PERIODIC PULSE TESTING IN POROUS MEDIA WITH NONLINEAR PERMEABILITY DEPENDENCE VERSUS PRESSURE

Alexander Gavrilov, Marat Ovchinnikov and Galiya Kushtanova
Kazan Federal University, Institute of Physics, Kremlyovskaya, Kazan, Russian Federation
E-Mail: kushtanova-galya@mail.ru

ABSTRACT
The filtration pressure waves propagations in nonlinear media are considered for two models pressure dependences of permeability. The differences of the amplitudes and phases hydrodynamic periodic pulse testing signals are calculated for nonlinear and analogous linear situations. These differences can reach unity and tens of percents. Obtained results can be used in correction the values of the filtration parameters.

Keywords: porous and fractured porous media, nonlinear filtrationof fluids, periodic pulse testing.

1. INTRODUCTION
Hydrodynamic study of reservoir filtration parameters by non-stationary methods is an important part of the oil production process [1]. Nonlinear filtration of liquids and gases in porous media has been the subject of many papers [2-6]. However, there is no systematic study of the nonlinearity effects in porous media under hydrodynamic perturbations propagation in the form of filtration harmonic pressure waves [7-8]. So, the propagation of nonlinear filtration waves in saturated porous media remains poorly understood. At the same time, nonlinearity definitely takes place with the pressure changes in the fluid-saturated formations. In general, many parameters depend on pressure: porosity, density, viscosity, and the permeability dependence of pressure have a greater influence on the filtration process, especially in fractured media where the permeability of fractures media can vary in a wide range [10-11].

An example of a natural full-scale experiment with the realization of filtration pressure waves for a fractured-pore collector is shown in Figure-1.

Here, the primary disturbances of the production rate were set on the injection well in the form of quasi-rectangular pulses. Corresponding pressure changes (Pₙ) with respect to the initial one appeared on this disturbing well and in fact the harmonic pressure changes was observed on the reacting well. The reservoir itself «produces Fourier analysis», since the high-frequency pressure oscillations are effectively attenuated in the media under consideration.

It should be noted that effects of nonlinearity are attributed often to the effect of formation heterogeneity, for instance, in experiments such as build-up pressure test. This leads to the need for correction of the filtration parameters values. In practice it is also important to monitor the limiting values of wells downhole pressures to avoid the irreversible deformations of the reservoir.

Therefore, it is of interest to see if the form of the hydrodynamic signal changes under nonlinearity conditions. Is it possible to experimentally detect and estimate the degree of nonlinearity of the medium? How much will differ the calculated values of the filtration parameters near wellbore and in interwellspace in the framework of linear and nonlinear models?

2. MODEL
It is customary to use the Darcy linear law for the describing the filtration in porous media

$$ \vec{v} = -\frac{k}{\mu} \ \text{grad} \ P $$

and the diffusivity equation in the form

$$ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r P \right) \right) = \frac{\partial}{\partial t} \left( P + \frac{\partial P}{\partial t} \right). $$

Here P - pressure, k – permeability, w – filtration velocity, \( \mu \) – viscosity, \( \chi \) – diffusivity coefficient.

Using the Barenblatt hypothesis [12] and the Warren and Ruth model [13-14] for fractured porous media we can enter the time dimension parameters \( \tau \) and write the diffusivity equation in the form

$$ \frac{Z}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \left( P + \frac{\partial P}{\partial t} \right) \right) = \frac{\partial}{\partial \tau} \left( P + \frac{\partial P}{\partial \tau} \right). $$
It should be noted that in natural conditions the together action of both in homogeneity and nonlinearity is often encountered. Therefore, in this work an attempt is made to estimate the influence of nonlinearity only for homogeneous layers by a computational experiment. It is suggested that only the permeability depends essentially on the pressure and the dependence of the remaining parameters on the pressure is insignificant. The dependence of the permeability on pressure was specified in the form of two models

\[ k(p) = k_0 \exp(\alpha(p - p_0)) \]  

and

\[ k(p) = k_0 \frac{1}{2} \left[ 1 + \tanh(\beta(p - p_0)) \right] \]  

The second model is more suitable for describing the permeability changes in fracture porous media. Here \( \alpha \) and \( \beta \) are parameters of media.

Then equations (2) and (3) will be written as

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( \chi(k(p)) r \frac{\partial P}{\partial r} \right) = \frac{\partial P}{\partial t}, \]  

and

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( \chi(k(p)) r \frac{\partial}{\partial r} \left( P + \tau \frac{\partial P}{\partial t} \right) \right) = \frac{\partial}{\partial t} \left( P + \tau \frac{\partial P}{\partial t} \right) \]  

The computational experiment was a solution of equations (6) and (7) for the case of single-phase radial filtration. The vertical well rate \( q(t) \) varies according to the harmonic law \( q(t) = q_0 \sin(\omega t) = q_0 \sin(2\pi t / T) \), where \( \omega \) is the frequency and \( T \) is the period. Using harmonic pressure wave’s procedure in practice, it is possible to calculate the transmissibility coefficient and other hydraulic conductivity parameters. To do this, we need to calculate the amplitudes and phases of the pressure change functions with Fourier analysis.

3. RESULTS

We used a model of a homogeneous and isotropic formation with a thickness \( h = 1 \) m and a permeability \( k_0 = 1 \) D (darcy) saturated with a homogeneous liquid with viscosity \( \mu = 1 \) mPa·s as a basic variant for numerical calculations. The unperturbed values of the diffusivity coefficient and the transmissibility coefficient was \( \chi = 2.5 \) m²/s and \( \epsilon = kh/\mu = 10^7 \) D·m/Pa·s, respectively. The initial pressure in the formation was constant and equal to 10 MPa. The conditional observation point was at a distance \( r = 308 \) m.

The calculation of the pressure in the reservoir was carried out according to the implicit four-point scheme for the case of the porous reservoir and the five-point scheme for the fractured-porous reservoir. An example of calculations is shown in Figure-2 in the form of the pressure changes in the time in the disturbing and reacting wells by setting three groups of periods of the flow rate change at the disturbing well with 6, 12 and 24 hours, respectively.

In this example and all subsequent ones the baseline values of the reservoir parameters were chosen to be the same for correct comparisons of changes in pressure dynamics caused only by the influence of the nonlinearity.

![Figure-2. Periodic pressure changes on disturbing (P₁) and reactive (P₂) wells in computational experiments.](image_url)

The absolute values of the pressure also have significance in the nonlinearity conditions. Obviously, a decrease in permeability will lead to a decrease in the diffusivity, the form of the hydrodynamic signals will change, and near the disturbing well where the pressure changes will be maximum.

Let us consider the relative changes in the pressure amplitudes under nonlinearity of the type (4) near the disturbing well for various values of the parameter \( \alpha \). Calculations have shown that the relative amplitude variation dependence is well described by the power function

\[ \frac{P_{\text{cr}} - P_{\text{c}}}{P_{\text{c}}} = a_1 \alpha^{b_1}, \]  

where \( a_1 = 8.90 \) and \( b_1 = 1.157 \) (\( R^2 > 0.99 \)) for the considered model. Here \( P_{\text{c}} \) is the value of the calculated pressure amplitude for \( \alpha = 0 \), and \( P_{\text{cr}} \) is for nonzero \( \alpha \). There is no observed dependence \( \frac{P_{\text{cr}} - P_{\text{c}}}{P_{\text{c}}} \) on the perturbation period \( T \).

Let us further consider the relative change in the pressure amplitudes under conditions of nonlinearity of the type (4) in the disturbing well for different values of the parameter \( \alpha \) in case of the fractured-porous formation model (7). In this case, the dependence of the relative amplitude variation is well described also by the power function

\[ \frac{P_{\text{cr}} - P_{\text{c}}}{P_{\text{c}}} = a_2 \alpha^{b_2}, \]  

where \( a_2 = 13.79 \) and \( b_2 = 1.212 \) (\( R^2 > 0.99 \)).

For model (6), let us now consider the dependence of the pressure amplitudes ratio on the
perturbing and reacting wells \( P_r / P_c \) at different periods of impact on the formation and different values of the nonlinearity parameter \( \alpha \) of type (4). The results are shown in Figure-3. An increase in the values of this ratio with an increase in the period is observed, however, the differences associated with an increase in the values of the parameter \( \alpha \) are small.

\[
\frac{P_r}{P_c} = (c_{41} \alpha + c_{42})T + (c_{43} \alpha + c_{44}),
\]

where \( c_{41} = 1.173 \cdot 10^{-6}, c_{42} = 2.881 \cdot 10^{-7}, c_{43} = 1.299 \cdot 10^{-2}, c_{44} = 3.104 \cdot 10^{-3} (R^2 > 0.99) \). The insignificant deviations of the amplitude ratio in question mean that the influence of nonlinearity in the main part of the formation is small and is concentrated mainly near the disturbing well.

In Figure-4 the relationship between \( P_r / P_c \) for a model of a fractured-porous formation with a nonlinearity of the type (5) is shown.

An increase in this ratio is observed; however, as in the previous case the differences between the curves with increasing parameter \( \beta \) are small.

The dependence of the amplitude change \( P_r / P_c \) on the period \( T \) and the parameter \( \beta \) is well described by a function

\[
\frac{P_r}{P_c} = (c_{51} \beta + c_{52}) \left( \frac{T}{T_0} \right)^2 + (c_{53} \beta + c_{54}).
\]

where \( c_{51} = 2.90 \cdot 10^{-3}, c_{52} = 6.565 \cdot 10^{-4}, c_{53} = 1.273 \cdot 10^{-4}, c_{54} = 7.982 \cdot 10^{-3} (R^2 > 0.99) \). Insignificant deviations in the ratio of amplitudes means that the influence of nonlinearity in the main part of the formation is small and but concentrated near the disturbing well.

The analogous dependence of the amplitude variation for the model (7) and nonlinearity of the type (5) is well described by the function

\[
\frac{P_r}{P_c} = (c_{61} \beta + c_{62}) \ln(T) + (c_{63} \beta + c_{64}).
\]

where \( c_{61} = 2.055, c_{62} = 0.119, c_{63} = 5.443, c_{64} = 0.245 (R^2 = 0.84) \).

The calculated values of the phase differences between the flow rate and the pressure in the disturbing well and the reacting well are insignificant although they can be differentiated. This is shown in Figure-5 for nonlinearity of the type (4) of the model (6). Here \( \phi_{qq}, \phi_{qq} \) is the phase difference between the flow rate and the pressure in the disturbing and reacting wells, respectively.

We see that the difference is a \( 10^{-1}-10^{-3} \) of the radians. This is a small but still detectable quantity.

5. DISCUSSIONS AND CONCLUSIONS

The presence of a nonlinearity of the form (4) or (5) leads to a decrease in the calculated effective permeability and hydraulic conductivity in the wellbore space when the pressure falls. Here the more the flow rate the more the pressure drop, the greater the permeability and, as a consequence, the transmissibility and diffusivity in the wellbore space falls.
A quantitative estimate shows that the differences between the values of the calculated parameters in the linear and nonlinear models are unity and tens of percent. This fact can definitely serve as a diagnostic sign for determining the degree of nonlinearity.

At the same time, in the far zone from the disturbing well the changes in the transmissibility and the diffusivity coefficients are relatively small because the small deviation of the absolute value of the pressure from the initial one at large distances from the disturbing well and the influence of nonlinearity decreases.

Thus, the influence of nonlinearity can be significant and this requires an adjustment of the values of the obtained filtration parameters of the seams especially for near-well space research.

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REFERENCES


