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A MATHEMATICAL MODELING AND SIMULATION OF NON LINEAR ORDINARY DIFFERENTIAL EQUATIONS USING HPM

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ABSTRACT

In this paper, we investigated the approximate analytical solution of non linear ordinary differential equations system using Homotopy Perturbation Method (HPM). These analytical solutions are representing a simple form of the system's description allowing easy curve fitting to experiment. Numerical simulations are represented to provide the detailed results.

Keywords: non linear ordinary differential equation, mathematical modeling, HPM, numerical simulation.

INTRODUCTION

The system of nonlinear ordinary differential equation is investigated in various aspects [1]. Now a day's much attention paid to ordinary differential equations. In this model they will see how the analytic is approximate solution and they will study about the x, y and z like free load [7]. There are many models are played to explain this method like SIR, SIRS, etc. Homotopy perturbation method is one of the main tools to find the approximate solution or analytically approximate solution of the model. In this situation to study about the nonlinear ODE is very important [13].

First in 1993, Perelson, Krischner and De Boer have proposed an ordinary differential equation model [2, 3]. In this model it consists of four phenomena such as the x, y and z. Initially this model is very different. This model played a significant role in the development of the nonlinear in various aspects. In other words is included the initial and boundary values of the non linear system [8 - 10]. The main target of is to find the analytic solutions of the non linear ODE [18].

In the system we have to investigate the numerical solutions [11, 12]. This model is generally called as approximation of the real world problems. Finally in this paper we will find an approximate solution of the model by using the Homotopy Perturbation method (HPM) [14, 15]. Some method of finding numerical solutions is discussed in [16, 17]. The amount of x, y and z is the key role for the system is affected by Non linear system [19].

Nomenclature

x Number of first compartment
y Number of second compartment
z Number of third compartment

Here s, d, a, β, ρ, δ, q and c all are constants

Mathematical modeling

The system nonlinear ordinary differential equation model:

$$\frac{dx}{dt} = s - dx + ax - \beta xz + \rho y \tag{1}$$

$$\frac{dy}{dt} = \beta xz - \delta y - \rho y \tag{2}$$

$$\frac{dz}{dt} = qy - cz \tag{3}$$

In order to find the approximate solution of the above system, we need the following initial and boundary conditions:

$x(0) = 0$, $y(0) = 0$, $z(0) = 0$ and
$x(0) = 32$, $y(0) = 20$, $z(0) = 3208$.

Homotopy Perturbation Method (HPM)

Homotopy perturbation method is involved many non linear problems in Engineering sciences and Applied sciences [4, 6]. This Homotopy perturbation method is used in diffusion equation, non linear dynamical system, Blasius equation and many differential equations like Burger's equations, Volterra's integro differential equations. In Appendix A, we provide a convenient way to obtain analytic or approximate solution for a wide variety of problems arising in different fields [20]. This method is universally accepted for solving non linear differential equations [5]. This is one of the simple and easy method implementation of non linear differential equations. Here we consider the parameter p as a small parameter.

The solution of the above system by using Homotopy perturbation method is

$$(1-p)\left(\frac{dx}{dt} - s + dx - ax\right) + p \left(\frac{dx}{dt} - s + dx - ax + \beta xz - \rho y\right) = 0 \tag{4}$$

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$$(1-p)\left(\frac{dy}{dt}+\delta y+\rho y\right)+ p\left(\frac{dy}{dt}-\beta xz+\delta y+\rho y\right)=0 \tag{5}$$

$$(1-p)\left(\frac{dz}{dt}+cz\right)+p\left(\frac{dz}{dt}-qy+cz\right)=0 \tag{6}$$

Comparing the coefficient of p, we get

$$x_0=(30-s/d)e^{-dt}+\frac{s}{d} \tag{7}$$

$$y_0=400e^{-(\delta+\rho)t} \tag{8}$$

$$z_0=600e^{-ct} \tag{9}$$

$$x_1=32e^{-At}+\frac{2A}{s}\left(1-e^{-At}\right)+ \frac{A(30-s/d)}{A-d}\left(e^{-At}-e^{-dt}\right)+\frac{s}{d}\left(e^{-At}-1\right) \tag{10}$$

$$+\frac{600\beta(30-s/d)}{A-c-d}\left(e^{-At}-e^{-(c+d)t}\right)+ \frac{600\beta(s/d)}{A-c}\left(e^{-At}-e^{-ct}\right)+\frac{400\rho}{A-\mu}\left(e^{-\mu t}-e^{-At}\right)$$

$$y_1=20e^{-Bt}+\frac{600\beta(30-s/d)}{B-c-d}\left(e^{-(c+d)t}-e^{-Bt}\right) +\frac{600\beta(s/d)}{B-c}\left(e^{-ct}-e^{-Bt}\right)$$

$$z_1=3200e^{-ct}+\frac{400q}{c-\mu}\left(e^{-\mu t}+e^{-ct}\right) \tag{11}$$

Where

$A=d-a$;
$B=\delta+\rho$;
$\mu=\sigma+\rho$.

The Homotopy perturbation method is gives the approximate analytical solution of the above system we consider as is

$$x(t)=(30-s/d)e^{-dt}+\frac{s}{d}+32e^{-At}+\frac{2A}{s} \left(1-e^{-At}\right)+\frac{A(30-s/d)}{A-d}\left(e^{-At}-e^{-dt}\right)$$

$$+\frac{s}{d}\left(e^{-At}-1\right)+\frac{600\beta(30-s/d)}{A-c-d}\left(e^{-At}-e^{-(c+d)t}\right) +\frac{600\beta(s/d)}{A-c}\left(e^{-At}-e^{-ct}\right)+\frac{400\rho}{A-\mu}\left(e^{-\mu t}-e^{-At}\right)$$

$$y(t)=400e^{-\mu t}+20e^{-Bt}+\frac{600\beta(30-s/d)}{B-c-d} \left(e^{-(c+d)t}-e^{-Bt}\right)+\frac{600\beta(s/d)}{B-c}\left(e^{-ct}-e^{-Bt}\right)$$

$$z(t)=600e^{-ct}+3200e^{-ct}+\frac{400q}{c-\mu}\left(e^{-\mu t}+e^{-ct}\right)$$

Numerical simulation

Here we can study the effect of general nonlinear ODE system where the rate of change of first compartment with respect to time, the rate of change of second compartment with respect to time and the rate of third compartment with respect to time is discussed. In Figures 1, 2 and 3 When $\beta=0.0002, 0.0006, 0.0010, \delta=1$ and $\rho=0.02, 0.03, 0.04$ we discussed how the 'x' is increased in the non linear system. And when $\beta=0.0002, 0.0006, 0.0010, \delta=1$ and $\rho=0.02, 0.06, 0.10$ we discussed the rate of change of 'y' how is increasing from time by time in Figures 4, 5 and 6. Finally we discussed the rate of change of 'z' after some time the remaining 'z' how it will become actively 'y' when $\delta=2,3,4$ in Figures 7 and 8.

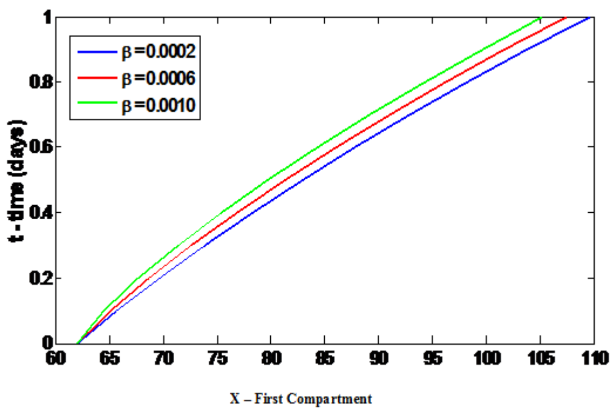

Figure-1. Parameter estimation of first compartment for β = 0.0002, 0.0006, 0.0010 & δ = 1.

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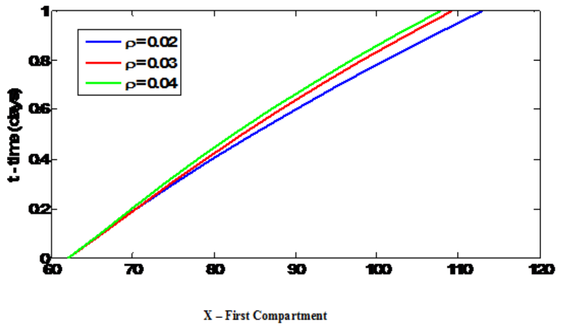

Figure-2. Parameter estimation of first compartment for $\rho = 0.02, 0.03, 0.04$.

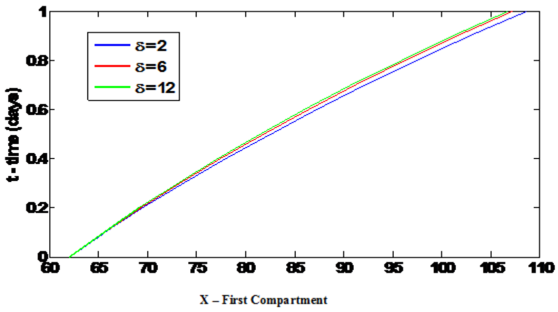

Figure-3. Parameter estimation of first compartment for $\beta = 0.002$ & $\delta = 2,4,12$.

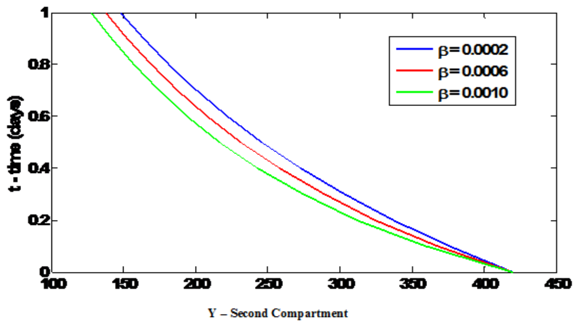

Figure-4. Parameter estimation of second compartment for $\beta = 0.0002, 0.0006, 0.0010$ & $\delta = 1$.

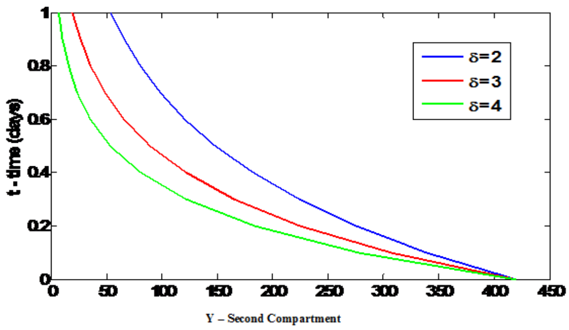

Figure-5. Parameter estimation of second compartment for $\beta = 0.002$ & $\delta = 2,3,4$.

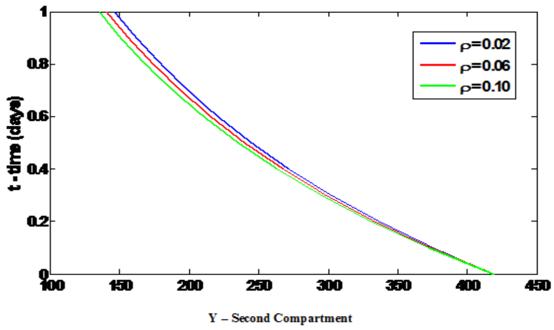

Figure-6. Parameter estimation of second compartment for $\rho = 0.02, 0.06, 0.10$.

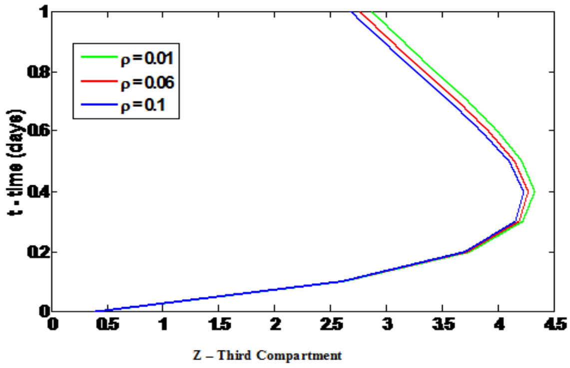

Figure-7. Parameter estimation of third compartment for $\rho = 0.01, 0.06, 0.10$.

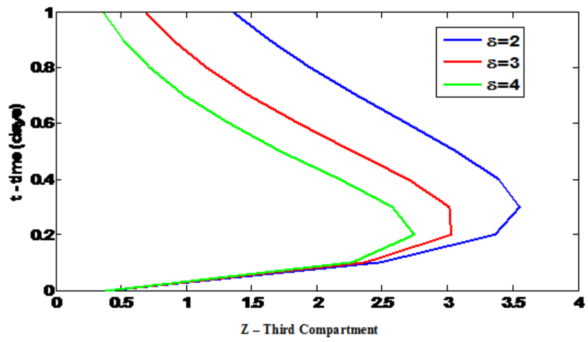

Figure-8. Parameter estimation of third compartment for $\delta = 2,3,4$.

CONCLUSIONS

Here we have studied the first, second and third compartment model and the analytical solution of the non linear differential equation of the system by using Homotopy Perturbation method. The rate of change of x, y and z like with respect to time is investigated in numerical simulation. Homotopy perturbation method is simple and easy to solve non linear differential equations. Finally we found the approximately analytical solution of three nonlinear ODE systems.

Appendix A - Homotopy Perturbation Method (HPM)

HPM method has used to find the limitations of traditional perturbation methods. It can take full advantage of the traditional perturbation techniques to solve various non - linear equations. To make clear idea about this method, we will consider the following function.

$$K_0(q) - f(s) = 0, s \in \Phi \quad \text{(A1)}$$

Using the boundary conditions of the HPM

$$A_0\left(q, \frac{\partial q}{\partial n}\right) = 0, n \in \Omega \quad \text{(A2)}$$

Where, K_0 is a differential operator, A_0 is a boundary operator and $f(s)$ is known analytical function and Ω is the boundary of the domain Φ .Now the Equation (A1) can rewrite as

$$M(q) + R(q) - f(s) = 0 \quad \text{(A3)}$$

By using the techniques of HPM

$$\begin{aligned} Z(c,p) &= (1-p)[M(c) - M(q_0)] \\ &+ p[K_0(c) - f(s)] = 0 \end{aligned} \quad \text{(A4)}$$

$$\begin{aligned} Z(c,p) &= M(c) - M(q_0) + pM(q_0) \\ &+ p[R(c) - f(s)] = 0 \end{aligned} \quad \text{(A5)}$$

Here q_0 is an initial value of first equation and $p \in [0,1]$ is an embedding parameter. Using the boundary conditions in the above equation, we get

$$Z(c,0) = M(c) - M(q_0) = 0 \quad \text{(A6)}$$

$$Z(c,1) = K_0(c) - f(s) = 0 \quad \text{(A7)}$$

The equation (A5) becomes linear when $p = 0$ and becomes non linear $p = 1$. The process will change to zero if $M(c) - M(q_0) = 0$ to $K_0(c) - f(s) = 0$.

The embedding parameter p is very small and the solution can be written as

$$c = c_0 + pc_1 + p^2 c_2 + ... \quad \text{(A8)}$$

The approximate solution of equation first is when $p = 1$

$$q = \lim_{p \to 1} c = c_0 + c_1 + c_2 + ... \quad \text{(A9)}$$

Conflict of Interest
There is no conflict of Interest.

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