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CONSTRUCTING AN INFORMATION MATRIX FOR MULTIVARIATE DCC-MGARCH (1, 1) METHOD

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ABSTRACT

The analytic form of Fisher Information Matrix (IM) for DCC-MGARCH (1, 1) was suggested. After that, it was applied for simplifying the general algorithm: the statistical hypothesis about constant correlation matrix usage was put forward and statistical verification was made. IM was employed for Russian share market: to do investigations the five equilibrium portfolios was compounded from four different shares in each case. Computations made showed that there are three types $T1$–$T3$ of trading days on the market and day type changing from $T1$ to $T2$ and vice versa is happening over the time moments $T3$. Moreover, the clustarisation effect of multivariate volatility that was investigated by scientists from all around the world in the univariate case was discovered and described.

Keywords: fisher matrix, multivariate conditional dynamic correlation DCC-MGARCH method.

INTRODUCTION

The analytic form of Fischer`s information matrix for econometric algorithm DCC-MGARCH (1, 1) was found. Also here a statistical hypothesis about constant correlation matrix was put forward and its statistical verification was made. Information matrix is using for econometric research of Russian stock market. Clustering effect of multivariate volatility which was confirmed by other scientists was found.

Nowadays mathematical description and statistical data processing of information received as a result of functioning of the stochastic systems, is based on well-studied one-dimensional laws of probability. Because of substantial growth of the quantity of such systems and complicating their internal structure, one-dimensional distributions can`t be applied properly to find a solution with given accuracy in a short time. As the result, there is a huge interest in description of whole system behavior without the reduction into the one-dimensional subtasks. It facilitates understanding of internal structure processes inside the systems and it let us to avoid error that occurs in factorization process. Thanks to multivariate methods, scientists hope to build more accurate forecast of probability`s behavior and evolution of complicated systems.

There are a lot of books which can help to find information about constructing multivariate econometric methods and to research a structure and properties of multivariate data. Let us to distinguish some of them. The first are works [1-5] which are a base of all researches and works [6, 7] which tell us about 20 years experience of using methods. Because of increasing quantity of multivariate algorithms, we should check out the quality of estimated coefficients and prove their unbiasedness, effectiveness and consistency. In addition, it is required to construct the confidence intervals for these estimates or even for functions of them. This becomes particularly relevant in light of examining and considering in researchers' investigations of more complex econometric parameters like risk measures VAR, CVAR, ES, market price of risk, asymmetry coefficients (to detect leverage effect), kurtosis and others. DCC-MGARCH(1,1) is also very useful for data processing in computer based systems of student learning, training and testing, especially when fuzzy-logic elements in mixed diagnostic tests [19, 20] are used.

Despite the fact that the theory of information matrices has been developed more than eighty years ago starting with the fundamental works of Fischer in 1922, its application to the study of properties of multidimensional economic algorithms significantly limited. The cycle of modern works on the subject opens work [8]: to simulate the behaviour of financial markets as US, Japan, Germany, Britain, France, Italy, was calculated two-dimensional information matrix coefficients method GARCH(1,1) using a constant correlation matrix and built the so-called test information matrix (IM-test) to check the stationarity hypothesis about correlation matrices in the financial market. Then in [9], to describe the likely future values of eight shares held US highly compute Fisher matrix. It was assumed that the initial model satisfies to multivariate methods MGARCH (1, 1) with a time-varying diagonal covariance matrix. Finally, in [10] DCC-MVGARCH (1, 1) with elliptical probability distribution function of standardized residuals and a time-dependent correlation matrix the expectations of informants were calculated.

In this paper we construct Fisher information matrix for one of the most common multidimensional economic methods in our days - the algorithm DCC-MGARCH (1, 1) [6, 7, 11]. It is needed for finding such matrices, primarily, as a necessary part of computing estimates of unknown parameters for the multivariate family-GARCH algorithms in a real time, for example, using the process of scoring [17] and not by a classical method of maximum likelihood. Secondly, this matrix can be used to the proof of efficiency of the found estimations as the following marginal ratio is held:

$$\lim_{n\to\infty} \sqrt{n}\left(\hat{\Theta} - \Theta\right) = \bar{Y} \sim N\left(0, J_{\Theta}^{-1}(\Theta)\right)$$

where $J_\Theta(\Theta)$ - an information matrix; $\hat{\Theta}$ - assessment for a vector of parameters Θ; n – a number of data. At last, its usage allows us to reduce a number of the coefficients estimated by DCC-MGARCH (1, 1): the assumption about correlation matrices which are poorly changing in time allows us to simplify DCC-MGARCH to CCC- MGARCH.

Further, in our work the hypothesis (equation. (6)) moved out and critical statistics was built (equation. (10)). In conclusion, information matrix is used for making the research of the Russian stock market, for which five different portfolios consist of four assets are created, everything with the fixed and equal shares in the portfolio. Shares are chosen randomly, but on condition that the trades with them took place on MICEX (www.micex.ru) during all fixed period. The first portfolio is constituted from shares of the company Lukoil, Surgutneftgaz, Rostelecom, RAO UES, with close prices from January 2, 2000 till October 27, 2006 (only 1701 values) are taken. The second portfolio is built from shares of the companies Norilsk Nickel MMC, Aeroflot, the AvtoVAZ, Mosenergo, with close prices from October 31, 2001 till March 23, 2007 (only 1361 values) are taken. The third portfolio consists of shares of the companies Baltic, Rosneft, Rosbank, Polyus Gold, with close prices from August 23, 2006 till March 24, 2007 are taken (all the 144th value). The fourth portfolio is built from shares priced from February, 2003 till March, 2007 (only 1025 values). At last, the fifth portfolio is constituted from shares of the companies RITEK, MTS, Sibirtelekom, Tatneft, with close prices from February, 2004 till March, 2007 (only 730 values) are taken. All the data were provided by the companies RBC (http://export.rbc.ru) and FINAM (http://www.finam.ru).

THE MODEL

Let us construct Fisher information matrix for DCC-MGARCH (1, 1) with standardized residuals, satisfying to a multivariate normal distribution. Let $\{u_t\}$ is a multivariate series of log returns, $u_t = (u_{1t}, u_{2t}, \ldots, u_{Kt})^T$, calculated for some set of asset prices $y_t = (y_{1t}, y_{2t}, \ldots, y_{Kt})^T$:

$$u_{it} = \ln(y_{it}) - \ln(y_{i,t-1}),\ t = 1,\ldots,T,\ i = 1,\ldots,K,$$

where K is a total number of assets in a portfolio.

Assuming that multivariate time series $\{u_t\}$ has conditional heteroscedasticity, $t = 1,\ldots,T$, we suppose that its conditional expected value equal to zero:

$$E(u_{it} | F_{t-1}) = 0,\ i = 1,\ldots,K,$$

and conditional variances in fixed moment of time t determined as

$$D(u_t | F_{t-1}) = H_t,$$

where $H_t = (-h_{ijt}-)$ is symmetric, positive definite, covariance matrix KxK, consisting of variances $h_{iit} = \sigma_{it}^2$, $i = 1,\ldots,K$ and covariance's $h_{ijt} = \sigma_{ijt}$, $1 \le i < j \le K$; $\bar{F} = (F_n)_{n \ge 0}$ is a filtration, determined by σ-subalgebras F_n such that $\Theta = (\theta_1,\ldots,\theta_N)$, if $m \le n$. Also, let's assume, that u_t have conditionally Gaussian multivariate distribution law, i.e.

$$u_t = H_t^{1/2} \bar{\varepsilon}_t,$$

where $H_t^{1/2}$ is a Cholesky decomposition for H_t, column vector $\bar{\varepsilon}_t \sim N(0, I_K)$ and I_K is an identity KxK-matrix.

Let variances σ_{it}^2, $i = 1,\ldots,K$ satisfy to autoregressive dependence like one-dimensional GARCH (1, 1) process [12] for every fixed index i:

$$\sigma_{it}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i u_{i,t-1}^2, \tag{1}$$

where $\sigma_{i,0}^2 = \text{const}$, $u_{i,0} = \text{const}$, $\omega_i \ge 0, \alpha_i \ge 0, \beta_i \ge 0$ are some parameters and $\alpha_i + \beta_i < 1$, $t = 1,2,\ldots,T$.

After finding volatilities σ_{it}^2, the off-diagonal elements σ_{ijt} of covariance matrix H_t can be determined from expressions as follows:

$$\sigma_{ijt} = \rho_{ijt} \sigma_{it} \sigma_{jt},\ 1 \le i < j \le K, \tag{2}$$

where ρ_{ijt} are the coefficients of positive definite correlation matrix Γ_i, participating in the expansion of H_t=$D_t\Gamma_tD_t$ and D_t is a diagonal matrix with elements σ_{it}.

Let's combine unknown parameters expressed in equations (1) and (2) into one vector with size $N = 3K + TK(K-1)/2$ to be determined:

$$\Theta = (\omega_1, \alpha_1, \beta_1, \ldots, \omega_K, \alpha_K, \beta_K, \rho_{12t}, \rho_{13t}, \ldots, \rho_{1KT}, \rho_{23T}, \ldots, \rho_{K-1,KT}).$$

By assumption that log returns u_t satisfy to normal distribution law, the evaluation of θ_i, $i = \overline{1,N}$ is conducted by maximum likelihood method with conditional probability functions of Gaussian distribution

$$f_t = (2\pi)^{-K/2} \det{}^{-1/2}(H_t) \exp\left(-\frac{1}{2} u_t^T H_t^{-1} u_t\right),$$

which were calculated in T vectors of observations u_1, u_2,...,u_t, . Log likelihood function is as follows

$$l \equiv l(\Theta) = \sum_{t=1}^{T} l_t = \sum_{t=1}^{T} \ln f_t \,, \tag{3}$$

where $l_t = -\frac{K}{2}\ln 2\pi - \frac{1}{2}\ln(\det H_t) - \frac{1}{2}u_t^T H_t^{-1} u_t$.

Discarding constant term $\left(-\frac{K}{2}\ln 2\pi\right)$ and taking into account, that $H_t = D_t \Gamma_t D_t$, equation (3) takes the final form:

$$l_t = -\frac{1}{2}\ln|D_t\Gamma_t D_t| - \frac{1}{2}u_t^T D_t^{-1}\Gamma_t^{-1}D_t^{-1}u_t =$$

$$= -\frac{1}{2}\ln|\Gamma_t| - \frac{1}{2}\sum_{j=1}^{K}\ln\sigma_{jt}^2 - \frac{1}{2}u_t^T D_t^{-1}\Gamma_t^{-1}D_t^{-1}u_t \,,\; t = 1,\ldots,T. \tag{4}$$

In general case, when u_t satisfy to any probability law, to obtain stable estimates of the vector of parameters $\hat{\Theta}$ it is good to use maximum likelihood method with function l_t too, due to execution of asymptotic ratio [13, 14]:

$$\lim_{n\to\infty}\sqrt{n}\left(\hat{\Theta}_n - \Theta\right) = \bar{Y} \sim N\left(0, J_\Theta^{-1}(\Theta)\right), \tag{5}$$

where $J_{ij}(\Theta) = E\left(\frac{\partial l(\Theta)}{\partial\theta_i}\cdot\frac{\partial l(\Theta)}{\partial\theta_j}\right)$, $i, j = \overline{1, N}$.

Note, that even if we use small quantity of multivariate time series values u_t, $1 \le t \le T$, a number of estimated coefficients θ_i, $i = \overline{1, N}$, in DCC-GARCH(1,1) will be enormous. Moreover, assessments of the correlation and covariance matrices found by maximum likelihood method are not necessary positive define [1, 3]. So, we should modify DCC-MGARCH(1,1) . Suppose that for time-varying elements of the correlation matrix ρ_{ijt} the next ratio is fair:

$$\rho_{ijt} = \rho_{ij} + \delta_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} \,,\; 1 \le i < j \le K \,,\; 1 \le t \le T \,, \tag{6}$$

where $\varepsilon_{i,t} = u_{i,t}\sigma_{it}^{-1}$ are standardized residuals (noises), $\varepsilon_t = D_t^{-1}H_t^{1/2}\bar{\varepsilon}_t$ and ρ_{ij} are some elements of fixed correlation matrix $\Gamma = \Gamma_s$ computed at chosen by researcher time t=s.

According to equation. (6), matrices Γ_t, $1 \le t \le T$, changes slowly over the time t and can be replaced by the sum of constant correlation matrix Γ and some white noise. Consequently, dimension of vector Θ reduces significantly to $N = K^2 + 2K$. And it takes form like

$$\Theta = \left(\omega_1, \alpha_1, \beta_1, \ldots, \omega_K, \alpha_K, \beta_K, \rho_{12}, \rho_{13}, \ldots, \rho_{K-1,K}, \delta_{12}, \delta_{13}, \ldots, \delta_{K-1,K}\right). \tag{7}$$

Let get some conditions under which disturbed covariance matrix $\bar{H}_t = D_t\bar{\Gamma}_t D_t$ with constant matrix $\bar{\Gamma}_t$ whose elements are defined by equation. (6), will be positive define. Let formulate and prove the theorem 1.

Theorem 1: Let $\Delta = (-\delta_{ij}-)$, $\delta_{ii} = 0$, $i, j = 1,..,K$, is a semi-definite matrix with equal to zero coefficients on the main diagonal, $\varepsilon = diag(\varepsilon_{1,t-1},\ldots,\varepsilon_{K,t-1})$ is a diagonal disturbed matrix with the elements of one sign (minus or plus). Then $\bar{H}_t$ is positive definite.

Proof:

Obvious, that

$$\bar{H}_t = D_t\bar{\Gamma}_t D_t = D_t(\Gamma + \varepsilon\Delta\varepsilon)D_t = D_t\Gamma D_t +$$
$$= D_t\Gamma D_t + D_t\varepsilon\Delta\varepsilon D_t = H_t + A_t \,,$$

where H_t is non-disturbed positive definite covariance matrix, $A_t = D_t\varepsilon\Delta\varepsilon D_t = (\varepsilon D_t)^T\Delta(D_t\varepsilon)^T = \varepsilon D_t\Delta D_t\varepsilon$ is a noise. Let us look closer at the matrix A_t.

Let matrix $\varepsilon = diag(\varepsilon_{1,t-1},\ldots,\varepsilon_{K,t-1})$ is composed from positive standardized residuals. Then εD_t is positive definite, that is A_t, as the product of positive definite and semi-definite matrices, will be positive semi-definite.

Let matrix $\varepsilon = diag(\varepsilon_{1,t-1},\ldots,\varepsilon_{K,t-1})$ is composed from negatively standardized residuals. Then $(-\varepsilon D_t)$ is positive definite. So, because

$$A_t = -(-\varepsilon D_t)\Delta(-(-\varepsilon D_t)) = (-\varepsilon D_t)\Delta(-\varepsilon D_t),$$

then A_t is positive definite as it was proved above. So, under conditions of theorem 1, A_t is always positive definite. Let us prove, that $\bar{H}_t = H_t + A_t$ is positive definite. Actually, according to the definition, for any vector $V \in R^K, V \neq 0$ we have:

$$V^T\bar{H}_t V = V^T(H_t + A_t)V = V^T H_t V + V^T A_t V.$$

Since $V^T H_t V > 0$, $V^T A_t V \ge 0$, then $V^T\bar{H}_t V > 0$, which completes the proof of the theorem 1.

Remark: If matrix ε is composed from the residuals of different signs, then A_t will be uncertain matrix. And proof of positive definiteness $\bar{H}_t$ will be failed. Note more, that assuming $\delta_{ij} = 0$, $1 \le i < j \le K$, in equation. (6), the algorithm DCC–MGARCH(1,1) converts to known method CCC–MGARCH(1,1) [2].

The indisputable advantage of the latest algorithm is the use of a single, constant in time, correlation matrix Γ for modeling the values of H_t:

$$H_t = D_t\,\Gamma\,D_t \,,$$

which not only leads to a reduction in the number of estimated parameters in the vector Θ, but also greatly

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ease the calculation procedure. Indeed, in the case of conditional normality of random variables $u_t \sim N(0, H_t | F_{t-1})$ maximum likelihood estimate $\hat{\Gamma}$ for Γ is always calculated as the sample mean of standardized residuals:

$$\hat{\Gamma} = T^{-1} \sum_{t=1}^{T} D_t^{-1} u_t u_t^T D_t^{-1} \qquad (8)$$

and $\hat{\Gamma}$ is almost surely positive definite.

Further, (8) allows us to simplify the equations. (3)-(4) and write equation. (3) in the form (assuming that the permanent members discarded) as follows:

$$l = -\sum_{t=1}^{T} \ln|D_t| - \frac{T}{2} \ln\left(\sum_{t=1}^{T} D_t^{-1} u_t u_t^T D_t^{-1} \right).$$

Therefore, the number of coefficients to be estimated by the method of maximum likelihood is reduced to $N = 3K$:

$$\Theta = (\omega_1, \alpha_1, \beta_1, \ldots, \omega_K, \alpha_K, \beta_K).$$

To check the validity of equalities $\delta_{ij} = 0$, $1 \le i < j \le K$, put forward the statistical hypothesis $\overline{H}_0 : \delta_{ij} = 0$, $1 \le i < j \le K$, about the constancy of correlation matrices in the expansion of $H_t = D_t \Gamma_t D_t$ having $Q = (K^2 - K)/2$ independent constraints. Suppose there is an alternative hypothesis $\overline{H}_1 : \delta_{ij} \ne 0$, $1 \le i < j \le K$.

To construct a critical statistics γ, we use an analogue of the asymptotic relation (5):

$$\lim_{n \to \infty} \sqrt{n}\left(l(\Theta) - l(\hat{\Theta})\right) = \overline{Y}, \qquad (9)$$

where $\nabla_\Theta l(\Theta) = \left(\frac{\partial l(\Theta)}{\partial \theta_1}, \frac{\partial l(\Theta)}{\partial \theta_2}, \ldots, \frac{\partial l(\Theta)}{\partial \theta_N} \right)$, $\hat{\Theta}$ - estimation for the vector in (7), found by the maximum likelihood method, and $\overline{Y} \sim N\left(0, \nabla_\Theta l(\Theta) J_\Theta^{-1}(\Theta) \nabla_\Theta^T l(\Theta)\right)$. Relation (9) is hold as a result of the validity of Slutsky's lemma about the limit transition under the sign of a continuous function $l(\Theta)$. Further, since $\nabla_\Theta l(\Theta) \cdot J_\Theta(\Theta) \cdot \nabla_\Theta^T l(\Theta)$ is the sum of the squares of normally distributed random variables, it has χ^2-distribution with Q degrees of freedom. Therefore, γ can be selected as follows [18]:

$$\gamma = \hat{S} \cdot J_\Theta^{-1}(\hat{\Theta}) \cdot \hat{S}^T, \qquad (10)$$

where $\hat{S} = \nabla_\Theta l(\hat{\Theta})$.

The hypothesis $\overline{H}_0$ is accepted with a confidence level $(1-\alpha)$, if $\gamma < \chi^2_{\alpha/2}(Q)$ and rejected otherwise.

Note that the information matrix $J_\Theta(\Theta)$ is useful not only for the calculation of the critical statistics (10) and test the null hypothesis about the constancy of correlation matrices in the method of DCC-MGARCH (1, 1). It can also be used in the construction of confidence interval estimates of risk marginal values of VAR_α (more about calculating the VAR_α can be found, for example, in [7, 15]), as the following inequalities (marked by "$\le$" refers to a total ordering on set of real vectors from R^K):

$$\hat{VAR}_\alpha - Z_{r/2} n^{-1/2} I_{VAR}^{1/2}(\hat{\Theta}) \le VAR_\alpha \le \hat{VAR}_\alpha + Z_{r/2} n^{-1/2} I_{VAR}^{1/2}(\hat{\Theta}) \qquad (11)$$

where $I_{VAR}(\Theta) = grad_\Theta(g_\alpha(\Theta)) J_\Theta^{-1}(\Theta) grad_\Theta^t(g_\alpha(\Theta))$, α - significance level, $g_\alpha(\Theta)$–quantile function for the probability law $F(x, \Theta)$ that defines the multidimensional distribution of random variable ξ with realizations $\{u_t\}$, $1 \le t \le T$, $Z_{r/2}$ is a vector-quantile of the standard multivariate normal distribution with probability $r/2$ and $(1-r)$ is the probability with which the confidence parallelepiped covers the theoretical value of VAR_α.

Other possible usages of Fisher-information matrix, as well as its basic properties are considered in detail in the monograph [16].

Let us construct an information matrix $J_\Theta(\Theta)$, for which we should find a vector-gradient $\nabla_\Theta l(\Theta)$ and calculate the corresponding expected values.

Differentiating variances σ_{it}^2, determined in accordance with (1), on $\omega_i, \alpha_i, \beta_i$, it is easy to prove that the following recurrent relations take place:

$$\frac{\partial \sigma_{it}^2}{\partial \omega_i} = 1 + \alpha_i \frac{\partial \sigma_{i,t-1}^2}{\partial \omega_i};$$

$$\frac{\partial \sigma_{it}^2}{\partial \alpha_i} = \sigma_{i,t-1}^2 + \alpha_i \frac{\partial \sigma_{i,t-1}^2}{\partial \alpha_i};$$

$$\frac{\partial \sigma_{it}^2}{\partial \beta_i} = u_{i,t-1}^2 + \alpha_i \frac{\partial \sigma_{i,t-1}^2}{\partial \beta_i},$$

(11)

$$\frac{\partial \sigma_{i,0}^2}{\partial \omega_i} = 0; \frac{\partial \sigma_{i,0}^2}{\partial \alpha_i} = 0; \frac{\partial \sigma_{i,0}^2}{\partial \beta_i} = 0, t = 1, \ldots, T, i = 1, \ldots, K.$$

Since $\frac{\partial \Gamma_t^{-1}}{\partial \theta_s} = -\Gamma_t^{-1} \frac{\partial \Gamma_t}{\partial \theta_s} \Gamma_t^{-1}$, the coordinates of the gradient vector $\nabla_\Theta l(\Theta)$, are depended on the function l_t from (4) and are expressed as follows [18]:

$$\frac{\partial l(\Theta)}{\partial \omega_i} = \sum_{t=1}^{T} \frac{(d_{it}\varepsilon_{it} - 1)}{2\sigma_{it}^2} \cdot \frac{\partial \sigma_{it}^2}{\partial \omega_i},$$

$$\frac{\partial l(\Theta)}{\partial \alpha_i}=\sum_{t=1}^{T}\frac{(d_{it}\varepsilon_{it}-1)}{2\sigma_{it}^2}\cdot\frac{\partial \sigma_{it}^2}{\partial \alpha_i},$$

$$\frac{\partial l(\Theta)}{\partial \beta_i}=\sum_{t=1}^{T}\frac{(d_{it}\varepsilon_{it}-1)}{2\sigma_{it}^2}\cdot\frac{\partial \sigma_{it}^2}{\partial \beta_i},$$

$$\frac{\partial l(\Theta)}{\partial \rho_{ij}}=\sum_{t=1}^{T}\left(d_{it}\,d_{jt}-\rho_t^{ij}\right),$$

$$\frac{\partial l(\Theta)}{\partial \delta_{ij}}=\sum_{t=1}^{T}\left(d_{it}\,d_{jt}-\rho_t^{ij}\right)\varepsilon_{i,t-1}\varepsilon_{j,t-1}\ ,\ 1\le i<j\le K\ , (12)$$

where $d_t=(d_{1t},d_{2t},\ldots,d_{Kt})^T=\Gamma_t^{-1}D_t^{-1}u_t$, d_{it} is an i-th component of the vector d_t, ρ_t^{ij} are the elements of the inverse matrix Γ_t^{-1} and $\varepsilon_t=D_t^{-1}u_t$ with $\varepsilon_0=0$.

Note that from the independence of variances σ_{1t}^2, $\sigma_{2t}^2,\ldots,\ \sigma_{Kt}^2$ in equation. (1) follows that the derivatives $\frac{\partial l}{\partial \omega_s}$ and $\frac{\partial l}{\partial \omega_k}$, $\frac{\partial l}{\partial \alpha_s}$ and $\frac{\partial l}{\partial \alpha_k}$, $\frac{\partial l}{\partial \beta_s}$ and $\frac{\partial l}{\partial \beta_k}$;...; $\frac{\partial l}{\partial \omega_s}$ and $\frac{\partial l}{\partial \beta_k}$, if $s\ne k$, are also independent in equation. (12), and their corresponding information matrix coefficients $J_{ij}(\Theta)=E(\partial l/\partial\theta_i)\cdot E(\partial l/\partial\theta_j)=0$ since the expectations of all informants are equal to zero. At the same time, $\partial l/\partial\rho_{ij}$ and $\partial l/\partial\delta_{ij}$ are dependent from each other and cannot be expressed by other derivatives on the another model parameters. So, it concludes that matrix $J_\Theta(\Theta)$ has a block form and consists of submatrices A_s, $s=\overline{1,(K+1)}$, located on the main diagonal. Submatrices A_s are defined as the mathematical expectations of the product of partial derivatives with respect to all possible combinations of the parameters from the sets $\{\omega_1,\alpha_1,\beta_1\},\ldots,\{\omega_K,\alpha_K,\beta_K\}$, $\{\rho_{12},\ldots,\rho_{K-1,K},\delta_{12},\ldots,\delta_{K-1,K}\}$.

So, submatrices A_s have the form as follows:

$$A_s=E\left(\partial l/\partial\theta_i^{(s)}\cdot\partial l/\partial\theta_j^{(s)}\right),$$
$$\theta_i^{(s)},\theta_j^{(s)}\in\{\omega_s,\alpha_s,\beta_s\},\ i,j=\overline{1,3}\,,\ s=\overline{1,K}\,;$$
$$A_{K+1}=E\left(\partial l/\partial\theta_i^{(K+1)}\cdot\partial l/\partial\theta_j^{(K+1)}\right),$$
$$\theta_i^{(K+1)},\theta_j^{(K+1)}\in\{\rho_{12},\ldots,\rho_{K-1,K},\delta_{12},\ldots,\delta_{K-1,K}\},$$
$$i,j=\overline{1,K(K-1)}.$$

Matrices $A_1,\ldots,A_K$ were calculated in [9]:

$$a_{ij}^{(s)}=\sum_{t=1}^{T}\sum_{q=1}^{K}\frac{1}{2\sigma_{qt}^4}\frac{\partial\sigma_{qt}^2}{\partial\theta_i^{(s)}}\frac{\partial\sigma_{qt}^2}{\partial\theta_j^{(s)}},$$

where $a_{ij}^{(s)}$ are the elements of A_s, $i,j=\overline{1,3}$, $s=\overline{1,K}$.

Definition of the functional form of the relationships for the elements of submatrix A_{K+1} is made for the first time. Here we present only the final result, for calculations we use equation. (12). The auxiliary matrices $C_t=(-c_{ij}-)=\Gamma_t^{-1}D_t^{-1}$, $B_t=(-b_{ij}-)=H_t^{1/2}$ were introduced; under the sign of K-dimensional integral, which defines mean, variable $u=B_t y$ was substituted, where y is a new uncorrelated random variable, and possible fourth-order moments of the multivariate normal distribution were calculated. For example, the following expressions were derived:

$$\int_{R^K}u_i^4 f(u)du=3\sum_{s_1=1}^{K}b_{i,s_1}^4+6\sum_{s_1\ne s_2}b_{i,s_1}b_{i,s_2}\ ,\ i=\overline{1,K}\,;$$

$$\int_{R^K}u_i^2u_ju_q f(u)du=\sum_{s_1=1}^{K}\sum_{s_2=1}^{K}b_{i,s_1}^2b_{j,s_2}b_{q,s_2}\ ,\ i,j=\overline{1,K}\,,$$

where $f(u)=(2\pi)^{-K/2}\det^{-1/2}(H_t)\exp\left(-\frac{1}{2}u^TH_t^{-1}u\right)$ and $u=(u_1,u_2,\ldots,u_K)^T$.

Note also that $A_{K+1}=\begin{pmatrix}M_1 & M_2\\ M_2^T & M_3\end{pmatrix}$ is a block matrix with sub-matrices, which dimensions are $K(K-1)/2\times K(K-1)/2$. Block $M_1=\left(-m_{\lambda,\eta}^{(1)}-\right)$ is built from elements that are expectations $E(\partial l/\partial\theta_\lambda\cdot\partial l/\partial\theta_\eta)$, where $\theta_\lambda,\theta_\eta\in\{\rho_{12},\ldots,\rho_{K-1,K}\}$. Next submatrix $M_2=\left(-m_{\lambda,\eta}^{(2)}-\right)$ consists of the elements $E(\partial l/\partial\theta_\lambda\cdot\partial l/\partial\theta_\eta)$, where $\theta_\lambda\in\{\rho_{12},\ldots,\rho_{K-1,K}\}$, $\theta_\eta\in\{\delta_{12},\ldots,\delta_{K-1,K}\}$. At last, $M_3=\left(-m_{\lambda,\eta}^{(3)}-\right)$ have elements $E(\partial l/\partial\theta_\lambda\cdot\partial l/\partial\theta_\eta)$ with parameters $\theta_\lambda,\theta_\eta\in\{\delta_{12},\ldots,\delta_{K-1,K}\}$.

So, finally we have:

$$m_{\lambda,\eta}^{(1)}=\sum_{t=1}^{T}\sum_{p=1}^{4}F_p-\rho^{m,s}F_6-\rho^{i,j}F_7+\rho^{i,j}\rho^{m,s},$$

$$m_{\lambda,\eta}^{(2)}=m_{\lambda,\eta}^{(1)}\,\varepsilon_{m,t-1}\varepsilon_{s,t-1}\,,\ m_{\lambda,\eta}^{(3)}=m_{\lambda,\eta}^{(1)}\,\varepsilon_{i,t-1}\varepsilon_{j,t-1}\,\varepsilon_{m,t-1}\varepsilon_{s,t-1}\,,$$

$$F_1=\sum_{s_1\ne s_2}c_{is_1}c_{js_1}c_{ms_2}c_{ss_2}\left(3\sum_{s_3}b_{s_1s_3}^2b_{s_2s_3}^2+3\sum_{s_3\ne s_4}b_{s_1s_3}b_{s_2s_3}b_{s_1s_4}b_{s_2s_4}+\right.$$

$$\left.+\sum_{s_3\ne s_4}b_{s_1s_3}^2b_{s_2s_4}^2\right);\ F_2=3\sum_{s_1}c_{is_1}c_{js_1}c_{ms_1}c_{ss_1}\left(\sum_{s_2}b_{s_1s_2}^4+\right.$$

$$\left.+2\sum_{s_2\ne s_3}b_{s_1s_2}b_{s_1s_3}\right);F_5=\sum_{s_5\ne s_6}\left(b_{s_1s_5}b_{s_2s_6}+b_{s_1s_5}b_{s_3s_6}+\right.$$

$$\left.+b_{s_1s_5}b_{s_4s_6}+b_{s_2s_5}b_{s_3s_6}+b_{s_2s_5}b_{s_4s_6}+b_{s_3s_5}b_{s_4s_6}\right);$$

$$F_3=2\sum_{s_1}\sum_{s_2\ne s_3}c_{is_1}c_{js_1}c_{ms_2}c_{ss_3}\left[\sum_{s_4}\sum_{s_5}b_{s_1s_4}^2b_{s_2s_5}b_{s_3s_5}\right];$$

$$F_4=\sum_{s_1\ne s_2}\sum_{s_3\ne s_4}c_{is_1}c_{js_2}c_{ms_3}c_{ss_4}\left[3\sum_{s_5}b_{s_1s_5}b_{s_2s_5}b_{s_3s_5}b_{s_4s_5}+F_5\right];$$

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$$F_6 = \sum_{s_1}\sum_{s_2} c_{is_1} c_{js_1} b_{s_1 s_2}^2 + \sum_{s_1 \neq s_2}\sum_{s_3} c_{is_1} c_{js_2} b_{s_1 s_3} b_{s_2 s_3} ;$$

$$F_7 = \sum_{s_1}\sum_{s_2} c_{ms_1} c_{ss_1} b_{s_1 s_2}^2 + \sum_{s_1 \neq s_2}\sum_{s_3} c_{ms_1} c_{ss_2} b_{s_1 s_3} b_{s_2 s_3} ,$$

$$\lambda, \eta = \overline{1, K(K-1)/2} .$$

ECONOMETRIC ANALYSIS OF STOCK PRICES

We will use block information matrices $J_\Theta(\Theta)$, constructed above, for testing statistical hypothesis $\overline{H}_0$ about the constancy of the matrices of risky assets correlations on the Russian stock market. The hypothesis put forward in order to reduce the number of estimated parameters in the method DCC-MGARCH(1,1) and to simplify the structure of the calculated structure. Consider five different portfolios with four assets in each. The first portfolio (P1) forms from the common shares of Lukoil, Surgutneftegaz, Rostelecom, RAO UES, fixing close prices for the period from 02 January 2000 to 27 October 2006 (total 1701 share values). The second portfolio (P2) forms from the common shares of companies Norilsk Nickel, Aeroflot, AvtoVAZ, Mosenergo, take close prices from 31 October 2001 to 23 March 2007 (total amount is 1361 quotes). The third portfolio (P3) makes up from common shares of companies Baltika, Rosneft, Rosbank, Polyus Gold, taking close prices from 23 August 2006 to 24 March 2007 (total of 144 values). The fourth portfolio (P4) forms from the common shares of the companies RAO UES, Aeroflot, Sberbank, Transneft, take close prices from 01 February 2003 to 31 March 2007 (total amount of 1025 values). Finally, the fifth portfolio (P5) forms from the common shares of companies RITEK, MTS, Sibirtelecom, Tatneft, fix close prices for the period from 01 February 2004 to 31 March 2007 (total amount is 730 values).

Note that number of shares K=4 in (P1)–(P5) is chosen to ease the presentation of the results obtained in the computation.

Before we construct the information matrix $J_\Theta(\Theta)$ statistical hypothesis of the initial asset prices conditional heteroscedasticity has been checked for assets forming (P1)–(P5).

Table-1. Results of ARCH-test made for P1 shares.

Lag p	H_0	ARCH-test statistics	χ^2_α
Lukoil			
Lag p	**H_0**	**ARCH-test statistics**	χ^2_α
10	decline	214.47	19.67
15	decline	230.14	26.3
20	decline	244.15	32.7
Rostelecom			
Lag p	H_0	ARCH-test statistics	χ^2_α
10	decline	103.31	19.67
15	decline	107.44	26.3
20	decline	108.28	32.7
Surgutneftgaz			
Lag p	H_0	ARCH-test statistics	χ^2_α
10	decline	263.47	19.67
15	decline	266.26	26.3
20	decline	271.45	32.7
RAO UES			
Lag p	H_0	ARCH-test statistics	χ^2_α
10	decline	189.15	19.67
15	decline	253.19	26.3
20	decline	259.40	32.7

As it is known [17], that such research carry out by the ARCH-test (or Engle test) applied for residuals of time series with having put forward a null hypothesis H_0 about its conditional homoscedasticity. Since the one-dimensional process GARCH (p,q) is the local ARCH(p+q)-process, then a critical statistic obeys the chi-squared-distribution with (p+1) degrees of freedom.

Calculations made for portfolios (P1)–(P5) demonstrated the presence of heteroscedasticity (or the lack of homoscedasticity) in all the time series of portfolios. Due to the large amount of information received, we present results for (P1) only. We collect it in Table 1 with a significance level α=0.05. In such a way, the original assumption about the conditional heteroscedasticity of the multidimensional time series in all portfolios is confirmed, that allows us to find the variance σ^2_{it} in accordance with equality (1).

Since $\sigma^2_{1t},\ldots,\sigma^2_{Kt}$ are independent from each other, then for their finding the classic one-dimensional method GARCH (1, 1) [12] with lag k=20 was used.

Further, in accordance with equality (8) in every fixed moment of time t, t=1,…,T, estimations of maximum likelihood $\hat{\Gamma}_t$ for the conditional correlation matrices $\hat{\Gamma}_t = t^{-1}\sum_{s=1}^{t} D_s^{-1} u_s u_s^T D_s^{-1}$ were calculated. They will be symmetric and positive definite on its construction. Then $\hat{H}_t = D_t \hat{\Gamma}_t D_t$ will be positive definite covariance matrices and for them both the Cholesky decomposition $\hat{H}_t = B_t B_t^T$ and the inverse matrices $C_t = \hat{\Gamma}_t^{-1} D_t^{-1}$ are exist. Using B_t, C_t in blocks M_1–M_3, volatilities σ^2_{it} and recurrent relations (11), we get estimations of information matrices $J_\Theta(\hat{\Theta})$ in each time t, t=1,…,T. Further we find informant $\nabla_\Theta l(\hat{\Theta})$ by using (12) and compute inverse

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matrices $J_\Theta(\hat{\Theta})$, if they exist. At last we calculate test statistics (10) and compare it with known $\chi^2_{0.975}(6) = 14{,}45$ for accepting or declining general hypothesis $\overline{H}_0$.

The calculations of statistics made for stocks portfolios (P1)-(P5) revealed three main types of trading days on the Russian market. The first type (T1) includes days for which the hypothesis about constant matrix correlation is hold. And we can apply simplified econometric method CCC-MGACH(1,1). Next, the second type (T2) consists of the days for which the hypothesis $\overline{H}_0$ is rejected, and respectively, the more complex algorithm DCC-MGARCH(1,1) must be used. Finally, the third type (T3) is formed by days in which the determinant $\det\left(J_\Theta(\hat{\Theta})\right)$ is equal to zero, inverse matrix $J_\Theta^{-1}(\hat{\Theta})$ does not exist and statistics γ is not defined. Moments that enter into T3 are the most interesting for analysis because for them there is a linear dependence of the rows in the block matrix A_{K+1}. As a result, we find a functional connection between the various correlations of share quotes in some portfolios. Hence, some companies initially connected with each other more closely, so, the movement of their quotes come about the same events, i.e., thanks to the movement of the financial capital or covering the news etc. In addition, we can talk about how to use the insider information, as we observe coordinated buying or selling shares of portfolios (in other words, there are trading days that come about class T3 for all the portfolios (P1)–(P5) simultaneously).

The number of trading days of each type ~~founds~~ for the portfolios P1-P3 is shown in the Table-2.

Table-2. Distribution of trading days for P1-P5 portfolios.

	P1	P2	P3	P4	P5
*T*1	827	614	45	439	219
*T*2	813	611	67	500	454
*T*3	41	116	12	66	57

As follows from the results of the calculations presented in the Table-2, the hypothesis about the constancy of the covariance matrix in decomposition $H_t = D_t \Gamma_t D_t$ is rejected. Consequently, the usage of the CCC-MGARCH (1, 1) algorithm on entire time interval is not accepted. To obtain more accurate results we need to use a more sophisticated method DCC-MGARCH (1, 1). Meanwhile, the number of parameters to be estimated in the DCC-MGARCH (1, 1) can be significantly reduced if we take the assumption (6) about the nature of depending the elements of matrix Γ_t.

Having been calculating the critical statistics (10) for the portfolios (P1) - (P5) we discovered a curious effect: trade days T1 (T2), when the hypothesis of $\overline{H}_0$ is accepted (rejected), seek to follow each other, i.e. forming clusters. By analogy with the well-studied and described in the literature the effect of volatility clustering [7], which is valid for one-dimensional time series, the observed behaviour of the portfolios will be called generalized volatility clustering or clustering of covariance matrices.

Further, notice that a change of the type from T1 to T2 and from T1 to T2 goes through time points T3. Also T1 days are characterized by a moderate change in the daily volatility between 0% and 2% for each of the assets, whereas for T2 days its behavior is more stochastic: one-dimensional daily volatility exceeds 2%.

Proved no consistency of hypothesis about the use of an algorithm CCC-MGARCH(1,1) on the Russian stock market helped us to make a further econometric analysis of the behavior of portfolios (P1)–(P5) using DCC-MGARCH(1,1). Since this analysis is beyond the scope of this research work, let us cite just a few outcomes: built possible scenario of the future value of the portfolios (P1)–(P5) with the use of normal, α-stable [15] and STS-distribution [15,18], for the last of these distributions achieved the highest accuracy of calculations. For example, the volatility between the simulated and known values for all time series of portfolio (P1) didn't exceed 4.1%, for (P2) - 4.8%, for (P3) - 6.7% and for (P4) - 6.4%.

CONCLUSIONS

We found closed form of Fisher information matrix for DCC-MGARCH (1, 1) algorithm. It applies to Russian stock market investigation. Computations made allow us to discover three different time periods T1- T3. And changing class from T1 to T2 and from T1 to T2 goes through time points T3 only. At last we found 'clustering effect' of multivariate covariance matrices when covariance matrices with high (low) determinants tend to each other.

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