



# CAR ACCIDENTS SIMULATION OF THE INTERSECTION OF TWO ROADS IN THE MIXED TRAFFIC FLOW USING FUKUI-ISHIBASHI MODEL

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## ABSTRACT

The main idea behind this work is to present the accident probability the traffic at non-signalized intersection of two cross roads using Fukui-Ishibashi (FI) model. A comparative study of FI and Nagel-Schreckenberg (NaSch) models is presented. It is found in one hand, when vehicles distinguished only by their lengths, the car accidents start to occur above a critical density  $\rho_c$ . Furthermore, the increase of the fraction of long vehicles (FL) delays the occurrence of car accidents (increasing  $\rho_c$ ) and increases the risk of collisions when  $\rho > \rho_c$ . In other hand the plateau behavior in the fundamental diagram depend strongly to the fraction of long and short vehicles. In other side, the mixture of maximum velocities leads to the appearance of accidents at the intersection. Moreover, the decrease of the fraction of fast vehicles (Ff) increase the accident probability (Pac). Additionally, the influence of roads length was also studied. We found that the increase of the roads length, decrease the risk of collision. Comparing FI and NaSch models, it was found that the critical density and the accident probability are respectively smaller than NaSch ones. These results prove that FI model shows better results than NaSch in case of mixture lengths and velocities.

**Keywords:** FI model, intersection, car accidents, mixed traffic flow, road length.

## 1. INTRODUCTION

Nowadays CA [1-7] has become a well-established method to model, to analyze and to predict the behavior of a real road traffic. A well-established and popular CA model is proposed by Japanese scholar Fukui and Ishibashi's (FI) in 1996 [2]. It represents a one-dimensional cellular automaton traffic flow model. In addition, the control and optimization of the flow at the intersection is a major preliminary step to optimize the city networks [8] were the first to come up with a two-dimensional CA model for inner-city traffic systems. The first model for simulation of two roads that cross was however recommended in several works [9-11]. To model vehicle traffic realistically, on one hand, we must simulate the road with vehicles having different desired vehicles and lengths. In reference [12], the length of vehicles and their effect on traffic flow was examined. Another very important aspect to examine is car accidents on the road as this is directly related to the well-being driver behavior. It is highly important to simulate car accidents when studying the traffic flow.

Along this line of study, our objective in this paper is to study in some depth the characteristics of traffic flow, the probability of the occurrence of car accidents ( $P_{ac}$ ) considering the impact of mixing lengths in a single intersection with closed boundary condition, using a simplified version of NaSch model [2]. This paper has the following layout. In section 2, the model is introduced and formulated. In sections 3, the results of simulations of each controlling scheme are exhibited and discussed. Concluding end, the paper in section 4.

## 2. MODEL AND METHOD

### 2.1 The FI model

In this paper, two equal-length perpendicular roads (R1 and R2) that cross in the middle are considered (Figure-1). Both roads are divided into equal-sized cells. The movement of the vehicles is described as follows: the vehicles on R1 move from top to bottom, while the on R2 they move from left to right. Each car can take different values of velocity ranging from 0 to  $V_{max}$ .  $L$  is the common length of the road.  $D1$  and  $D2$  represent the distances between the car and the intersection cell in both roads (Figure-1).

In this work, the system is updated in a parallel manner according to the rules of Fukui and Ishibashi (FI) [2]. The main difference between NaSch and FI models is the absence of a velocity memory. All vehicles have an intrinsic velocity  $V_{max}$ . In each timestep, all cars attempt to move at the maximum velocity  $V_{max}$ .

The FI model [2] is defined by the following set of rules:

- R1 Acceleration:**  $V_n = \min(V_{max}, d_n)$ .
- R2 Randomization:** If  $V_n = V_{max}$ , then  $V_n \rightarrow V_{max} - 1$  with probability  $P_b$ .
- R3 Vehicle movement:**  $X_n \rightarrow X_n + V_n$ .

As usual,  $X_n$  and  $V_n$  denote the position and speed, respectively, of the  $n^{th}$  vehicle and  $d_n = X_{n+1} - X_n - 1$ , represent the number of empty cells in front of this car (headway).

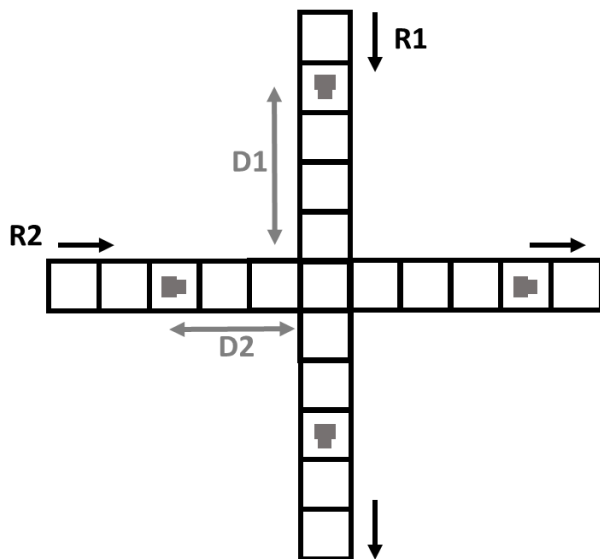


Figure-1. The scheme of the roads.

## 2.2 Mixture lengths

In this study, two types of vehicles distinguished by their lengths are considered. Type1 represents the short vehicle, designed by one cell, while the type2 denotes the long one and represented by two adjacent cells. The same maximum velocity ( $V_{\max}=3$ ) is chosen for both types of vehicles.  $F_L$  and  $F_{Sh}$  denote respectively the fraction of long and short vehicles ( $F_{Sh}+F_L=1$ ). The priority rules at the intersection of the two roads are described in the next sections.

## 2.3 Mixture velocities and overtaking rules

Here, two types of vehicles discriminated by their maximum velocities are considered. Type1 represent the slow vehicle with  $V_{\max}=1$ , while type2 denote the fast one with  $V_{\max}=5$ .  $F_f$  and  $F_s$  signify respectively, the fraction of the fast and slow vehicles ( $F_s+F_f=1$ ). In this case, the fast vehicles can overtake slow ones according to [12]. We summarise these rules as follow:

- C1: type2 (fast vehicle) follows type1 (slow vehicle).
- C2:  $Gap_1 < V_{\max}$ , where  $Gap_1$  denote the distance between the type1 and type2.
- C3:  $Gap_2 > \min(V_{type1}+1; V_{\max}) + 1$ , where  $Gap_2$  denote the number of empty sites in front of type1. It represents the minimal safety distance required for overtaking.
- C4: The overtaking velocity is:  $V_{type2} = \min(Gap_1+1+Gap_2, V_{\max})$ .
- C5:  $V_{\max} > 2(V_{\max}+1)$ , this inequality guides the choice of  $V_{\max}$  and  $V_{\max}$ ; it is obtained based on the conditions C2, C3, and C4.

## 2.4 Priority rules and car accidents condition

The priority at the intersection between two vehicles is established according to reference [13]. The closer vehicle goes first. If ( $D1=D2$ ), the vehicle with the faster speed goes first. If ( $D1=D2$  and  $V_1=V_2$ ), the vehicles move arbitrarily (with a probability  $P=0.5$ ). In all cases,

the not-priority vehicle decelerates (velocity becomes  $V_i=D1,2$ ). Whereas the priority vehicle moves with its velocity.

The simulation of the car accidents probability ( $P_{ac}$ ) is done according to reference [12]. When two vehicles reach the intersection at the same time, the accident happens with a probability  $p'$  (in this work,  $p'=0.1$ ).

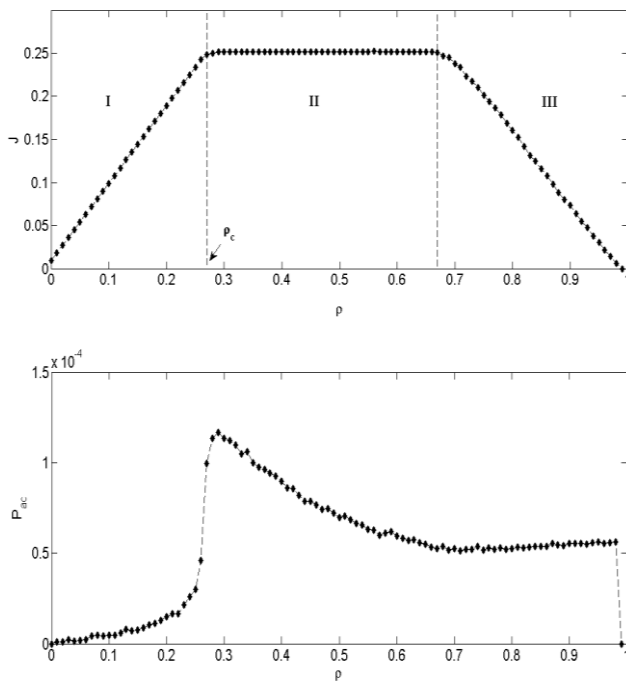
## 3. RESULTS AND DISCUSSIONS

### 3.1 Mixture lengths

We have taken the roads lengths equal to 7500m. Each cell is set to 7.5m which corresponds to  $L1=L2=1000$  cells. The fraction of the long vehicles is defined by  $F_L$  ( $0 < F_L < 1$ ). The number of iterations was set to  $2 \times 10^5$ . The system is updated for 500 time steps.  $J_1$  and  $J_2$  designate the flows of R1 and R2 respectively, and  $J$  represent the mean flow ( $J = (J_1+J_2)/2$ ).

Figure 2 shows the fundamental diagram and the accident probability ( $P_{ac}$ ) versus density. The mean flow  $J$  exhibits a three-regime behaviour: an increasing, a plateau and finally a decreasing portion. In the free flow regime, the flow increases with the increasing of  $\rho$ , after that, a plateau region is created caused by the intersection. Finally, in the third region, the flow decreases until zero. The same three regions are observed on the accident probability curve. In region I, almost no collision at the intersection cell, which agrees with the free flow regime seen on the fundamental diagram. In this case, all vehicles move with their maximum velocity and the collision probability is less than  $P_{ac}$ .

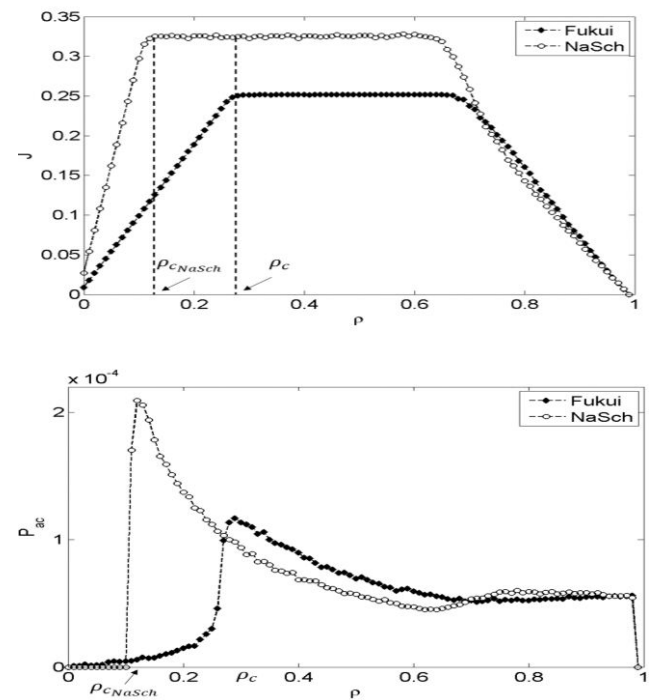
In region II,  $P_{ac}$  increases after a critical density  $\rho_c$  until it reaches its maximum. In this region, we observe that the accident probability decreases with increasing density, which agrees with the plateau region in the fundamental diagram. This behavior is explained by congestion, where the vehicles take long time to reach the intersection. In region III, the  $P_{ac}$  remains practically constant with low values until it vanishes for  $\rho=1$ .



**Figure-2.** The flow ( $J$ ) and Car accident probability  $P_{ac}$  versus density  $F_L=0.2$  and  $F_{Sh}=0.8$  and  $V_{max}=3$ ,  $P_b=0$ .

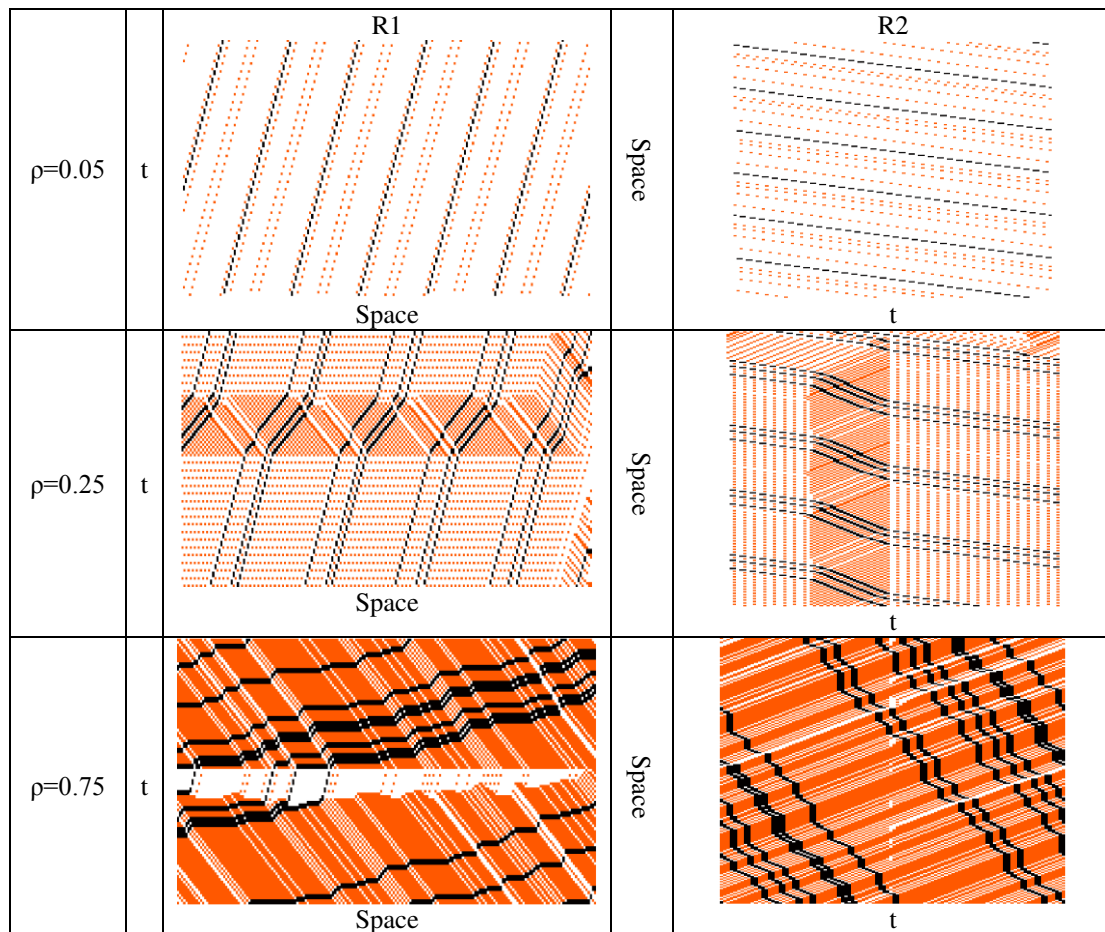
Figure-3 shows a comparison between FI and NaSch model in term of fundamental diagram and the accident probability ( $P_{ac}$ ) versus density. In the case of the fundamental diagram, we observe that the free flow regime in FI model is almost the double of NaSch one ( $\rho_c \sim 2 \rho_{cNaSch}$ ). The congestion region in FI model is smaller than NaSch one. This difference can be explained by the fact that in FI model the increase of speed of the vehicles is not necessarily gradual. After this region, the two models have the same behaviour. Based on the Accident Probability graph, we observe that maximum of the accident probability in the case of FI model is almost the half of NaSch one. From these results, we conclude

that FI model shows better results than NaSch in case of mixture lengths.



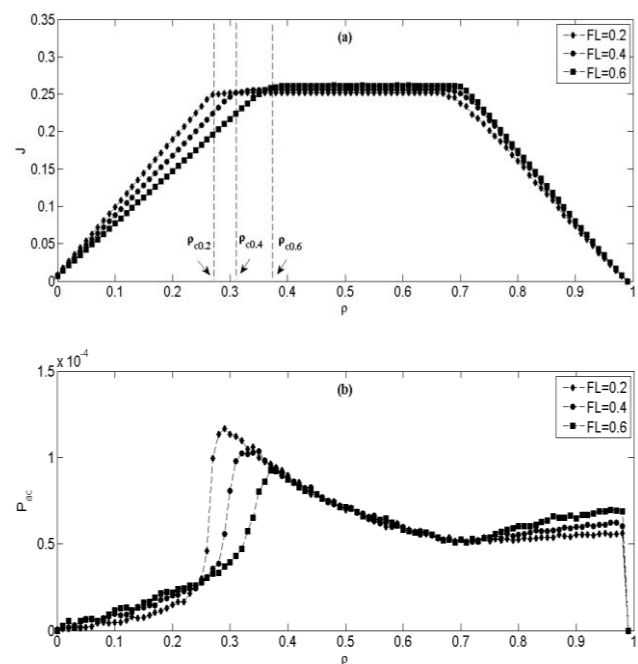
**Figure-3.** Comparison of FI and NaSch model: The flow ( $J$ ) and Car accident probability  $P_{ac}$  versus density  $F_L=0.2$  and  $F_{Sh}=0.8$  and  $V_{max}=3$ ,  $P_b=0$ .

Figure-4 shows the progress of the system in the space and time for the both roads in different regions. Black and red dots represent respectively long and short vehicles.



**Figure-4.** Space-time configurations for different values of  $\rho$ . Black and red dots present long and short vehicles respectively, with:  $F_L=0.2$ ,  $F_{sh}=0.8$  and  $V_{max}=3$ ,  $P_b=0$ .

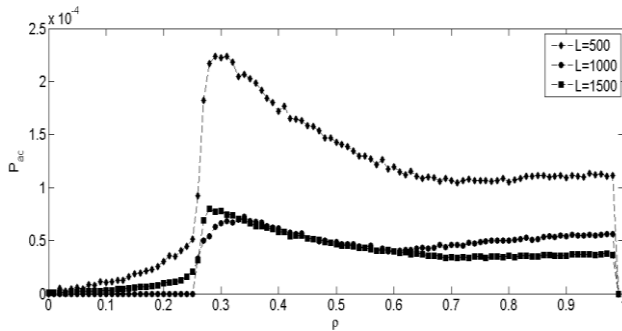
To study the impact of  $F_L$  on the flow and on the accident probability, the fundamental diagram and the accident probability for different values of  $F_L$  were plotted (Figure-5). In the regions I and II, the flow decreases with the increasing of  $F_L$ . This behavior is explained by the fact that the long vehicles move slowly contrary to short one. In the free flow regime, a linear relationship occurs between the decreasing flow and the critical density  $\rho_c$  and the increase of  $F_L$ . In the region III, the flow keeps approximately the same values. In the case of the accident probability (Figure-5(b)), the increase of  $F_L$  makes the region I broadest (increase of  $\rho_c$ :  $\rho_{c0.2} < \rho_{c0.4} < \rho_{c0.6}$ ) and increases  $P_{ac}$ . This behavior is explained by the fact that the intersection becomes busy for more time, which favors the occurrence of accidents.



**Figure-5.** Fundamental diagram (a) and car accident probability (b) versus density for different values of  $F_L$  with  $V_{max}=3$  and  $P_b=0$ .



To better understand the relationship between  $P_{ac}$  and the roads length  $L$ , we have plotted the  $P_{ac}$  versus  $L$  (Figure-6). We can see clearly this dependence. Indeed, with the increase of the roads length  $L$  by a factor of 50%, the accident probability decreased correspondingly by a factor of 170%. This means that car accidents happen more frequently as the length of the roads  $L$  decreases.

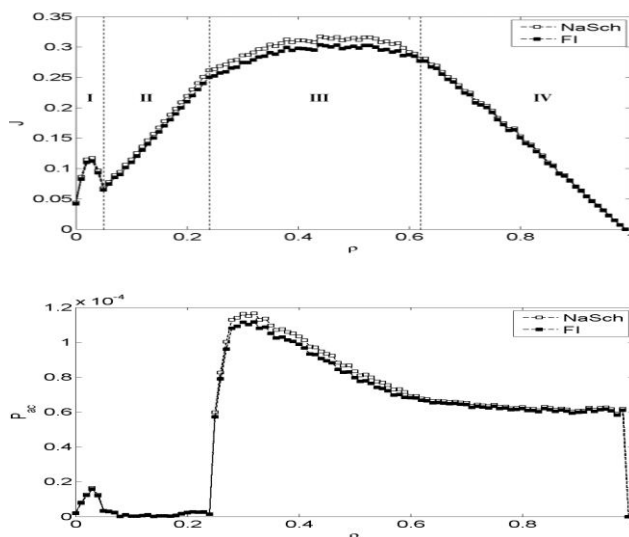


**Figure-6.** Car accident probability  $P_{ac}$  versus  $\rho$  for various values of  $L$ . ( $F_L=0.2$ ,  $V_{max}=3$  and  $P_b=0$ ).

The comparison of FI and NaSch [13] models shows better results in the case of FI model. Indeed, based on the fundamental diagram and the accident probability plots, we observe that the critical density  $\rho_c$  in FI model is bigger than NaSch model [13] one. Moreover, the  $P_{ac}$  in this model is smaller than NaSch one. Furthermore, the fundamental diagram shows a small plateau region in FI model compared to NaSch model. From these results, we conclude that FI model is better than NaSch [13].

### 3.2 Mixture velocities

In this section, the impact of mixture maximum velocities on the fundamental diagram and the accident probability ( $P_{ac}$ ) is investigated. Figure-7 represents the flow and the accident probability as a function of density.



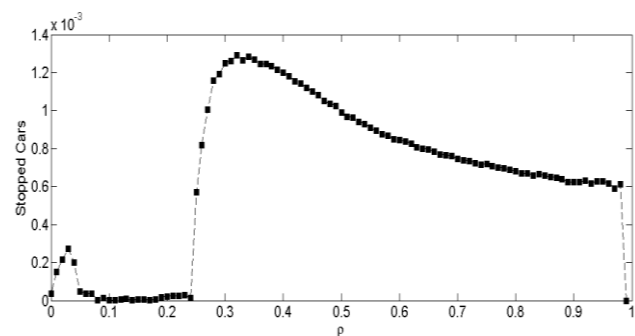
**Figure-7.** FI vs NaSch model: The flow ( $J$ ) and Car accident probability  $P_{ac}$  versus density ( $V_{fmax}=5$  and  $F_f=0.8$ ) and ( $V_{smax}=1$  and  $F_s=0.2$ ), with  $P_b=0$ .

Beside the three topologies founded [12], the fundamental diagram shows a plateau region in the intermediate densities because of the intersection. The possibility of overtaking decreases because of the priority rules at intersection which leads to a more reduction of the flow in the first topology.

Compared with the case of mixture lengths, the  $P_{ac}$  shows four regions, where we have a presence of collisions in the free flow regime ( $\rho < \rho_{c1}$ ). This region of collisions corresponds respectively to the first topology in the fundamental diagram and in the accident probability. In the second region,  $P_{ac}$  vanishes until a critical density  $\rho_{c2}$  and it suddenly increases to reach its maximum, then starts decreasing. For higher densities, the collision probability remains almost constant and it vanishes when  $\rho=1$ . The comparison between FI and NaSch model shows that both models have the same behavior. Moreover, the accident probability in FI model is less than NaSch one.

To better clarify the appearance of collision in the free flow regime ( $\rho < \rho_{c1}$ ), we have calculated the velocities before and after application of the priority rules and we have determined the stopped vehicles because of the intersection.

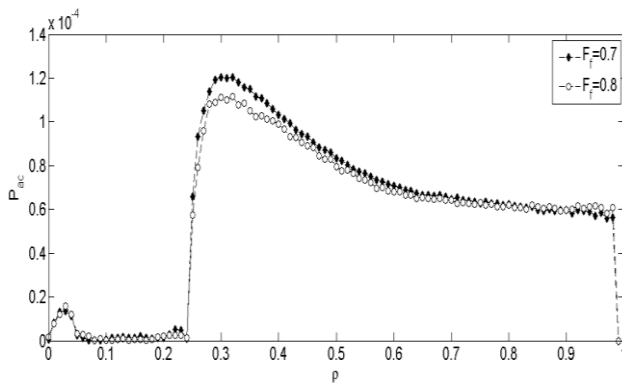
Figure-8 illustrates the blocked cars versus density. There are some cars that can be blocked because of the intersection even if in the free flow regime. This means that the collision can occur if drivers do not respect the priority rules. Now, we will analyze the effect of the fraction of fast vehicles ( $F_f$ ) on the accident probability  $P_{ac}$ .



**Figure-8.** Stopped car versus density ( $V_{fmax}=5$  and  $F_f=0.8$ ) and ( $V_{smax}=1$  and  $F_s=0.2$ ), with  $P_b=0$ .

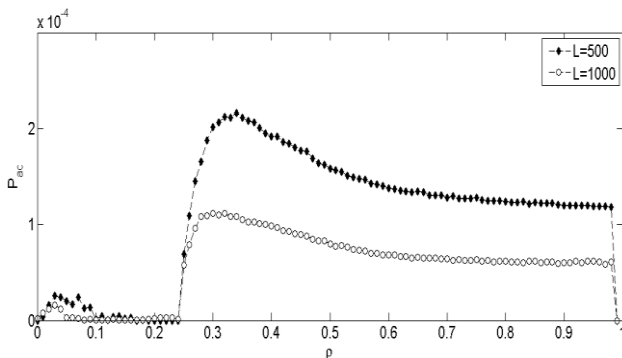
Figure-9 illustrates the accident probability versus density for two values of  $F_f$ . In the first region, the  $P_{ac}$  increases with decreasing of  $F_f$ . Decreasing  $F_f$  by a factor of 12.5%, makes increasing of  $P_{ac}$  by a factor of 8%. After that, this probability vanishes. In the intermediate densities, the increase of  $F_f$  reduces the accident probability in this region. Finally,  $F_f$  has no effect, where we have almost the same comportment of  $P_{ac}$  in both cases because of the high densities.





**Figure-9.** Car accident probability  $P_{ac}$  versus density for various values of  $F_f$  ( $V_{fmax}=5$  and  $F_f=0.8$ ) and ( $V_{smax}=1$  and  $F_s=0.2$ ), with  $P_b=0$ .

The relation between the accident probability and roads length variation is shown in Figure-10.

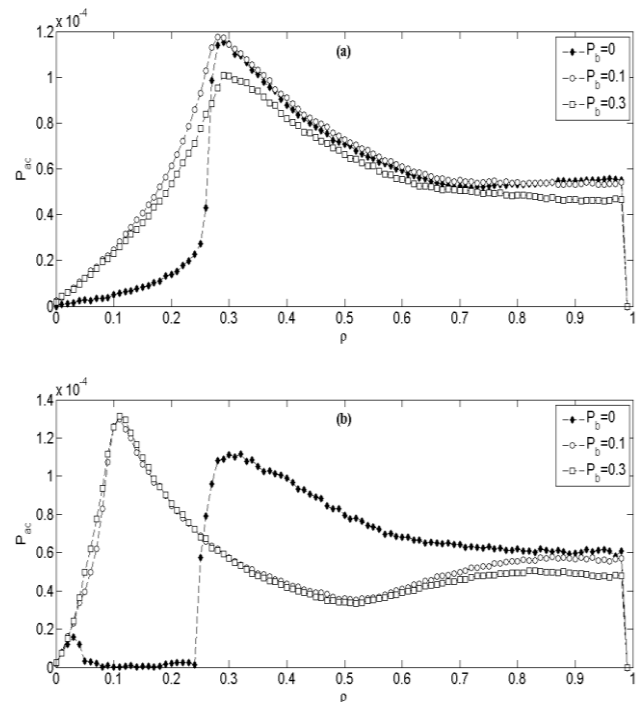


**Figure-10.** Car accident probability  $P_{ac}$  versus density for various values of  $L$ . ( $V_{fmax}=5$  and  $F_f=0.8$ ) and ( $V_{smax}=1$  and  $F_s=0.2$ ), with  $P_b=0$ .

The length variation in mixture velocities has the same effect as that of mixture lengths, where the accident probability increases with decreasing of roads length. Figure-10 shows that when we increase the road length  $L$  by a factor of 100%, the  $P_{ac}$  decreases by the same factor. Contrary to the mixture length case, the comparison of FI and NaSch [13] models don't show a significant improvement in term of  $P_{ac}$  and the Flow. In this case, the critical density  $\rho_c$  is slightly bigger than NaSch [13] one. Moreover, the  $P_{ac}$  is smaller than NaSch [13] one. The plateau region in the fundamental diagram is smaller than NaSch [13] model.

### 3.3 Impact of $P_b$ on $P_{ac}$ in the Both Cases

To study the impact of braking probability we plotted the probability of occurrence of car accident as a function of density for different values of  $P_b$  (Figures 11(a) and 11(b)).



**Figure-11.** Car accident probability  $P_{ac}$  versus density for different values of  $P_b$ . (a): the case of the mixture lengths, with ( $F_L=0.2$ ,  $F_{sh}=0.8$  and  $V_{max}=3$ ); (b): the case of mixture maximum velocities, with ( $V_{fmax}=5$  and  $F_f=0.8$ ) and ( $V_{smax}=1$  and  $F_s=0.2$ ).

The obtained results prove that the  $P_b$  has the same effect on  $P_{ac}$  in the both cases (mixtures lengths and mixtures velocities). For the values of  $P_b \neq 0$ , the accidents start from the free flow regime ( $\rho_c = 0$ ) in contrast to the deterministic case. Moreover,  $P_b$  has almost completely changed the behavior of  $P_{ac}$  in the mixture velocities case, because the braking probability influences on the overtaking which leads to this change. In the jamming phase, the  $P_{ac}$  decreases with the increasing of  $P_b$  because in this region the density is higher, and vehicles require more time to reach the intersection and when we increase  $P_b$  they require even more time which leads to the decrease of  $P_{ac}$ .

## 4. CONCLUSIONS

In this work, the probability of the occurrence of car accidents at the intersection of two roads without respect the priority rules, has been presented. The impact of mixture lengths on the flow and accident probability ( $P_{ac}$ ) has been explored. Based on the results of the mixture length, we remark that the accidents start to occur above a critical density  $\rho_c$ , and the increase of the fraction of long vehicles  $F_L$  increases the risk of collisions when  $\rho > \rho_c$ .

In the case of the mixture velocities, the accidents probability can be occurring even in the free flow regime. Furthermore, the increase of the fraction of fast vehicles ( $F_f$ ) leads to the decrease of  $P_{ac}$ . Moreover, the decrease of the roads length provokes an increase of the accident probability ( $P_{ac}$ ) in both the cases.



Comparing FI and NaSch models, it was found that the critical density and the accident probability are respectively smaller than NaSch ones in the case of mixture lengths. In the case of the mixture velocities, we observe that both models have the same behaviour. This study proved that FI model leads to better results than the NaSch one.

## REFERENCES

- [1] S. Wolfram. 1986. Theory and Applications of Cellular Automata. World Scientific Publishing. pp. 485-557.
- [2] M. Fukui and Y. Ishibashi. 1996. J. Phys. Soc. Jpn. 65, pp. 1868-1870.
- [3] M. Fukui and Y. Ishibashi. 1997. J. Phys. Soc. Jpn. 66, pp. 3683-3684.
- [4] K. Nagel and M. Schreckenberg. 1992. J. Phys. I, 2, 2221.
- [5] R. Barlovic, L. Santen, A. Schadschneider and M. Schreckenberg. 1998. Euro. Phys. J. B, 5, pp. 793-800.
- [6] K. Nishinari and D. J. 2000. Phys. A, 33, 7709.
- [7] S. Sakai K. Nishinari and S. Iida. 2006. A New Stochastic Cellular Automaton Model on Traffic Flow and Its Jamming Phase Transition. Journal of Physics A: Mathematical and General. 39, 15327.
- [8] S. Kokubo, J. Tanimoto and A. Hagishima. 2011. A New Cellular Automata Model Including a Decelerating Damping Effect to Reproduce Kerner's Three-Phase Theory. Physica A: Statistical Mechanics and Its Applications. 390, pp. 561-568.
- [9] O. Biham, A. A. Middleton and D. A. Levine. 1992. Phys. Rev. A 46, R6124.
- [10] T. Nagatani. 1993. J. Phys. A: Math. Gen. 26, 6625.
- [11] Benyoussef H., Chakib and H. Ez-Zahraouy. 2003. Phys. Rev. E 68, 026129.
- [12] H. Ez-Zahraouy, K. Jetto and A. Benyoussef. 2006. Chin. J. Phys. 44, 486.
- [13] R. Marzoug, H. Ez-Zahraouy and A. Benyoussef. 2015. Inter. J. M. Phys. C. 26(1).