VOL. 13, NO. 9, MAY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

NULL STEERING USING PHASE SHIFTERS

Maha Abdulameer Kadhim
Department of Electronics, Middle Technical University (MTU), Technical Instructors Training Institute, Baghdad, Iraq
E-Mail: Maha.kahdum@gmail..com

ABSTRACT

The importance of wide band nulling arise from the fact that the array pattern is frequency dependent and the direction of arrival of the undesired signal is not always located exactly. The proposed method for wide band nulling is based on the minimax approximation. The algorithm depends on finding a set of new phases to reject a wide sector in the side lobe while not seriously affecting the other pattern characteristics. The method combines the advantages of the phase only techniques and the flexibility of the minimax algorithm. The phase-only nulling can provide the pattern synthesis at lower cost while the Minimax provides the designer with the ability to control the null depth and the side lobe level to obtain an acceptable signal to interference noise ratio. The method is tested for both linear and planar arrays and is proved to be effective for any initial pattern. The disadvantage of this method is that it is not capable of obtaining two nulls symmetrically distributed around the main beam which is not considered to be a common practical case.

Keyword: communication, planar arrays, steering, mainbeam, null bands.

1. INTRODUCTION

Most work concerned with adaptive nulling has been directed towards maximizing a performance index such as signal-to-noise and interference ratio (SN1R) or antenna gain indices [1]. The main advantage of adaptive arrays is their capability of interference suppression by reducing the side lobe levels at the direction of interfering signals while keeping the main beam pointing towards the desired signal. The adaptive nulling mentioned previously gave a great importance to the synthesis techniques since the design of such adaptive systems includes many questions concerning the optimality of the system, the degrees of freedom, the perturbations of the pattern, and the depth of the null [2]. Another major reason that gave importance over the synthesis techniques studies is the increasing pollution of the electromagnetic environment.

2. LINEAR ANTENNA ARRAYS

Most of the communication systems involve the use of linear arrays. Such arrays are sometimes essential because of their special properties and high capabilities. For example, the signal to noise ratio can be enhanced using an array and the weak signals that are embedded under the noise can be magnified and detected [3].A uniform antenna array consists of N identical antennas arranged along a straight line with equal interelement spacing with identical excitations in magnitude and may encounter a progressive phase shift. Such array is shown in Figure-1, and the array factor can be written as:

$$F = 1 + e^{-jkdi\,sin\theta} + e^{-jkd2\,sin\theta} + \cdots + e^{-jkdN\,sin\theta} \qquad (1)$$

Where d_1, d_2... d_N are the elements' spacing relative to the first element and k is the wave number= $2\pi/\lambda$. Assuming that the number of elements equal 2N and setting u sin(θ) ,where θ is the scanning angle from broadside, the array factor is Written as

$$F = \sum_{n=1}^{2N} e^{jd_n ku} \qquad (2)$$

Where

$$d_n = d\left(n - \frac{2N+1}{2}\right) \qquad (3)$$

This function is plotted in Figure-2 for 2N = 20 and d = O.A. The pattern has a main beam directed towards the desired signal while a group of sidobes lay around this main beam at lower power. The pattern consists of a total of 20 nulls in the domain u = [-1 1].

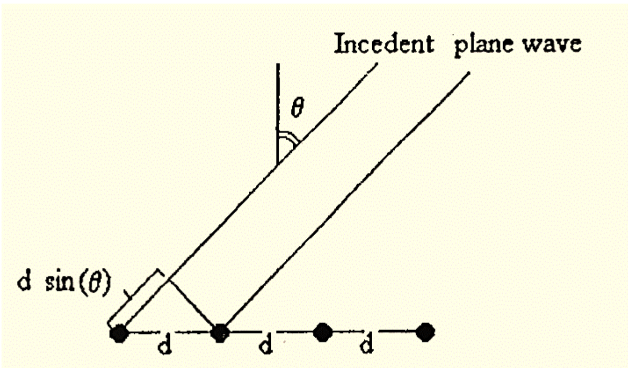

Figure-1. Phase delay between element excitations in a linear array.

The beamwidth of the pattern has two definitions; the first null beamwidth which is measured between the first two nulls around the main beam, and the half power beamwidth FPBW which is measured between the half power points.

VOL. 13, NO. 9, MAY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

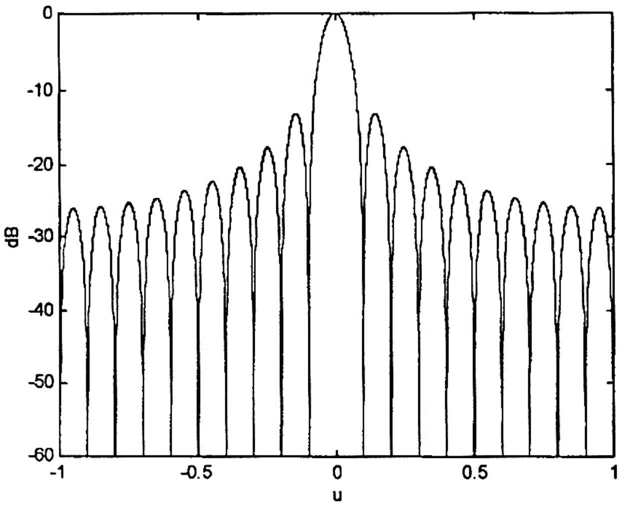

Figure-2. Radiation pattern of 20 elements uniform array.

The received or the transmitted signals at the array elements are modified by a weight vector to enable the control of the radiation pattern. One of the most popular applications is the phase scanning system used in Electronic Scanning Radar System where the main beam position scanning is accomplished by varying the phase of the separate elements of a linear array in a predetermined manner [4].

The weighting vector applied to the elements of the array is also used in the radiation pattern shaping. Radiation patterns with lower sidelobe level can be obtained by weighted illumination. This lower sidelobe level is obtained at the expense of a loss in the directivity [5].

3. ANTENNA ARRAY SYNTHESIS TECHNIQUES

Antenna pattern synthesis took a new direction towards the null steering algorithms. Although this method is considered to be the most costly choice [6], it offer excellent control over the antenna pattern. Several studies have addressed this method and many proposed techniques where studied. The early studies [7] used to find the complex weighting values that control the array pattern using analogue correlation between the sum channel and each element channel [8].

Null steering can be achieved by controlling the current amplitudes only which very much simplify the adaptive system. Vu (1984) achieved null steering using amplitude only control method which limits the number of rejected signals to half the number of the array elements [9].

The amplitude only method allows the designer to put two symmetrical nulls with linear solutions to the problem. Mismar and Ismail (1995) presented a linear programming of an array by controlling the current amplitudes [10].

4. ELEMENT POSITION CONTROL

Ismail and Dawoud proposed a method that uses LMS algorithm approximation to place nulls in the radiation pattern to suppress interference and undesired signals by controlling the element position [11].

5. PHASE ONLY CONTROL METHOD

This technique is a very attractive choice since it provides the control of the phases of the elements at no additional cost [12].The procedure used for calculating the phases of the array uses a linear programming algorithm. This method is called the Minimax algorithm which depends on solving an over determined system of linear equations where several constraints are imposed upon the pattern to achieve a wide band sector below the side lobe level, while trying to maintain the side lobe level and the main beam characteristics within certain tolerance [13].

6. RESEARCH PROCEDURE

The goal of this algorithm is to find a set of phases ϕ that minimizes (p). The work done by HAUPT uses the gradient search minimization technique for narrow band adaptive nulling in monopulse Radar. The proposed algorithm imposes single or multiple wide band nulls in the array pattern by minimizing the power due to signals arriving in a certain band represented in the equation (4)

$$P = P(A, D, \Phi) = \left| \sum_{m=1}^{M} \sum_{n=1}^{N} \gamma_{mn} a_n e^{j\phi n} e^{jdn\, k\, um} \right|^2 \quad (4)$$

Where
$A = [\, a_1, a_2, \dots, a_N]$, Amplitude weight vector,
$D = [d_1, d_2, \dots, d_N]$, element spacing in wavelengths,
$\phi = [\phi_1, \phi_2, \dots, \phi_N]$, adaptive phase settings,
γ_{mn} represented to electric field at element

Now we will assume one desired signal to arrive at u_0 while an undesired signal orgroup of signals at a sector $u_{null} = [u_{11} \quad u_{12}]$, γ_{0n} and $\gamma_{1n}$are the electric field at element (n) due to the desired and undesired signals respectively[14], the value of $\gamma_{0n}$is normalized and γ_{1n} is assumed as a fraction of one, so a ratio E is defined as follows:

$$\frac{\gamma_{1n}}{\gamma_{0n}} = R_n \quad (5)$$

Now, since the power is calculated over a region, the continuous summation over the undesired band is actually an integration, so the power can be rewritten as

$$P = \left| \sum_{n=1}^{N} a_n e^{j\phi\, n} e^{jd_n\, k\, us} + \int_{u11}^{u12} \sum_{n=1}^{N} R_n a_n e^{j\phi n} e^{jd_n ku} du \right|^2 (6)$$

Assuming that the signals are uncorrelated then the power consists of two components; the power due to the desired signal P_s, and the power due to the undesired signal P_{ud}.

VOL. 13, NO. 9, MAY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

$$P = P_s + \left| \int_{u11}^{u12} \sum_{n=1}^{N} R_n a_n e^{j\phi n} e^{jd_n k u} du \right|^2 \qquad (7)$$

$$P_{ud} = \left| \sum_{n=1}^{N} R_n e^{j\phi n} \frac{e^{idnku12} - e^{jdnku11}}{jd_n K} \right|^2 \qquad (8)$$

The equations for the gradient search algorithm is given by:

$$\phi^{(k+1)} = \phi^{(k)} - \alpha \nabla P\left(\phi^{(k)}\right) \qquad (9)$$

Where the gradient of P is given by:

$$\nabla P\left(\phi^{(k)}\right) = \left[\frac{\partial P(\phi^{(k)})}{\partial \phi_1}, \frac{\partial P(\phi^{(k)})}{\partial \phi_2}, \dots, \frac{\partial P(\phi^{(k)})}{\partial \phi_N}\right] \qquad (10)$$

$$\frac{\partial P}{\partial \phi_n} = \frac{P_n(\phi + \Delta\phi) - P_n(\phi)}{\Delta\phi} \qquad (11)$$

Where Δϕ is the phase shifter step size, a constant [15], An α that gives the minimum undesired power (∇P). limit exists on the value ofα,represented in equestion (12)

$$\alpha = \frac{\Delta\Phi}{\sqrt{\left(\sum_{n=1}^{N}\left(\frac{\Delta P n}{\Delta\Phi}\right)^2\right)}} \qquad (12)$$

The gradient search algorithm depends on calculating the new phases that will put a wide null in the antenna array pattern using equation 7 which requires equations 10, 11, and 12 to be calculated at every iteration. The algorithm can be summarized as Figure-3.

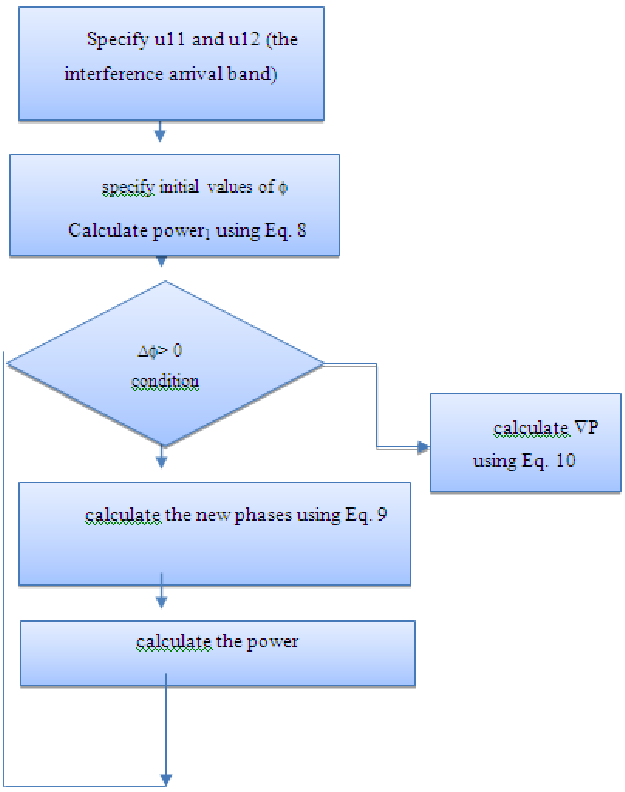

Figure-3. Steps of algorithm program.

7. RESULTS AND DISSECTIONS

The gradient search algorithm is used to calculate the phases required for the minimum power due to undesired signal so a null is formed in the band where the interfering signal is expected to arrive. The algorithm described above is programmed with different initial values of Δϕ chosen by the program. Several examples are considered in the following discussion.

First, an interference is considered to arrive at u = 0.85 with relative bandwidth of 5%. The gradient search algorithm is used to put a null in the band between u11=0.829 and u22 = 0.871. Figure-4 shows the output of the algorithm for Δϕ=0.25

A value of Δϕ= 0.012 will not adequately form the nulls as shown in Figure-2: the null depth in the suppressed band is -42.74 dB while in the case of Δϕ = 0.25 the null depth is - 45.7 dB. But it is obvious that the perturbations in the initial pattern are higher for larger values of Δϕ.

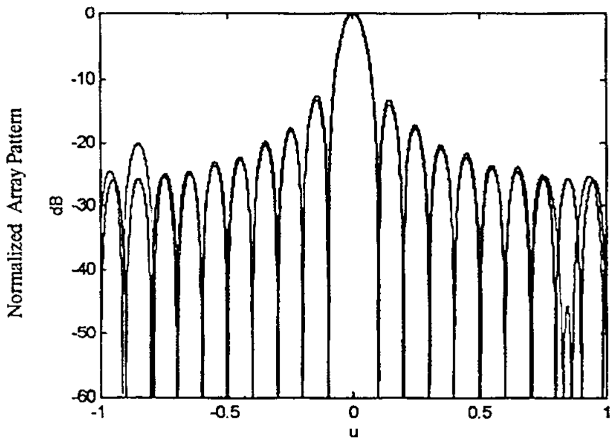

Figure-4. Linear array pattern with a wide band null for 20 element uniform imposed u = 0.85 (RED), the undesired signal has a relative bandwidth of 5%. The gradient search algorithm is used with Δϕ= 0.25.

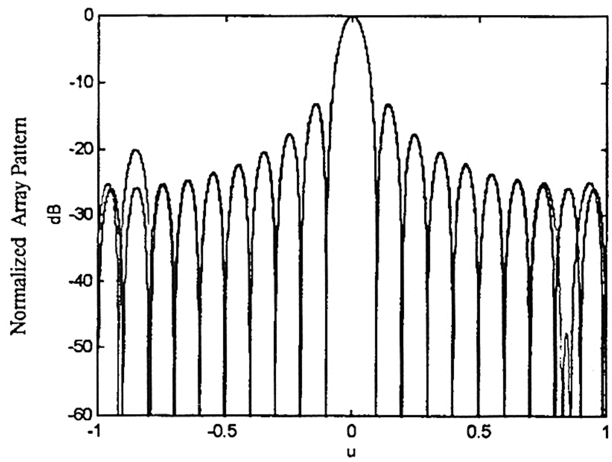

Figure-5. Linear array with a wide band null A for 20 element uniform imposed, u = 0.85 (RED). The undesired signal has a relative bandwidth of 5%. The gradient search algorithm is used with Δϕ= 0.012.

VOL. 13, NO. 9, MAY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

In the case of higher null bands, the maximum null depth that the algorithm can impose in the pattern is reduced. For example, if the same pattern is considered, with a relative bandwidth of 10%, the gradient search algorithm reduces the level of the side lobe at the prescribed location from - 25.7 dB to - 35.3 dB at $\Delta\phi$ = 0.35 This is shown in Figure-6.

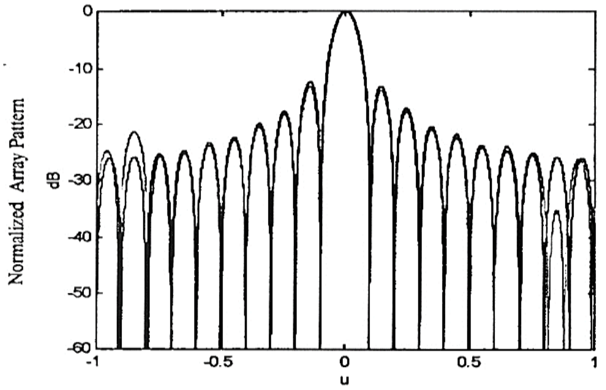

Figure-6. Linear array pattern with a wide band null for 20 element uniform imposed u = 0.85 (RED). The undesired signal has a relative bandwidth of 10%. The gradient search algorithm is used with $\Delta\phi$= 0.35.

Now, another example is to be considered where a null is to be imposed in a band closer to the main beam. The interfering signal is considered to arrive at u = 0.35. Two cases of the relative bandwidth of the signal is considered. The results are shown in Figures (7) and (8). Figure-9 shows the case where the interference signal has a relative bandwidth of 5% so the suppressed band is u = [0.341 0.359] and the maximum null depth obtained in this case is -46 dB at $\Delta\phi$ = 0.015 while in the initial pattern the sidelobe level at that band was -20.35 dB.

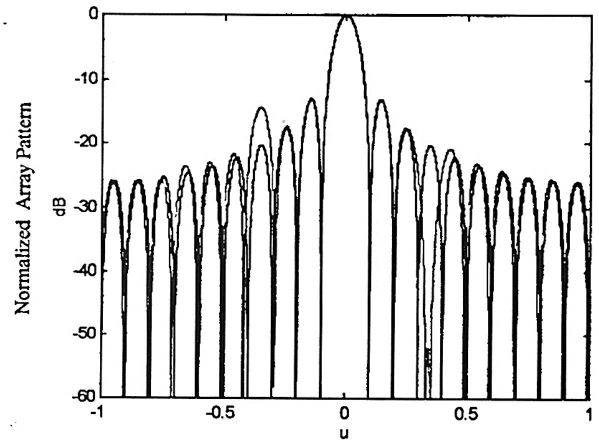

Figure-7. Linear array pattern with a wide band null for 20 element uniform imposed u = 0.35 (RED). The undesired signal has a relative bandwidth of 5%. The gradient search algorithm is used with $\Delta\phi$= 0.015.

In Figure-8 the relative bandwidth is 10% and the band to be rejected is u = [0.332 0.368]. In this case the null depth is -38.3 d13 at $\Delta\phi$= = 0.015.

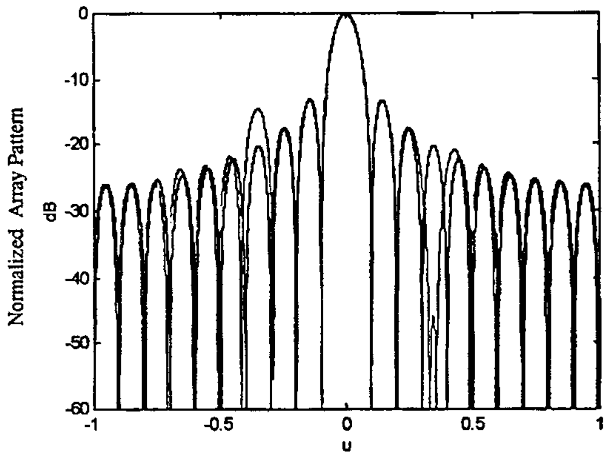

Figure-8. Linear array pattern with a wide band null for 20 element uniform imposed u = 0.35 (RED). The undesired signal has a relative bandwidth of 10%. The gradient search algorithm is used with $\Delta\phi$ = 0.015.

The next example will show the ability of this algorithm to impose multiple nulls in the array pattern. The algorithm first adjust the phases to obtain the first null. This perturbed pattern is considered as the initial pattern for the second nulling process so the algorithm modifies the phases again to put another null while trying to minimize the perturbations keeping the position of the first null nearly unchanged. Figure-10 shows three nulls located at u = -0.75, -0.55, and 0.35 with relative bandwidth of 5%.

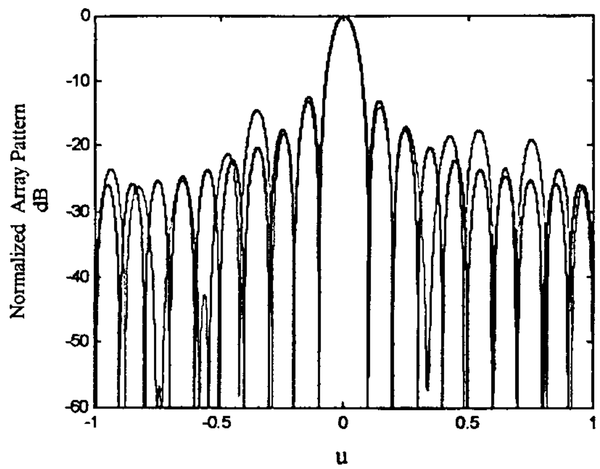

Figure-9. A 20 element uniform linear array pattern with three wide band nulls imposed at u = -0.75, -0.55, and 0.35 (RED). The undesired signals have a relative bandwidth of 5%. The gradient search algorithm is used with $\Delta\phi$ = 0.25.

- Phase Only Wide Band Nulling For Planar Arrays.

VOL. 13, NO. 9, MAY 2018 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Planar arrays are widely used in the radar and sonar applications. They have very interesting properties that made them essential for many applications. With planar arrays higher levels and lower side lobe directivity. Planar array that will be considered is rectangular-shaped of discrete sensor elements arranged in the x - y plane. Figure-10 shows a planar array with 2N X 2N, where the coordinate origin is chosen at the center of the pattern.

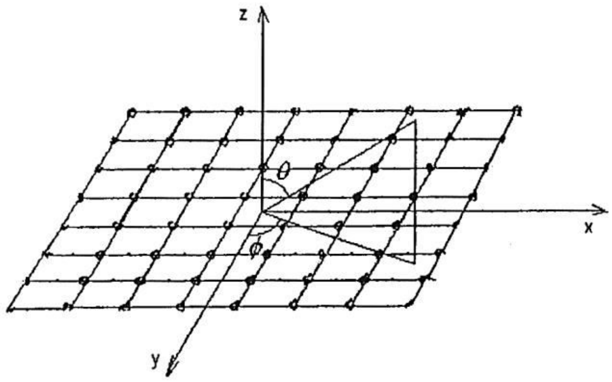

Figure-10. Rectangular planar array with isotropic elements.

Where ϕ(m, n) is the phase of the array element m, n.

Under the assumption that the phase perturbation is very small, the exponential function can be written using the first two terms of the series:

$$F = \sum_{m=1}^{2N_x} \sum_{n=1}^{2N_y} I(m,n)\left(1 + j\phi(m,n)\right) e^{jd_m u} e^{jd_n v} \quad (13)$$

$$F = \sum_{m=1}^{2N_x} \sum_{n=1}^{2N_y} I(m,n)\, e^{jd_m u} e^{jd_n v} + j \sum_{m=1}^{2N_x} \sum_{n=1}^{2N_y} I(m,n)\, \phi(m,n) e^{jd_m u} e^{jd_n v} \quad (14)$$

Now, due to the odd symmetry Of d_m, and d_n, the perturbed pattern can be rewritten as:

$$F = 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n) cos(d_m u)\, cos(d_n v) + j \sum_{m=1}^{2N_x} \sum_{n=1}^{2N_y} I(m,n)\, \phi(m,n) e^{jd_m u} e^{jd_n v} \quad (15)$$

To obtain a real function, the phases of the array elements are chosen to have certain symmetry to cancel the j operator. Assuming that the phases are even symmetric about the y-axis and odd symmetric about the x-axis, it can easily be proven that the perturbed pattern becomes

$$F = 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n) cos(d_m u)\, cos(d_n v) - 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\, \phi(m,n) cos(d_m u) sin(d_n v) \quad (16)$$

As a consequence of the difficulties encountered with a polar coordinate representation of a planar array beam pattern, it is convenient to introduce the transformation

u = sin θ cos ϕ
v = sin θ sin ϕ

Where the ranges of u, and v are the interval [-1, 1] [8]. Now a two dimensional representation for the planar arrays can be introduced, where contour lines are used to give information about the pattern characteristics. Figure-11 and Figures 12, 13, show the three dimensional view and the contour lines for a uniform planar array. The pattern is even symmetric about the u and the v axis.

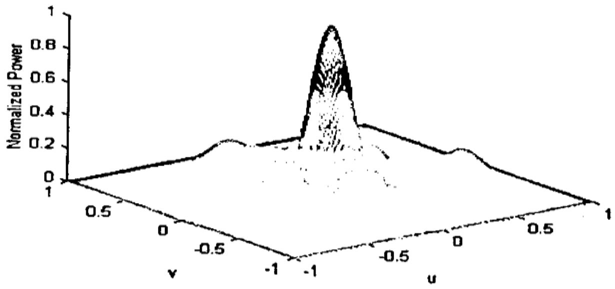

Figure-11. Three-dimensional plot of the initial pattern of a 10x 10 uniform planar array.

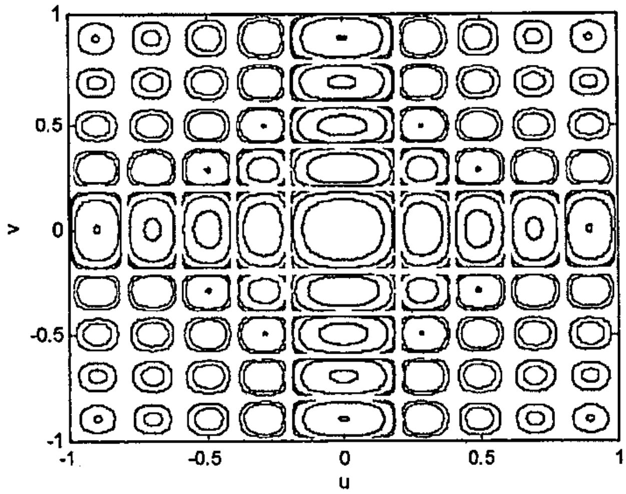

Figure-12. Contour lines for 10 x 10 planar array pattern with unity current excitations.

ARPN Journal of Engineering and Applied Sciences

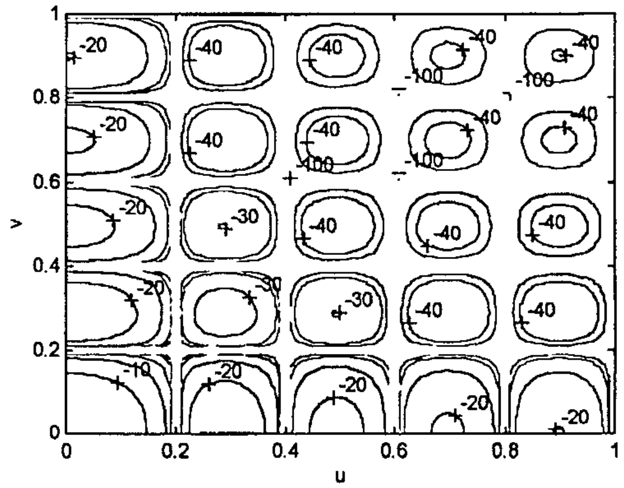

Figure-13. The first quarter contour lines for 10 x 10 array pattern with unity current excitations.

The problem now can easily be formulated n the same way as in the Lear array case. First we consider a desired pattern D.

$$D = \begin{cases} F_0(u,v) & u,v \in \mathit{Main\ Beam} \\ 0 & u,v \in \mathit{Side\ Lobe} \\ 0 & u,v \in \mathit{Null\ sector} \end{cases} \quad (17)$$

And the error function is again defined as

$$E = W|D - F| \quad (18)$$

This error function is again given different weights in the different regions.

$$\mathrm{E} = \mathrm{W_{mb}} \left|4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \mathrm{I(m,n)}\ \phi\mathrm{(m,n)cos(d_m u)sin(d_n v)}\right| \quad (19)$$

In the side lobe and the null sector the error is written as:

$$E = W_{sl} \left| -4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ cos(d_m u) sin(d_n v) + 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ \phi(m,n) cos(d_m u) sin(d_n v) \right| \quad (20)$$

$$E = W_{null} \left| -4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ cos(d_m u) sin(d_n v) + 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ \phi(m,n) cos(d_m u) sin(d_n v) \right| \quad (21)$$

To make an approximation of the perturbed pattern F(u,v) to the desired pattern we minimize a positive tolerance such that

$$|E(u,v)| \leq \delta \quad (22)$$

The main objective is to have a suppressed sector at the band were the interference is expected to arrive while trying to keep the mainbeam characteristics as in the initial pattern within a certain tolerance 6. So the resulting perturbed pattern is expected to be in the following form

$$F(u,v) = \begin{cases} F_0(u,v) \pm \delta_1 & u,v \in \mathit{Main\ Beam} \\ 0 \pm \delta_2 & u,v \in \mathit{null\ sector} \\ 0 \pm \delta_3 & \mathit{elsewhere} \end{cases} \quad (23)$$

where $\delta_2 << \delta_3$

The weighting function accordingly is defined as

$$W(u,v) = \begin{cases} \frac{\delta}{\delta_1} & u,v \in \mathit{Main\ Beam} \\ \frac{\delta}{\delta_2} & u,v \in \mathit{null\ sector} \\ \frac{\delta}{\delta_3} & \mathit{elsewhere} \end{cases} \quad (24)$$

The problem now can be formulated as a two dimensional problem and we can deal with it in the same way for linear array. The problem now is:

$$\mathit{Minimize}\ g =$$

$$\left[\phi_{1,1}, \phi_{1,2}, \dots, \phi_{1,N_y}, \phi_{2,1}, \phi_{2,2}, \dots, \phi_{N_x N_y}, \delta\right] * \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix} \quad (25)$$

Subject to the following constraints

1-In the main beam region:

$$\frac{\delta}{\delta_1} \left| 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ \phi(m,n) cos(d_m u) sin(d_n v) - \delta \leq 0 \right| \quad (26)$$

2- In the null sector and the sidelobe regions

$$\frac{\delta}{\delta_2} \left| 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ cos(d_m u) cos(d_n v) + 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} I(m,n)\ \phi(m,n) cos(d_m u) sin(d_n v) \right| - \delta \leq 0 \quad (27)$$

$$\frac{\delta}{\delta_3} \left| 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \mathrm{I(m,n)\ cos(d_m u) cos(d_n v)} + 4 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \mathrm{I(m,n)}\ \phi\mathrm{(m,n) cos(d_m u) sin(d_n v)} \right| - \delta \leq 0 \quad (28)$$

8. CONCLUSIONS

We have presented two techniques for wide band nulling. The first method uses the gradient search algorithm to impose single or multiple wide band nulls. It depends on minimizing the power due to undesired signals in a certain band. The method has the ability to achieve wide band nulls directly without imposing several narrow band nulls.

The second proposed method uses the MLNIMAX technique which is based on linear programming solution for an over determined system of linear equations. It calls the built in linear programming routines in the optimization toolbox in MATLAB software. The method approximates the perturbed pattern

ARPN Journal of Engineering and Applied Sciences

to a desired one by giving different tolerance values for the suppressed sector.

Minimax algorithm perturbs the phases of the initial pattern to obtain wide suppressed sectors. The phase only algorithm comes out with very interesting results; the algorithm shows a high flexibility in error control in the perturbed pattern so it becomes the user's choice to compromise between side lobe and depth of the null while keeping the main beam characteristics unchanged within a very small tolerance.

Several examples are examined to investigate the relations among the null depth and the mainbeam and side lobe perturbations. It turns out that the shift in the main beam increases as the null depth increases and at the same time the perturbations in the final pattern depend also on bow close the suppressed sector to the main beam. However the method can impose a wide band null without highly disturbing the pattern. Compared to the gradient search algorithm the proposed technique achieves deeper nulls and lower pattern distortion.

The Minimax technique also has the ability to utilize the available degrees of freedom to impose multiple wide band nulls without highly affecting the HPBW or the side lobe level. Unlike the gradient search algorithms, the minimax algorithm calculates the new phases to simultaneously impose several null bands at there prescribed locations which is faster than the alternative nulling used in gradient search algorithm. The proposed algorithm also have the ability to drive the null depth at a certain band lower than the other bands in a manner to keep the SLL and the mainbeam characteristics not seriously changed.

The algorithm was also used to impose wide band nulls in a rectangular planar array. Two initial patterns are examined, the uniform and the Chebyshev. The algorithm was able to impose symmetric and non-symmetric wideband nulls in the array pattern. As was shown by the contour lines of the array pattern the method achieves two symmetric nulls about the u axis while the side lobe at the opposite side of the pattern suffers from a rise in its level. This rise can be reduced by reducing the null depth.

REFERENCES

[1] Applebaum S. P. 1996. Adaptive Arrays. IEEE Trans. Anten. Propag. AP-24: 585-598.

[2] Baird C. A. and Rassweiler G. G. 2000. Adaptive Sidelobe Nulling Using Digitally Controlled Phase-Shifters. IEEE Trans. Anten. Propag. AP-24(5): 638-649.

[3] Balanis C. 2006. Antenna Theory Analysis and Design. Haper and Row publishers, New York, USA.

[4] Brookner Eli. 2008. Practical Phased - Array Antennas Systems. Artech House, London, U.K.

[5] Deford J. F. 2009. Phase-Only Synthesis of Minimum Peak Sidelobe Patterns for Linear and Planar Arrays. IEEE Trans. Anten. Propag. 36(2): 191-201.

[6] Balanis C. A. 2007. Antenna theory: analysis and design. 2nd Ed., New York, John Wiley & Sons.

[7] Barlevy A. S. and Rahmat-Sami Y. 2001. Characterization of electromagnetic band-gaps composed of multiple periodic tripods with interconnecting vias concept, analysis and design. IEEE Trans. Antennas Propagat. 49(3).

[8] Ismail, T. H. and Dawoud, M. M. 2000. Stutzman W. I. and G. A. Thiele G. A. Antenna theory and design. John Wiley & Sons, New York,USA.

[9] Mismar M. J. and Ismail T. H. 1999. mt. J. Electronics. 78(2): 409-415.

[10] Monzingo R. A. and Miller T. W. 2010. Introduction to adaptive arrays. John Willy and Sons, New York, USA.

[11] Pillal S. K. 2005. Array Signal Processing. Springlar-Verlag, New York.

[12] Shore R. A. 2009. Nulling at Symmetric Pattern Location with Phase Only Weight Control. IEEE Trans. Anten. Propag. AP-32(5): 530-533.

[13] Shore R. A. 2006. A proof of The Odd-Symmetry of the Phases for Minimum Weight Perturbation Phase-Only Null Synthesis. IEEE Trans. Anten. Propag. AP-32(5): 528-530.

[14] Shpak D. J. 1996. A Method for the Optimal Pattern Synthesis of Linear Arrays with Prescribed Nulls. IEEE Trans. Anten. Propag. 44(3): 286-294.

[15] Steyskal H., Shore R. A. and Haupt R. L. 2003. Methods for Null Control andTheir Effect on Radiation Pattern. IEEE Trans. Anten. Propag. AP-34(3): 404-409.