) = \frac{D_0}{\delta} \ In[1 + \delta(T - t)], t_1 < t \le T \quad (5)$$

Maximum amount of demand backlogged per cycle is obtained by putting $t = T$ in equation (5). Therefore,

$$S = -I(t) = \frac{D_0}{\delta} \ In[1 + \delta(T - t)] \quad (6)$$

Hence $Q = W + S = a\left(t_1 + \frac{\theta t_1^3}{6}\right) + b\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right) + c\left(\frac{t_1^3}{2} + \frac{\theta t_1^5}{10}\right) + \frac{D_0}{\delta} \ In[1 + \delta(T - t)]$ (7)

The inventory holding cost per cycle is

$$HC = C_1 \int_0^{t_1} I(t)dt = C_1\left[a\left(\frac{t_1{}^2}{2} + \frac{\theta t_1^4}{12}\right) + b\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right) + c\left(\frac{t_1^4}{4} + \frac{\theta t_1^6}{18}\right)\right] \quad (8)$$

The deterioration rate per cycle is

$$DC = C_2\left[W - \int_0^{t_1} I(t)dt\right] = C_2\theta\left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10}\right] \quad (9)$$

The shortage cost per cycle is

$$SC = C_4\left[-\int_{t_1}^{T} I = (t)dt\right] = C_4 D_0\left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10}\right] \quad (10)$$

The opportunity cost lost sales per cycle is

$$OC = C_5 D_0\left[T - t_1 - \frac{1}{\delta} \ In[1 + \delta(T - t_1)]\right] \quad (11)$$

Therefore the average total cost per unit time per cycle = (HC + DC + OC + SC + opportunity cost due to lost sales) / length if the ordering cycle, i.e.,

$$TC = TC((t_1, T) = \frac{C_1}{T}\left[a\left(\frac{t_1{}^2}{2} + \frac{\theta t_1^4}{12}\right) + b\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right) + c\left(\frac{t_1^4}{4} + \frac{\theta t_1^6}{18}\right)\right] + \frac{C_2\theta}{T}\left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10}\right] + \frac{C_3}{T} + \frac{(C_4+\delta C_5)}{\delta T}[\mathrm{T} - t_1] - \frac{(C_4+\delta C_5)}{\delta^2 T}\left[In\left(1 + \delta(T - t_1)\right)\right] \quad (12)$$

Our aim is to determine the optimal values of t_1 and T in order to minimize the average total cost per unit time, TC.

Using calculus, we now minimize TC. The optimum values of t_1 and T for the minimum average cost TC are the solutions of the equations

$$\frac{\partial(TC)}{\partial t_1} = 0 \text{ and} \frac{\partial(TC)}{\partial T} = 0 \quad (13)$$

Provided that they satisfy the sufficient condition

$$\frac{\partial^2(TC)}{\partial t_1{}^2} > 0, \frac{\partial^2(TC)}{\partial^2 T^2} > 0 \text{ and} \frac{\partial^2(TC)}{\partial t_1{}^2} \cdot \frac{\partial^2(TC)}{\partial^2 T^2} - \left(\frac{\partial^2(TC)}{\partial t_1 \partial T}\right)^2 > 0$$

Equation (13) implies

$$\frac{\partial(TC)}{\partial t_1} = \frac{C_1}{T}\left[a\left(\frac{t_1{}^2}{2} + \frac{\theta t_1^4}{12}\right) + b\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right) + c\left(\frac{t_1^4}{4} + \frac{\theta t_1^6}{18}\right)\right] + \frac{C_2\theta}{T}\left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10}\right] - \frac{(C_4+\delta C_5)D_0(T-t_1)}{T(1+\delta(T-t_1))} = 0 \quad (14)$$

and

$$\frac{\partial(TC)}{\partial T} = \frac{1}{T}\left[\frac{(C_4+\delta C_5)D_0(T-t_1)}{T(1+\delta(T-t_1))} - \frac{C_1}{T}\left[a\left(\frac{t_1{}^2}{2} + \frac{\theta t_1^4}{12}\right) + b\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right) + c\left(\frac{t_1^4}{4} + \frac{\theta t_1^6}{18}\right)\right] + \frac{C_2\theta}{T}\left[\frac{at_1^3}{6} + \frac{bt_1^4}{8} + \frac{ct_1^5}{10}\right] + \frac{C_3}{T} + \frac{(C_4+\delta C_5)}{\delta T}[\mathrm{T} - t_1] - \frac{(C_4+\delta C_5)}{\delta^2 T}\left[In\left(1 + \delta(T - t_1)\right)\right]\right] = 0 \quad (15)$$

Now t_1^* and T^* are obtained from the equations (14) and (15) respectively. Next, by using t_1^* and T^*, we can obtain the optimal economic order quantity, the optimal maximum inventory level and the minimum average cost per unit time from equations (7), (3) and (12) respectively.

NUMERICAL EXAMPLE

Let us consider the following example to illustrate the above developed model, taking $a = 12$, $b = 2$, $c = 1.5$, $C_1 = 0.5$, $C_2 = 1.5$, $C_3 = 3$, $C_4 = 2.5$, $C_5 = 2$, $D_0 = 8$, $\theta = 0.01$ and $\delta = 2$ (with appropriate units).

The optimal values of t_1^* and T^* are $t_1^* = 0.0021$ and $T^* = 1.7028$ units and the optimal total cost per unit time $TC^* = 65.7428$ units.

SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for t_1^*, T^* and total cost per unit time TC^* with respect to the changes in the values of the parameters a, b, c, θ, C_1 and C_4. The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed. The last column of the Table-1 gives the % changes in TC^* as compared to the original solution for the relevant costs.

ARPN Journal of Engineering and Applied Sciences

Table-1. Sensitivity analysis of different parameters.

Parameter	% change in parameter	t_1^*	T^*	TC^*	% Change in TC^*
a	+25	0.0026	1.7052	65.9401	0.30
	+5	0.0028	1.7108	66.4408	1.06
	-5	0.0028	1.7146	66.7870	1.59
	-25	0.0030	1.7164	66.9472	1.83
b	+25	0.0023	1.7264	67.9152	3.30
	+5	0.0024	1.7028	65.7306	-0.02
	-5	0.0024	1.6972	65.2280	-0.78
	-25	0.0025	1.6856	64.1754	-2.38
c	+25	0.0025	1.7028	65.7265	-0.02
	+5	0.0027	1.7027	65.7093	-0.05
	-5	0.0029	1.7026	65.6921	-0.08
	-25	0.0034	1.6787	63.5214	-3.38
C_1	+25	0.0039	1.9377	88.3558	34.40
	+5	0.0055	1.7036	65.6766	-0.10
	-5	0.0077	1.6932	64.6482	-1.66
	-25	0.0085	1.6701	63.0694	-4.07
C_4	+25	0.0018	1.2155	20.3862	-68.99
	+5	0.0016	1.2159	18.4688	-71.91
	-5	0.0016	1.2161	17.5066	-73.37
	-25	0.0015	1.2167	15.5873	-76.29
θ	+25	0.0022	1.7028	65.7388	-0.01
	+5	0.0022	1.6972	65.2320	-0.78
	-5	0.0022	1.6856	64.1875	-2.37
	-25	0.0022	1.6638	62.2431	-5.32

Observations

From table (1), the following points are observed.

- ${t_1}^*$, T^* and TC^* increases with decrease in the value of the parameter a.
- ${t_1}^*$increases while T^* and TC^* decreases with decrease in the value of the parameters bc and C_1.
- ${t_1}^*$andTC^*decreases while T^*increases with decrease in the value of the parameterC_4.
- ${t_1}^*$remains constant while T^* and TC^* decreases with decrease in the value of the parameterθ.

CONCLUSIONS

Many researchers have discussed about the constant rate of deterioration in each cycle which is not appropriate in the real world. So in the present paper we have considered the variable rate of deterioration along with time dependent Quadratic demand. It is seen thatamong the different demand patterns, the most realistic approach is to consider the quadratic demand pattern because it represents both accelerated and retarded growth in demand.To meet to the demand, the shortages are partially backlogged. The proposed model can be further extended by considering the stock dependent demand or stochastic demand.We can also incorporate discounts, inflation and permissible deferral in payments etc.

REFERENCES

[1] Anchal Agarwal and S.R. Singh. 2013. An EOQ Inventory model for two parameter Weibull Deterioration with time dependent demand and shortages. International Journal of Engineering Research and Technology. 2: 1193-1200.

[2] Anil Kumar Yadav and Meenakshi Yadav. 2015. An Inventory model for deteriorating goods with time dependent Quadratic Demand with partial backlogging. International Journal of Mathematical Archieve.6: 168-171.

[3] R.Begum, S.K.Sahu and R.R.Sahoo. 2012. An Inventory Model for Deteriorating Items with Quadratic demand and partial backlogging. British Journal of Applied Science and Technology.2: 112-131.

[4] H.J.Chang and C.Y.Dye. 1999. An EOQ model for deteriorating items with time varying Demand and partial backlogging. Journal of the Operational Research Society. 50: 1176-1182.

[5] R.B.Covert and G.S.Philip. 1973. An EOQ Model with Weibull Distribution Deterioration. AIIE Transactions. 5: 323-326.

[6] W.A.Donaldson. 1977. Inventory Replenishment policy for a linear trend in Demand - an analytical solution. Operational Research Quarterly.(28): 663-670.

[7] P.M.Ghare and G.P.Schrader. 1963. A model for an exponentially decaying inventory. Journal of Industrial Engineering. 14: 449-460.

[8] K. Jeyaraman and C.Sugapriya. 2008. Determining a common production cycle time for an EPQ model with non - instantaneous deteriorating items allowing price discount using permissible delay in payments. ARPN Journal of Engineering and Applied Sciences.3: 26-30.

[9] L.Y.Ouyang, K.S.Wu and M.C.Cheng. 2005. An Inventory model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging. *Yougoslav* Journal of Operations Research. 15: 277-288.

[10] R.Palani and M.Maragatham. 2016.EOQ model for controllable deterioration rate and time dependent demand and Inventory holding cost. International Journal of Mathematical Trends and Technology. 39: 245-251.

[11] PanditJagatananda Mishra, Trilokyanath Singh and Hadibandu Pattanayak. 2016. An EOQ Model for deteriorating items with Linear Demand, Salvage Value and Partial Backlogging. International Journal of Applied Engineering Research. 11: 6479-6484.

[12] Peter Shaohua Deng, Robert H J.Lin & Peter Chu. 2007.A note on the inventory models for deteriorating items with ramp type demand rate. European Journal of Operational Research.178:112-120.

[13] Ramen Patel. 2013. An Inventory model for Deteriorating items with Quadratic Demand, Partial backlogging under inflation and Permissible delay in payments. International Journal of Emerging trends in Engineering and Development. 6: 508-529.

[14] K.Rangarajan and K.Karthikeyan. 2016. EOQ model for Non- Instantaneous deteriorating Items with Cubic deterioration and Cubic Demand. Asian Journal of Research in Social Sciences and Humanities. 6: 2099-2111.

[15] T.Singh and H.Pattanayak. 2013. An EOQ Model for Deteriorating Items with Linear Demand, Variable Deterioration and Partial Backlogging. Journal of Service Science and Management.6: 186-190.

[16] Swapan Kumar Manna and Kripasindhu Chaudhuri. 2014. An order level Inventory model for a deteriorating item with Quadratic time-varying demand, shortage and partial backlogging. ARPN Journal of Engineering and Applied Sciences. 9: 692-698.

[17] R.Venkateswarlu and R. Mohan. 2011. Inventory models for Deteriorating Items with Time Dependent Quadratic Demand and Salvage value. International Journal of applied Mathematical Sciences. 5: 11-18.

[18] Vinod Kumar Mishra and Lal Sahab Singh. 2010. Deteriorating Inventory Model with Time dependent Demand and partial backlogging. *Applied Mathematical Sciences*. 4: 3611-3619.

[19] T.M.Whitin. 1957. Theory of inventory management. Princeton University Press. Princeton, NJ. pp.62-72.

[20] H.L.Yang. 2000. Two warehouse partial backlogging inventory models for deteriorating items under inflation. International Journal of Production Economics.(103): 362-370.