



# PREDICTING STRENGTH OF PILLARS IN FRACTURED ROCK MASS DURING DEVELOPMENT OF APATITE-NEPHELINIC ORES

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## ABSTRACT

A method of predicting the strength characteristics of fractured rock massif with the help of numerical modeling by the finite elements method in the Simulia Abaqus software suite has been proposed with respect to the conditions of the deposit of apatite-nephelinic ores in the Rasvumchorr Plateau of JSC "Apatit" in the Kirovsk Area of the Murmansk region. Geometric and geomechanical models of solids in the fractured rock mass have been built, where blockiness is taken into account in explicit form, and for modeling the conditions of contact interaction, nonlinear strength criterion of Barton is used. The sequences of creating a numerical geomechanical model of rock mass have been considered, and schemes of virtual testing have been specified. Studying the behavior of the model of pillars under load was aimed at defining the scale effect of tensile strength limit on uniaxial compression (bearing capacity) of the pillars in the fractured rock mass. The article also studied the anisotropy of strength and deformation characteristics of the rear sight and the effect of the ratio of sides on its carrying capacity. The proposed approach has been qualitatively compared to the existing empirical correlations.

**Keywords:** apatite-nephelinic ores, rock mass, pillar, deformation, fracture, mathematical modeling, geomechanical model.

## 1. INTRODUCTION

Study of the state of rock mass should include: establishing belonging to certain geological rock complex, detailed study of the mechanical and physical characteristics of certain homogeneous blocks of rocks, characteristics of the plots composed of rocks that are essentially different in their composition and properties, structural characteristics of the original rock mass [1]. The feature of pillars in fractured rock mass is their complex mechanical behavior due to the fact that the rock mass is a geological formation that is represented by a set of blocks of one or more types of rock mass separated by cracks of various nature [4]. Thus, the strength and deformation properties of a pillar are determined by the presence of defects and disruptions in it, and their characteristics [3].

Methods of calculating mechanical characteristics of the fractured rock mass may be divided into direct and indirect [10]. Direct methods include experiments in the laboratory and in the field [11], [12]. The test results obtained in the laboratory conditions for limited dimensions of samples do not reflect the real properties of the rock mass, the dimensions of which greatly exceed those of laboratory samples. On the other hand, samples of larger dimensions may be tested. It is worth noting that this process requires more labor, higher costs and is technically difficult to implement, [2] since the possibilities of choosing dimensions of the rock mass for testing are limited. Another important disadvantage of this approach is the invisibility of the initial cracks and uncertainty of the boundary conditions, which eliminates the possibility of determining the required dependencies. Indirect methods are divided into analytical [24], empirical [21]-[23], and numerical [17], [18], with each having its advantages and disadvantages.

The empirical approach allows only limited conservative estimation based on previous construction experience, moreover, it has no mathematical basis, but

today it is widely used, as allows to obtain quantitative indicators of the strength and deformation properties of rock mass based on qualitative characteristics. Although this method is used in the world practice, its use makes it impossible to present the anisotropy of properties in the tensor form [13]-[16].

Analytical methods of analysis reduce uncertainty in assessing the properties of the fractured rock mass, and allow displaying anisotropy of properties, compared to the empirical methods. However, the main disadvantage of the analytical methods is the impossibility of accounting for geometrically complex fracture systems, which are typical for real fractured rock mass.

In recent decades, the focus of researchers' attention has been gradually shifting to the numerical methods of determining mechanical parameters of the fractured mass, due to the active development of computer technologies and increasing their computing abilities [17]-[20]. The main goal of using numerical modeling methods with the use of mechanics of discrete environments is accounting in modeling for as many characteristics of fractured rock mass as possible for obtaining the most adequate results.

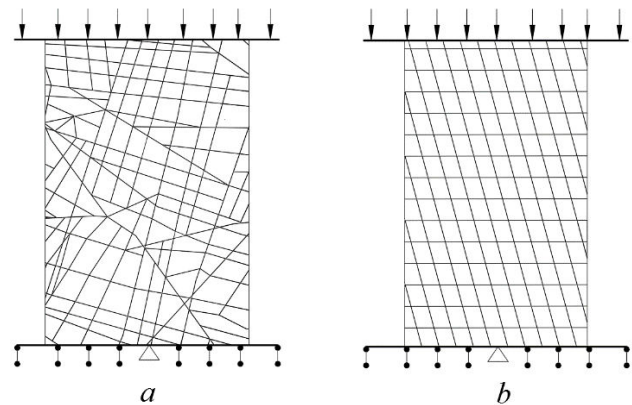
## 2. METHODS

To solve this problem, mathematical modeling was involved with the use of the finite element method performed in the Simulia Abaqus software suite. The Mohr-Coulomb elastic-plastic model was used for modeling the behavior of the rock mass. In solving the problem, it is supposed that fractured rock mass is destroyed due to the presence of cracks, along which the main shear deformation occurs. To describe the mechanical behavior along the interaction between blocks of rock mass, the nonlinear Barton strength criterion was used.



Complexity of the geological structure of fractured rock mass forces to consider not the rock mass itself, but its idealized reflection, which is a geomechanical numerical mathematical model. Building such a model requires schematization of the rock mass structure, its composition, physico-mechanical characteristics, interaction between blocks of rock mass. The numerical model of large-scale inhomogeneous mass was built in the following sequence: the structure of the fracture was determined and analyzed according to the engineering geological data, reflecting the characteristics of the rock mass geometric structure, and determining the shape of the blocks; the geometric shape of pillars was chosen for the model and the boundary conditions; based on the previous information, geomechanical model was built, which contained information about mechanical properties of the rock; block geometrical model of the pillars were built; the objective was calculated, and results were qualitatively assessed. It is believed that the blocks of rock mass that form the pillar do not reach their limit state when loaded, and the pillar is destroyed due to the shear along the formed weakening surfaces.

The Khibiny rock mass (the Rasvumchorr Plateau field) with the structure of blocks where pillars of various geometric structure are observed has been chosen for numerical experiments. For the purpose of studying the anisotropy of pillars mechanical characteristics in the fractured rock mass, models of two different 4 m wide and 8 m high fracture systems were built (Figure-1a, b). The fracture angle was rotated in 15-degree increments. To study the influence of pillar width on its carrying capacity at constant height (4 m), models were built with variation of the changed parameter from 3 to 7 meters in 1 m increments. To study the influence of pillar height on its carrying capacity at the constant width (4 m), models were built with the height varying from 3 to 10 meters in 1 m increments. To study the scale effect of mechanical characteristics of the studied object, models were built with the following dimensions: 0.5×1 m, 1×2 m, 1.5×3 m, 2×4 m, 2.5×5 m, 3×6 m, 4×8 m for fracture system 2 and for system 1 the ratio of the sides of the pillars varied as follows: 1:1, 1:1.5, 1:2. The typical calculation scheme (scheme of predetermined deformations) used in this work is shown in Figure-1. The lower part of the model is fixed in stationary state, the strain is transferred to the model of the rock mass via an absolutely rigid plate located on top, due to its movement down. Thus, the pillar located between the rigid plates is subjected to loads with subsequent deformation and redistribution of strains.

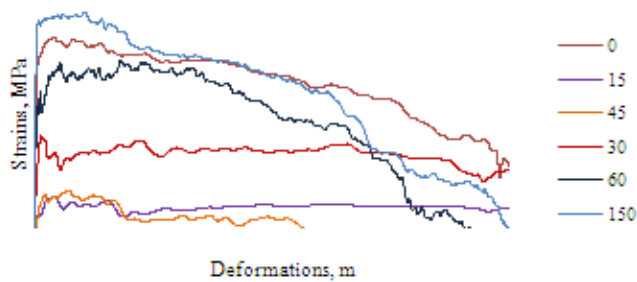


**Figure-1.** The schematic structural design for the numerical experiment for uniaxial compression of a pillar in fractured rock mass with block structure: *a* - fracture system 1; *b* - fracture system 2.

Within the framework of studying the physico-mechanical properties of intact rock at the laboratory, the following information about physico-mechanical characteristics was experimentally obtained:  $\rho=2,760 \text{ kg/m}^3$ , modulus of elasticity  $E=5 \cdot 10^4 \text{ MPa}$ , Poisson's ratio  $\nu=0.255$ , angle of internal friction  $\varphi=27^\circ$ , and grip  $c=30 \text{ MPa}$ . The following values of the parameters during interaction of blocks of rock mass along the contact were experimentally obtained: residual angle of internal friction was 27 degrees, grip was 795 PA, compressive strength of the crack wall material was 20 MPa, and the surface roughness of the blocks interaction surface was 18. The use of the Barton strength criterion for the contact surfaces of blocks of rock mass in simulation in the Abaqus software suite shows high convergence with the experimental data, the error is not more than 2 % [5].

### 3. RESULTS AND DISCUSSIONS

Some samples of the pillar exhibit nonlinear properties during deformation, but mainly accumulation of strains in the pillar occurs according to the linear dependence at relatively small deformations. The results of numerical experiments on 8 m high and 4 m wide pillars with fracture system 1 (Figure-1) are shown in the dependence of strain on deformations (Figure-2). As the pillar gets loaded, it accumulates strain in the areas where the plain of failure will occur in the future, and which allow connecting the area of load application to the area of pillar support, i.e. depending on the structure of the rock mass, various parts of the pillar will be loaded. Gradually, along with development, new shear microplanes are included into the process of deformation. Inclusion of new planes and deformation hardening continue until destruction of the pillar.

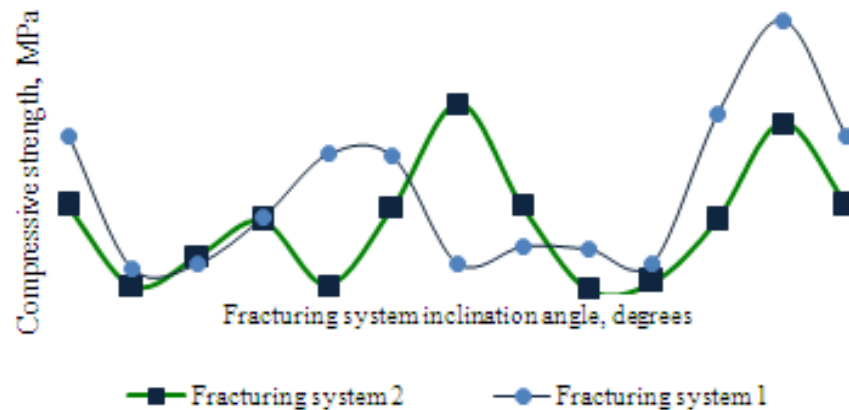


**Figure-2.** Diagrams of stress strains from deformation for pillars with various angles of fracture system inclination 1.

The destruction of the pillar occurs as a result of shear along the weakening surfaces inside the pillar. Until the moment of shear, strain is mainly accumulated

according to the linear dependencies, while deformation of the cracks in case of their mutual shear is in all cases characterized by significant nonlinearity. The microplanes of the shear are locations of strain concentration, the values of which do not reach the limit values of rock compressive strength, but are sufficient for the shear.

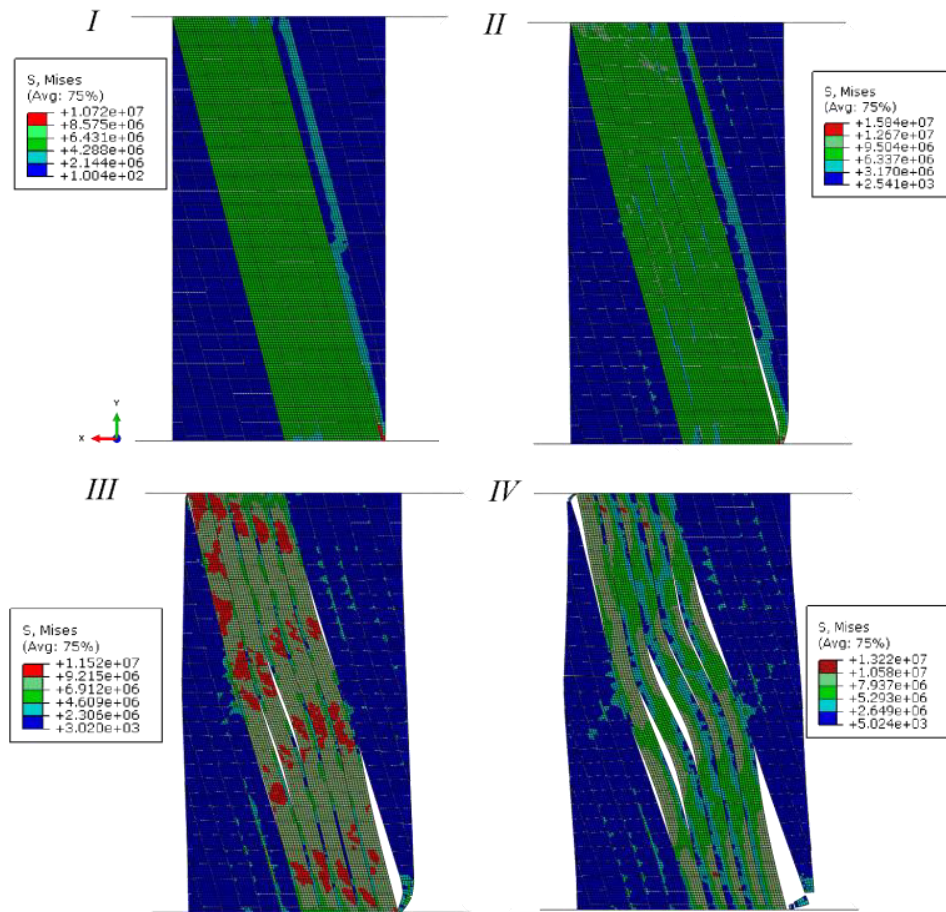
Understanding the fractured rock mass anisotropy properties is of practical importance (Figure-3 and Figure-6). Changing the angle of the main fracturing system results in changes of the mechanical characteristics of the pillar (strength under uniaxial compression, deformation modulus). Transition from the engineering-geological model to the geomechanical model was made with the use of auxiliary models that represent physical and mechanical properties of the structural elements.



**Figure-3.** Anisotropy of pillar compressive strength in fractured rock mass,

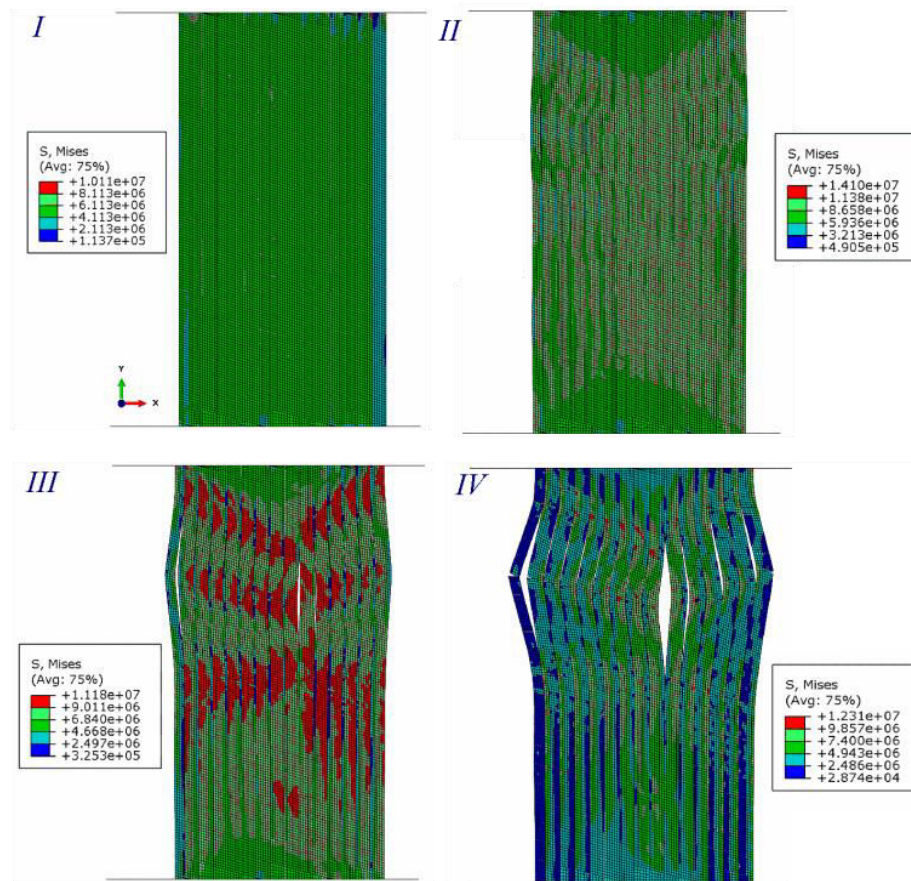
Let us look at the pillar compressive strength anisotropy curve with fracture system 2. Fundamentally important is the difference in the compressive strength of the pillar with cracks inclined to the horizontal platform at 0 and 90 degrees. The performed laboratory and numerical experiments with the system of fractures close to the layered one [4] show exactly the opposite behavior of the rock mass. This is mostly due to the fact that strength of the rock mass is determined by the nature of the bonds between elementary units that form it, and any change in the structure changes the properties. In order to understand the reason for the sharp difference between the strength and deformation properties of the same rock mass with differently oriented fracture systems, it is necessary to trace development of deformations and changes of strain that occur inside tested objects.

Figure-4 shows consequently recorded four stages of loading the pillar with the angle of inclination of the main crack system of 0 degrees at the same moments as in Figure-5, where plots of stress distribution and quality display of deformations in the pillar with the fracture angle of 90 degrees were consistently recorded. The pictures of strain plots show the difference of strain distribution in the pillar, if in the first case the strains occur not in all blocks, and a shear plane is formed, but only in those oriented at the angle of 60 degrees, while in the second case, strains are distributed throughout the entire pillar and at the pre-peak stage *III* the pillar is tensioned more and more evenly, which thereby increases its compressive strength. It is important to understand that cases when the load is applied with a perfectly horizontal plane in all cases that would not occur in natural conditions are considered.

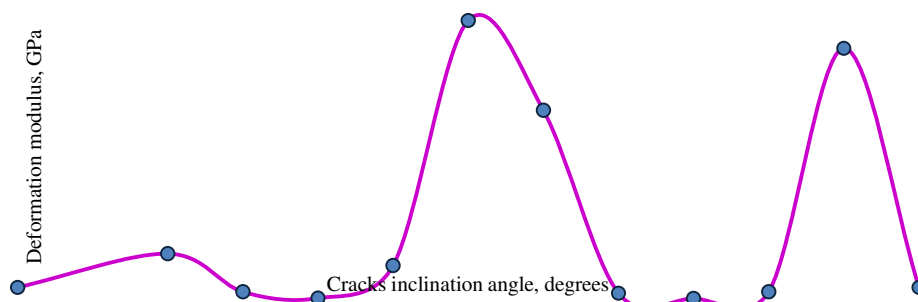


**Figure-4.** Stress distribution curves in the pillar with the fracture angle to the horizontal plane of 0 degrees (I, II, III, IV - time moments during the period of deformation).





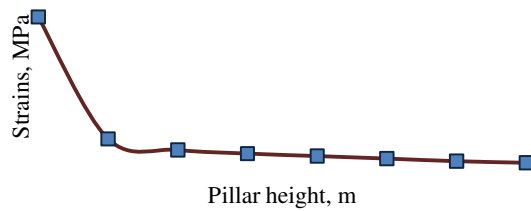
**Figure-5.** Stress distribution curves in the pillar with the fracture angle to the horizontal plane of 90 degrees (I, II, III, IV- time moments during the period of deformation).



**Figure-6.** Anisotropy of the deformation modulus upon uniaxial compression of the pillar in fractured rock mass with fracture system 2.

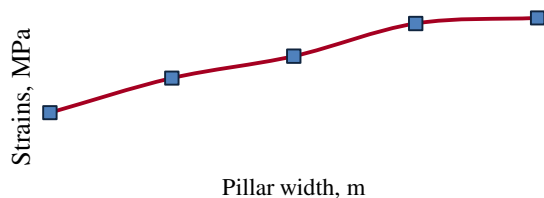
It is logical that in case of constant width with increasing height of the pillar, the bearing capacity of the pillar will decrease, but the nature of the changes is not uniform. A sharp decrease in the strength of uniaxial compression is observed when the height is increased from 3 to 4 m, while in the area of 5 to 10 m (Figure-7), a considerable decrease is observed. Such a description is relative, since the bearing capacity of the pillar falls down 6 times when the height is changed from 3 to 6 m, and 1.68 times, when the height is changed from 6 to 10 m. Knowing that the minimum compressive strength of the

ore is about 70 MPa, strength in the rock mass reduces almost 10 times in case of pillar width of 4 m and height of 5 m. From the practical point of view, during design of development of the mineral deposit, it is very important to understand the nature of dependency of the changes in the bearing capacity curve on the dimensions of the pillar.



**Figure-7.** Diagram of dependence of uniaxial compression strength of a pillar in fractured rock mass on its height with constant width in fracture system 2.

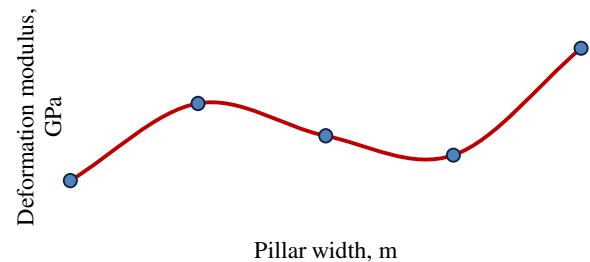
There is also a change in the bearing capacity of the pillar with constant height, but with variable width (Figure-8), but the nature of the curve differs from the previous curve. While in the previous case, there were sharp changes, in Figure-8, they are almost linear. This difference is explained by changes in the geometry of the fracturing, since in the second case, if width increases, formation of the shear platform in the pillar becomes much more complicated, which is confirmed by quantitative comparison of pillars' bearing capacity in Figure-7 and Figure-8.



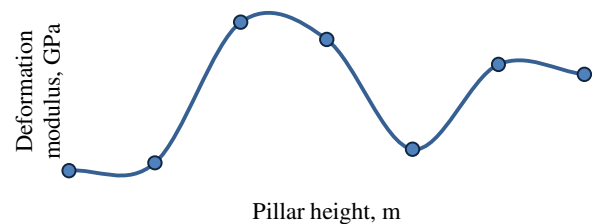
**Figure-8.** Diagram of dependence of uniaxial compression strength of a pillar in fractured rock mass on its width with constant height fracture system 2.

Separate consideration is required for the changes in the deformation modulus of pillars in fractured rock mass, since the nature of changes differs from the strength under uniaxial compression. This is explained mainly by different ability of pillars to accumulate strain without significant deformation - in various cases formation of shear surface requires different strains, since the length of the main fracturing system. Changes in the deformation modulus occur at interval as the width of the pillar (Figure-9) or height of the pillar (Figure-10) increases. In the first case, the changes are from 2.2 GPa to 8 GPa, in the second case - from 1.58 to 12.59 GPa. Thus, with greater compressive strength, a pillar in the fractured rock mass may have smaller modulus of deformation, for example, in the case when its entire height is 5 and 10 m (Figure-7), uniaxial compression strength in the second

case was almost two times smaller, and the deformation modulus was 4 times greater.

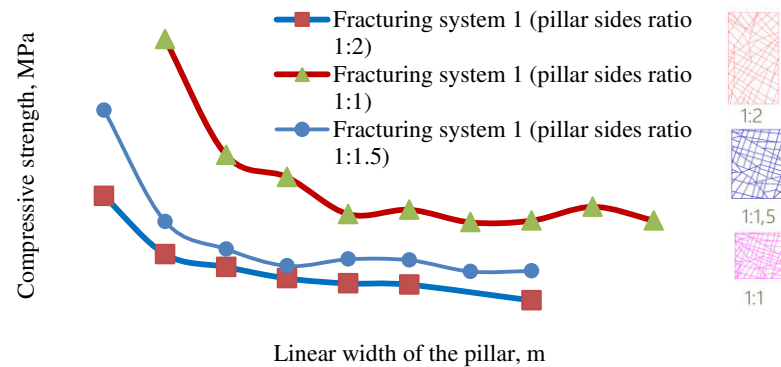


**Figure-9.** Diagram of dependence of deformation modulus under uniaxial compression of a pillar in fractured rock mass on its width with constant height in fracture system 2.



**Figure-10.** Diagram of dependence of deformation modulus under uniaxial compression of a pillar in fractured rock mass on its height with constant width fracture system 2.

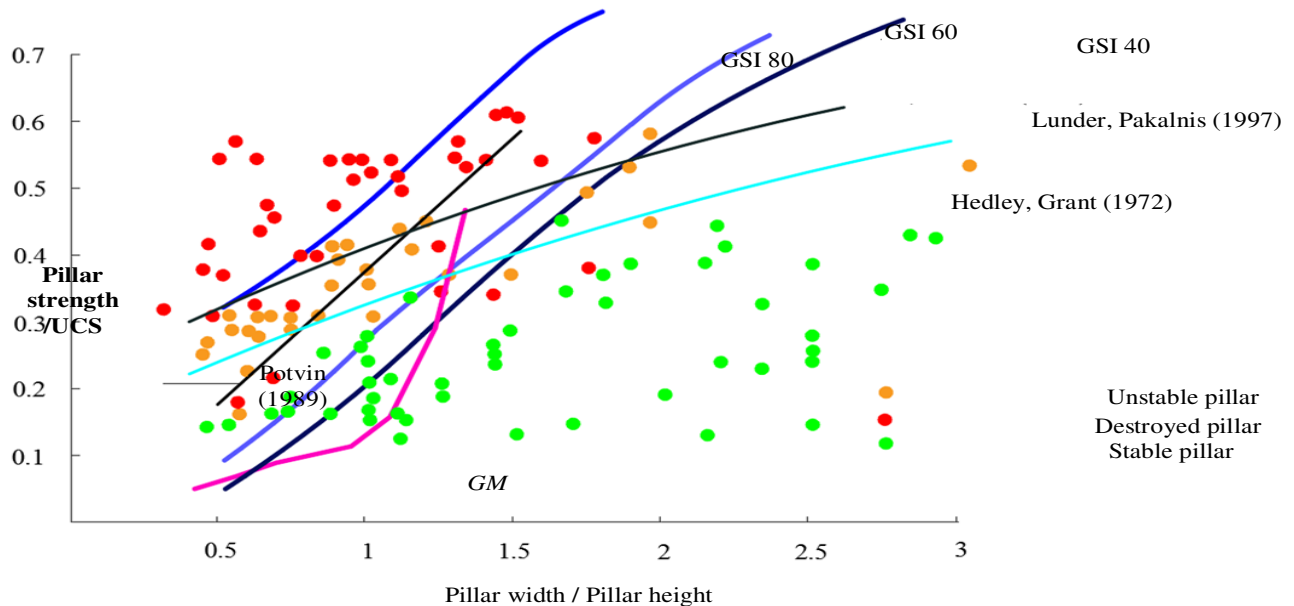
As the volume of the considered pillar increases, there is a change in the structure of the rock mass, which is quantitatively manifested by changing the ratio of pillar side to the average magnitude of the unit composing it, and changes in its mechanical characteristics. It has been established experimentally that geometrically similar area of the same fractured rock mass studied in the same conditions show different mechanical characteristics, which are described by the size-to-area function [7]-[9]. The influence of fracturing on the mechanical characteristics of the pillar is well illustrated by the results of numerical experiments for uniaxial compression (Figure-11). For the experiments with the chosen boundary conditions and fracturing, the ratio of pillar sides of 1:2, 1:1.5, 1:1, the elementary representative volume is the area of the rock mass with the width of 6 m. In case of further increasing the size of the model, the curve of compressive strength dependence on the dimensions becomes flatter (Figure-11).



**Figure-11.** Scale effect of pillar compressive strength in fractured rock mass.

The developed geomechanical model of a fractured rock mass qualitatively agrees with the theoretical and practical foundations developed to date, but the most important criterion of its adequacy for the use for practical purposes is its quantitative correlation with the results of observations and empirical dependencies obtained in course of mineral deposits development, and widely used in mining engineering. Such empirical correlations are [25-28], comparison of the results of the use of which is shown in Figure-12. The results obtained in this work are shown in Figure 12 by curve GM. It should be noted that in modeling, the rock mass was considered as a blocky environment, and the conditions were taken as the most unfavorable ones, therefore the curve is below the others. In work [25] 178 pillars in the rock mass were considered, and the authors empirically took into account that for thin pillars with the ratio of width to height less than one, pillar destruction starts at the strain equal to 1/3 of the compressive strength of the rock

mass. It has also found that all cases of destruction occur when the ratio of pillar width to height is less than 2.5 and the destruction mainly occurs when the pillar takes the form of an hourglass. But based on numerical simulation, one can make a conclusion that the pillar during destruction does not always take the form of an hourglass, and its definition depends on the structure of the rock mass itself. In general, by the results of the analysis, practical use of the proposed geomechanical model of the blocky rock mass for predicting the bearing capacity of the pillars is possible, because the dependency curve GM (Figure-12) is located in the zone where observations confirmed stability of pillars under the given boundary conditions. Further research requires clarification of the model adequacy for pillars with the width to height ratio more than 1.3. However, one should understand that each mineral deposit has unique initial data for solving the problem, and empirical relations for determining the bearing capacity of pillars will be different.



**Figure-12.** Comparison of the empirical dependencies of Lunder, Pakalnis [25], Hedley, Grant [26], Potvin [29] with the results of modeling in Phase 2 for indicators GSI 40, 60, 80 [27], and numerical experiments performed within the framework of this work (GM).

#### 4. CONCLUSIONS

Studying the behavior of rock mass and pillars is usually reduced to building deformation curves, on the basis of which the data are obtained about all rock mass characteristics that the engineer is interested in, but the curve doesn't allow to adequately assess such parameters of destruction as the degree and nature of the fracturing development, or establish the relationship between peculiarities of crack propagation and the characteristic points of curve (elastic strength, tensile strength, etc.) [6]. However, the use of finite element method for building a numerical mathematical geomechanical model of fractured rock mass or a pillar allows to track destruction and displacement of blocks during the process of loading, assess the number and size of cracks in the fracture, and trace the progress of dilatancy softening in case of inelastic deformation of samples.

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