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# A GENERALIZED MODEL FOR VISCOSITY AS A FUNCTION OF SHEAR RATE

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**ABSTRACT**

A new model for viscosity as a function of shear rate has been derived. This model contains three characteristics of fluid, which are zero-shear rate viscosity, infinite-shear rate viscosity, and non-Newtonian indication of fluid. With few assumptions, the model can be rearranged to model fluid either behaves as shear-thinning, shear-thickening, or Newtonian behaviour. In this study, it was implemented on a variety of drilling fluids by correlating the viscosity to the shear rate. By comparing to other well-known models, it shows a prediction with high R-squared value (≥ 0.99) better than power-law and Herschel-Bulkley.

**Keywords:** shear-thinning, shear-thickening, viscosity-shear rate model, non-Newtonian.

## 1. INTRODUCTION

Shear stress is in positive correlation with shear rate. In a linear relationship, a fluid under measurement is known to have a Newtonian behaviour. A slight deviation from this observation is considered non-Newtonian behaviour, and it can be categorized as either pseudoplastic (shear-thinning), dilatant (shear-thickening), or plastic. Shear stress is resulted from a ratio of force (e.g., Newton) over an area (e.g., $m^2$), and shear rate is, alternatively, known as the rate of deformation or velocity gradient (e.g., 1/s). A ratio on the amount of shear stress over a shear stress-generated velocity gradient is termed as viscosity. Hence, viscosity is an indication on the amount of resistant to flow. A fluid that requires a large amount of shear stress to cause a similar shear rate is more viscous than fluid which requires less.

In non-Newtonian fluid, viscosity changes with shear rate. When viscosity reduces with increasing shear rate, it is an indication of pseudoplastic behaviour, and when it increases with increasing shear rate, it is a dilatant behaviour. Viscosity itself is not limited to the influence of shear rate. Temperature in particular has a greater influence on viscosity than shear rate, as reported by Wan Nik et al. [1] on vegetable oils. In reality, viscosity is also affected by shear-time, composition, moisture, pressure, oil degradation, molecular weight, density, etc. [2-10]. As the current work is only intended to correlate between viscosity and shear rate, all the other parameters are not considered in this study.

There are many models in the literature addressing shear-thinning fluid. They are power-law, Cross, Carreau, Herschel-Bulkley, Al-Zahrani, Casson, Sisko, etc. [11]. The models are listed in sequence as followings, except Casson and Sisko:

$$\eta = K_P \gamma^{n_P - 1} \tag{1}$$

$$\eta = \eta_{\infty,\gamma} + \frac{\eta_{o,\gamma} - \eta_{\infty,\gamma}}{1 + (\alpha_c \gamma)^m} \tag{2}$$

$$\eta = \eta_{\infty,\gamma} + \frac{\eta_{o,\gamma} - \eta_{\infty,\gamma}}{\left[1 + (\lambda_c \gamma)^2\right]^N} \tag{3}$$

$$\eta = K_H \gamma^{n_H - 1} + \eta_{\infty,\gamma} \tag{4}$$

$$\eta = \frac{B}{\gamma}\left[\left(\frac{\gamma + A}{A}\right)^{n_a} - 1\right]^{(1/n_a)} \tag{5}$$

where: $\eta$, dynamic viscosity (Pa.s); $\gamma$, shear rate (1/s); $K_P$ and $K_H$, consistency index (Pa.s$^n$); $n_P$ and $n_H$, flow behaviour index (dimensionless); $\eta_{\infty,\gamma}$, viscosity at infinite-shear rate (Pa.s); $\eta_{o,\gamma}$, viscosity at zero-shear rate (Pa.s); $n_a$, $m$ and $N$, constant (dimensionless); $\lambda_c$ and $\alpha_c$, characteristic relaxation time (s); $A$, constant (1/s); and $B$, constant (Pa).

As claimed by Al-Zahrani[11], the mathematics of the many empirical models is difficult and rarely justifiable even based on the simplest problems. For this reason, a generalized model was developed by Al-Zahrani in correlating between shear stress and shear rate, for which, it can be transformed into viscosity versus shear rate. Al-Zahrani model has shown to model a variety of drilling fluids better than both the power-law and Herschel-Bulkley model. His work was extended to model the effect of temperature and wax percentage on crude oil, in addition to the effect of shear rate alone [2]. However, in the aspect of shear rate at a very low shear rate region, or to be known zero-shear rate viscosity, this model has not considered.

Therefore, the objective of the current study is to: (1) derive a generalized model for viscosity as a function of shear rate, which is able to estimate viscosity value at extreme conditions (i.e., viscosity at zero- and infinite-

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shear rate) and also to indicate the presence of oil non-Newtonian behaviour (i.e., flow behaviour index); and (2) compare proposed model with other published models.

**Derivation of viscosity versus shear rate**

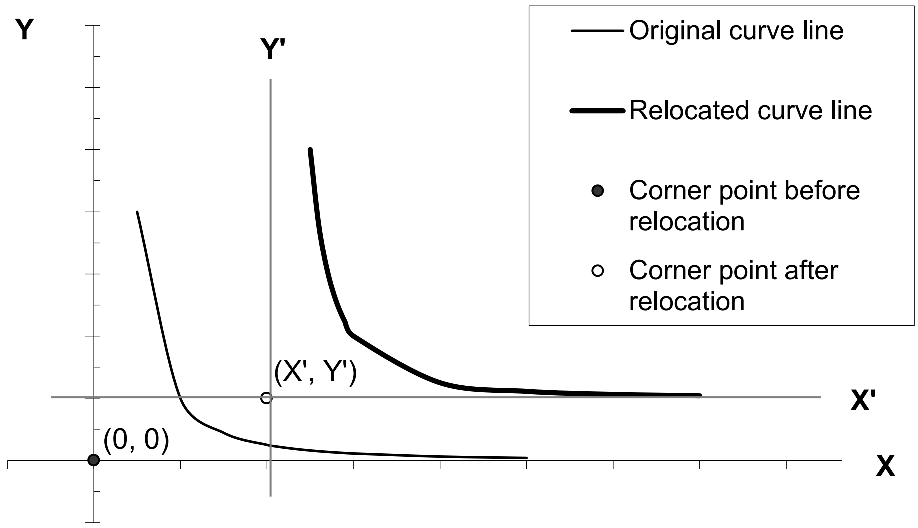


**Figure-1.** Graphical representation for both original curve line and relocated curve line.

As illustrated in Figure-1, viscosity ($Y$) decreases when shear rate ($X$) increased, with an unknown exponent ($n_G$),

$$Y \propto \frac{1}{X^{n_G}} \tag{6}$$

Thus,

$$Y = \frac{a}{X^{n_G}} \tag{7}$$

By allowing the relocation of corner point from $X = 0$ and $Y = 0$ to $X'$ and $Y'$, it becomes,

$$Y - Y' = \frac{a}{(X - X')^{n_G}} \tag{8}$$

When $X \rightarrow \infty$, $Y$ approximates infinite value ($Y_\infty$) as well, and hence,

$$Y - Y_\infty = \frac{a}{(X - X')^{n_G}} \tag{9}$$

When $X = 0$, $Y$ represents a value at zero-$X$ axis value. This value gives $Y_o$, and therefore, the equation can be rearranged into the following form,

$$a = (-X')^{n_G}(Y_o - Y_\infty) \tag{10}$$

By substituting Equation (10) into Equation (9), it gives the following,

$$Y = Y_\infty + \frac{Y_o - Y_\infty}{\left(-\frac{X}{X'} + 1\right)^{n_G}} \tag{11}$$

By letting $X = \gamma$, $Y = \eta$, and $\gamma' = -1$, therefore Equation (11) can be rewritten as following,

$$\eta = \eta^G_{\infty,\gamma} + \frac{\eta^G_{o,\gamma} - \eta^G_{\infty,\gamma}}{(\gamma + 1)^{n_G}} \tag{12}$$

Equation (12) is similar to both Cross and Carreau model. It estimates viscosity at zero-shear rate ($\eta^G_{o,\gamma}$), viscosity at infinite-shear rate ($\eta^G_{\infty,\gamma}$), and power constant ($n_G$), except characteristic relaxation time (i.e., $\lambda_c$,Carreau and $\alpha_c$, Cross). The relaxation time is an indication of Newtonian region right before the fluid started to behave as pseudoplastic in a continue increase of shear rate application.

Equation (12) has ability to model fluid characteristic that behaves other from shear-thinning behaviour. It can be extended to model shear-thickening and Newtonian fluid. For instance:

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(1) When $\eta_{\infty,\gamma}^{G} = 0$ and $n_G = -n_G^{'}$, Equation (12) can be transformed to model shear-thickening fluid,

$$\eta = \eta_o(\gamma + 1)^{n_G^{'}} \quad (13)$$

(2) When $n_G = 0$, it can be transformed to account for Newtonian fluid as following,

$$\eta = \eta_o \quad (14)$$

## 2. MATERIALS AND METHODS

The current modelling effort was solely based on published experimental data. The viscosity data at different shear rate were compiled from Al-Zahrani[11] research article that was based on a variety of drilling fluids. There were gel/waterfluid at 6, 8, 10, 12 and 28% (by weight) of Wyoming bentonite clay in water, and also, gelex water-based drilling fluid.

The shear rate dependence of drilling fluids was model with power-law, Cross, Carreau, Herschel-Bulkley, and Al-Zahrani as shown in Eqs. (1) to (5). Similar viscosity data were also curve-fitted by the current proposed model as shown in Eq. (12). All the model constants (i.e., $\eta_{o,\gamma}^{G}$, $\eta_{\infty,\gamma}^{G}$, $n_G$, $K_H$, $A$, $B$, etc.) were estimated from the experimental data by using nonlinear regression analysis in Mathematica software. Graphical illustration and mathematical calculation were carried out in Microsoft Excel.

## 3. RESULTS AND DISCUSSIONS

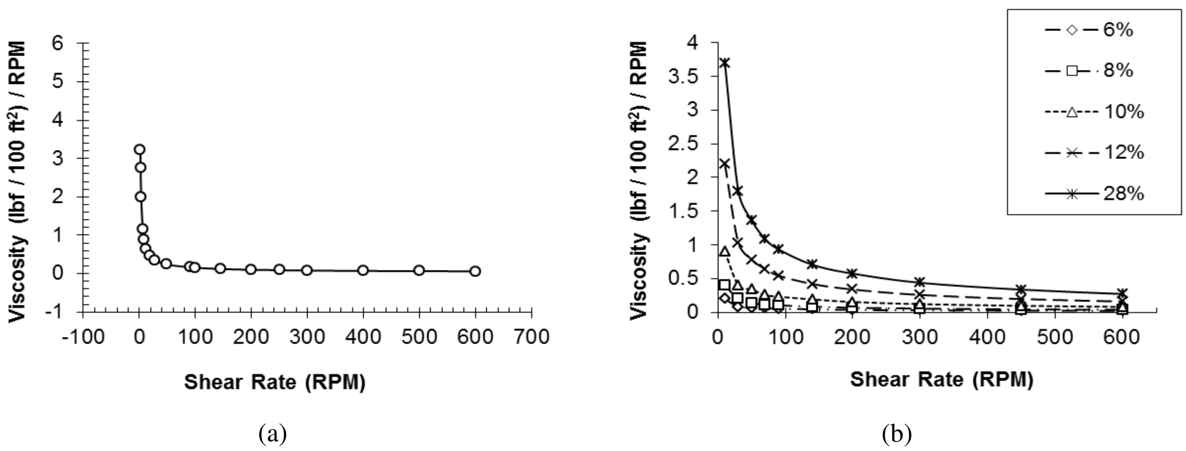



**Figure-2.** Viscosity as a function of shear rate: (a) Gelex water-based drilling fluid; and (b) Percentage bentonite in (gel) drilling fluids.Data retrieved from Al-Zahrani [11].

Figure-2(a) shows the viscosity of gelex water-based drilling fluid reduces when the shear rate of the fluid increased. A similar trend is also observed for bentonite in drilling fluids at different percentages (Figure-2(b)). It shows that a higher percentage of bentonite gives a higher viscosity value.

**Table-1.**Estimated constants for different drilling fluids by proposed model.

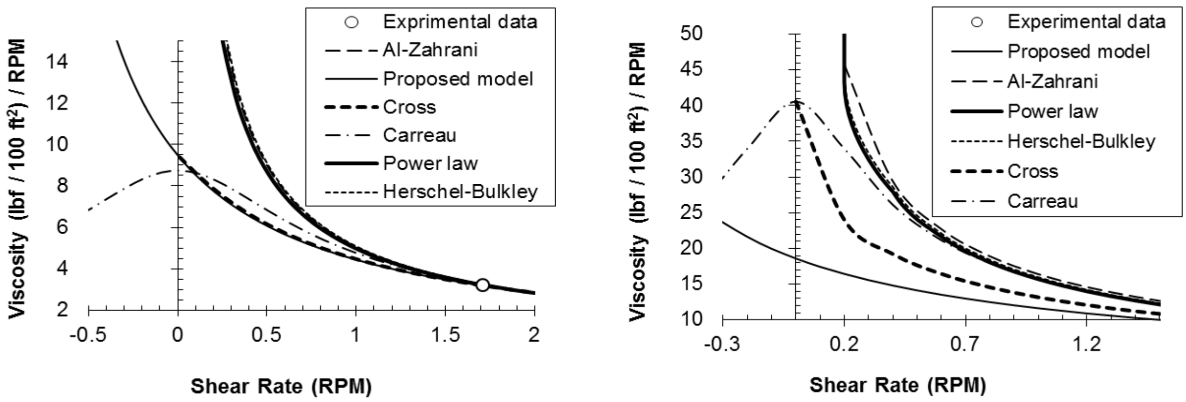
| | $\eta_{\infty,\gamma}^{G}$ | $\eta_{o,\gamma}^{G}$ | $n_G$ | R-squared |
|---|---|---|---|---|
| % weight of bentonite in drilling fluid | | | | |
| 6 | 0.0223 | 1.41 | 0.860 | 0.99 |
| 8 | 0.0217 | 2.20 | 0.731 | 1.00 |
| 10 | 0.0558 | 5.23 | 0.759 | 1.00 |
| 12 | 0.0571 | 11.8 | 0.711 | 1.00 |
| 28 | 0.0552 | 18.6 | 0.680 | 1.00 |
| Gelex water-based drilling fluid | | | | |
| | 0.0816 | 9.49 | 1.11 | 1.00 |

**Table-2.** Estimated constants for different drilling fluids by Al-Zahrani model.

| | $A$ | $B$ | $n_a$ | R-squared |
|---|---|---|---|---|
| % weight of bentonite in drilling fluid | | | | |
| 6 | 1 739 | 15.9 | 1.96 | 1.00 |
| 8 | 26 561 | 90.7 | 2.18 | 1.00 |
| 10 | 84 883 | 309 | 2.26 | 1.00 |
| 12 | 140 788 | 504 | 2.73 | 1.00 |
| 28 | 169 675 | 898 | 2.76 | 1.00 |
| Gelex water-based drilling fluid | | | | |
| | 157 | 8.69 | 6.00 | 1.00 |

The results of different drilling fluids that were curve-fitted to proposed model and Al-Zahrani model are tabulated in Tables 1 and 2. With high R-squared value of at least 0.99, it shows that the estimation of viscosity value from the proposed model is very near to the Al-Zahrani model estimation. In addition to the ability of the proposed model to provide accurate estimation, it contained more useful indication on fluid properties, for instance, viscosity at zero-shear rate, viscosity at infinite-shear rate, and power-law constant. Al-Zahrani model contained three constants of $A$, $B$, and $n_a$, for which these constants have not been described its relation to the fluid it models.

An important aspect that has not been considered in Al-Zahrani model is the viscosity at zero-shear rate. A similar problem is also noticed for power-law and Herschel-Bulkley model. As illustrated in Figures 3(a) and (b), only Cross, Carreau and the current proposed model have an intercept at viscosity-axis. However, the presence of characteristic relaxation time (i.e., $\lambda_c$ and $\alpha_c$) in Cross and Carreau model is likely to generate unreliable estimation of viscosity at zero-shear rate, due to either or a combination of: (1) limited availability of experimental data at low shear rate region to justify the begins of Newtonian region; and/or (2) nonexistence on the Newtonian region at very low shear rate region. Thus, fluid that has an insufficient experimental data at low shear rate region, and without the presence of Newtonian region at the low shear rate region, Equation (12) can be used to model viscosity at zero-shear rate ($\eta^G_{o,\gamma}$).




(a) (b)

**Figure-3.** Viscosity at very low shear rate region of: (a) Gelex water-based drilling fluid; and (b) 28% bentonite in (gel) drilling fluids.

## 4. CONCLUSIONS

The proposed model was found to model the experimental data with high R-squared value. The estimated constants from the model contained much more useful information than the present Al-Zahrani model, particularly at very low shear rate region. It was found that Al-Zahrani model is an improvement from Herschel-Bulkley model, and therefore, the current proposed model is an improvement to Al-Zahrani and the other models. This model can be extended from shear-thinning modelling to shear-thickening and Newtonian fluid, with some simple assumptions. Hence, this model can be used for viscosity prediction purposes, such as, to aid a proper design of pumping or injection of fluids, and other similar application.

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## REFERENCES

[1] Wan Nik, W.B., Ani, F.N., Masjuki, H.H., EngGiap, S.G., 2005. Rheology of bio-edible oils according to several rheological models and its potential as hydraulic fluid. Industrial Crops and Products 22: 249-255.

[2] Al-Zahrani, S.M., Al-Fariss, T.F., 1998. A general model for the viscosity of waxy oils. Chemical Engineering and Processing: Process Intensification. 37(5): 433-437.

[3] HaeRyu C., Chan Bae, Y., Hwan Lee, S., Yi S., Park Y.H. 1998. Rheological properties of hollow sphere loaded polymer melts. Polymer. 39(25): 6293-6299.

[4] Akdogan, H., McHugh, T.H., 1999. Twin screw extrusion of peach puree: Rheological properties and product characteristics. Journal of Food Processing and Preservation. 23(4): 285-305.

[5] Akinoso R., Igbeka J.C. 2006. Modelling of oil expression from sesame seed.Journal of Food Science and Technology. 43(6): 612-614.

[6] Armelin E., Marti M., Rude E., Labanda J., Llorens J., Aleman C. 2006. A simple model to describe the thixotropic behavior of paints. Progress in organic coatings.57: 229-235.

[7] Novak L.T. 2006. Entity-based eyring - NRTL viscosity model for mixtures containing oils and bitumens. Industrial and Engineering Chemistry Research. 45(21): 7329-7335.

[8] Benedito J., García-Pérez J.V., Carmen Dobarganes M., Mulet A. 2007. Rapid evaluation of frying oil degradation using ultrasonic technology. Food Research International.40 (3): 406-414.

[9] Ceriani R., Gonçalves C.B., Rabelo J., Caruso M., Cunha A.C.C., Cavaleri F.W., Batista E.A.C., Meirelles A.J.A. 2007. Group contribution model for predicting viscosity of fatty compounds. Journal of Chemical and Engineering Data. 52(3): 965-972.

[10] Ceriani R., Paiva F.R., Gonçalves C.B., Batista E.A.C., Meirelles A.J.A. 2008. Densities and viscosities of vegetable oils of nutritional value.Journal of Chemical and Engineering Data. 53(8): 1846-1853.

[11] Al-Zahrani S.M. 1997. A generalized rheological model for shear thinning fluids. Journal of Petroleum Science and Engineering.17: 211-215.