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CALIBRATION OF THE PARAMETERS OF THE KELVIN MODEL GENERALIZED IN THE NON-CONFINED STATIC FLUENCY TEST FOR ASFALTIC MIXTURES BY OPTIMIZATION

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ABSTRACT

Whit this article we show the calibration of parameters of a generalized kelvin model adapted for the simulation of specific data for the unconfined static creep test applied in the viscoelastic characterization of asphalt mixtures. The calibration is performed using the adaptation of an optimization system along with the placement method where the latter serves as the determination of the parameters of the Dirichlet series representative of the generalized kelvin model. The results obtained are very satisfactory and validate the correct formulation and easy implementation of the calibration system in worksheet.

Keywords: parameter calibration, creep, asphaltmix, optimization, dirichlet series.

1. INTRODUCTION

In the characterization of asphalt mixtures, the creep test is used as a fundamental test for the determination of viscoelastic behavior. To explain the experimental context indirectly involved in the subject of the article is explained below the essay in question.

The unconfined static creep test consists of an axial load test applied for a certain time and its respective discharge in the English school (recovery test); it is really a test of recovery. The dynamic machine (Figure-1]) addresses the creep test under the following conditions:

- Vertical applied compression: 100 kPa.
- Creep time: 3600 s.
- Recovery time: 1800 s.
- Temperature: 40 ° C.
- Sample diameter: 100 mm.
- Sample height of 65 mm.
- Number of samples for a series: 6.
- Preferred samples compacted by kneading in a rotary compressor.

The data that can be obtained from the test are:

- Max. Deformation (ε_{max}).
- Total vertical deformation at the end of the load (for 3600 s).
- Permanent deformation (ε_{perm}). Remaining deformation recorded at discharge end (for 5400 sec).
- Elastic deformation ($\varepsilon_{elast} = \varepsilon_{max} - \varepsilon_{perm}$).
- Creep rate. Flow rate measured at the last load of 1200 seconds. ($V\varepsilon = (\varepsilon 3600 - \varepsilon 2400) / 1200$ s).
- Stiffness module. Ratio of applied vertical tension and maximum deformation recorded to 3600 s.

2. METHODS

Below are presented, the function of formal model fluency Kelvin generalized, the Dirichlet series and method of placement.

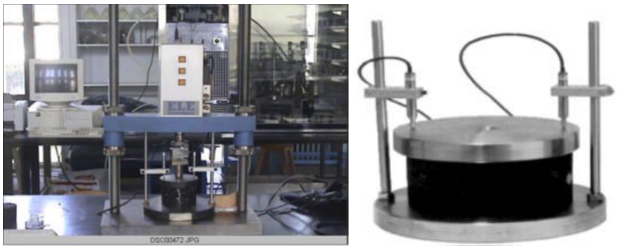

Figure-1. Assembly for creep test
Source: Article (Viscoelasticity, [8])

Figure-2 shows an example of a complete test, however, currently the American school does the so-called flow time test, with the difference that the test is only 100 seconds long and only performs the loading phase , which is strictly a fluency test, where the proof reports the creep compli-ance function.

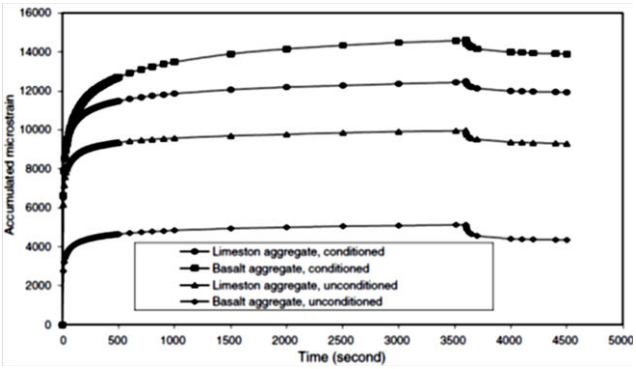

Figure-2. Curves obtained from the fluency test (English school).

2.1 Kelvin generalized model

The generalized Kelvin model can be mechanically represented as shown in Figure-3,

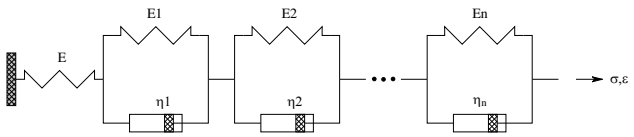

Figure-3. Mechanical model of Kelvin.

The model consists of the serial association of a spring element and n elements of Kelvin-Voigt. The deformation is then a sum that is described using Equation (1).

$$f(t) = \left[\frac{1}{E} + \sum_{i=1}^{n} \frac{1}{E_i}\left(1 - e^{\left(\frac{E_i}{\eta_i}t\right)}\right)\right] y(t), \quad (1)$$

The discrete spectrum of characteristic time in creep is described using Equation (2),

$$[\tau_f]_i = \frac{\eta_i}{E_i}, \quad (2)$$

Evaluating at t=0 and infinity we can obtained Equations (3) and (4),

$$f(0) = \frac{1}{E}. \quad (3)$$

$$f(\infty) = \frac{1}{E} + \sum_{i=1}^{i=n} \frac{1}{E_i}. \quad (4)$$

2.2 Dirichlet series and placement method (Schapery 1961, 1974)

The pavement design is based on mobile loads of short duration. The fluency function, therefore, is relevant and can be written as shown in equation (5),

$$fd(t) = \left[\frac{1}{E_0} + \sum_{i=1}^{i=n} \frac{1}{E_i}\left(1 - e^{\left(\frac{E_i}{\eta_i}t\right)}\right)\right] \quad (5)$$

The discrete time characteristic of creep is shown in Equation (6),

$$fd(t) = \sum_{i=1}^{i=n} FD_i e^{\left(-\frac{t}{[\tau_f]_i}\right)} \quad (6)$$

By comparing equations 5 and 6, with $[\tau_f]_n = \infty$, we find Equations (7) and (8),

$$FD_i = -\frac{1}{E_i} \quad (7)$$

$$FD_n = \frac{1}{E_0} + \sum_{i=1}^{i=n} \frac{1}{E_i} \quad (8)$$

The placement method is an approximate method for calculation and the updated response to a predetermined number of tabulated data pairs. Instead of determining two parameters E_i *and* $[\tau_f]_i$, several values of $[\tau_f]_i$ are taken arbitrarily, and the corresponding E_i is determined by the solution of a system of simultaneous equations.

The creep equation (5) is determined in practice based on the results of a creep test. 1000 creep tests were measured with at least 11 different times (*0.001, 0.003, 0.01, 0.1, 0.3, 1,3, 10, 30 and 100s*) on Federal Highway recommendation (FHWA, 1978) to cover all possible range of interest. This range of 0.001 to 100 s takes into account loads with short and long durations and also the Creep Compliance with the temperature.

Since moving loads are usually of short duration, typically characteristic low times are taken which an example can be taken as 0.01, 0.03, 0.1, 1, 10, 30 and ∞ seconds. If the creep function is specified for these seven times, the coefficients FD1 through FD7 can be determined from Equation (6) by the solution of a system of 7 simultaneous equations. If the fluency function is specified by 11 times then there are 11 equations and 7 unknowns (FD1, ..., FD7), so that to reduce the 11 equations to 7 equations multiply both sides of the system by the matrix transpose initial size 7 x 11, as shown in Equation (9),

$$\begin{bmatrix} e^{\left(-\frac{t_1}{[\tau_f]_1}\right)} & \cdots & e^{\left(-\frac{t_{11}}{[\tau_f]_1}\right)} \\ \vdots & 7x11\ Matriz & \vdots \\ e^{\left(-\frac{t_1}{[\tau_f]_7}\right)} & \cdots & e^{\left(-\frac{t_{11}}{[\tau_f]_7}\right)} \end{bmatrix} \begin{bmatrix} e^{\left(-\frac{t_1}{[\tau_f]_1}\right)} & \cdots & e^{\left(-\frac{t_1}{[\tau_f]_7}\right)} \\ \vdots & 11x7\ Matriz & \vdots \\ e^{\left(-\frac{t_{11}}{[\tau_f]_1}\right)} & \cdots & e^{\left(-\frac{t_{11}}{[\tau_f]_7}\right)} \end{bmatrix} \begin{Bmatrix} FD_1 \\ \vdots \\ FD_7 \end{Bmatrix} \begin{pmatrix} x \\ x \\ x \end{pmatrix} =$$
$$\begin{bmatrix} e^{\left(-\frac{t_1}{[\tau_f]_1}\right)} & \cdots & e^{\left(-\frac{t_{11}}{[\tau_f]_1}\right)} \\ \vdots & 7x11\ Matriz & \vdots \\ e^{\left(-\frac{t_1}{[\tau_f]_7}\right)} & \cdots & e^{\left(-\frac{t_{11}}{[\tau_f]_7}\right)} \end{bmatrix} \begin{Bmatrix} f_l(t_1) \\ \vdots \\ f_l(t_{11}) \end{Bmatrix} \quad (9)$$

The coefficients of the series (FD1, ..., FD7) and using equations (7) and (8) were used to determine the E_i and equation (2) for the determination of ηi. In practice, it is possible that there are oscillations of the curve caused by negative values of E_i, which they do not have a physical sense and in this sense historically have been numerous investigations, but as a contribution to the article, the authors present a restricted optimization scheme, adapted for the study of a case in the article.

3. RESULTS AND DISCUSSIONS

3.1 Presentation of the experimental data

Data used as a starting point correspond to a FLOW TIME test, these data come directly from a KENLAYER program file, and this is a modeling package that assists the design of pavements by theory of using elastic, viscoelastic and non-linear constitutive models for each of the layers.

The sample tested has dimensions of 100 mm in diameter and 66 mm in height.

Table-1. Data of creep and deformation function in time.

Point	t (s)	σ (kPa)	f_l (t) (1/kPa)	ε (mm/mm)
1	1	100	2.669E-05	2.67E-03
2	3	100	3.863E-05	3.86E-03
3	5	100	4.172E-05	4.17E-03
4	10	100	4.450E-05	4.45E-03
5	15	100	4.612E-05	4.61E-03
6	20	100	4.728E-05	4.73E-03
7	30	100	4.874E-05	4.87E-03
8	40	100	4.983E-05	4.98E-03
9	50	100	5.068E-05	5.07E-03
10	60	100	5.129E-05	5.13E-03
11	70	100	5.191E-05	5.19E-03
12	80	100	5.238E-05	5.24E-03
13	90	100	5.303E-05	5.30E-03
14	100	100	5.342E-05	5.34E-03

3.2 Determination of Dirichlet series parameters by the placement method

Calculations are performed in Excel using functions for matrix multiplication, transpose of an array and inverse matrix, in addition to conventional for different functions.

The following step-by-step describes the algorithm followed (not all described in detail):

a) Choosing the number of terms in the Dirichlet series: There was no defined criteria in the literature on how to choose how many terms to use (except the FHWA recommendation mentioned in the previous item), however, and taking into account that the placement method is an essentially least-squares method it may be possible that considerable numerical error propagations appear by increasing the size of matrices (in a first approach 8 terms were taken for series, however, oscillations appeared and E_i with negative values, a situation recognized in the literature), so a reasonable number can be 5, immediately defining the number of characteristic times to be used that would be 5, and which in turn defines that the generalized Kelvin model will have 4 KV elements, in short: n=4, Nk-v=4.

b) Choosing of characteristic times: One can choose to take as reference the own times of the data, preferably using the lower times that is where the faster changes in deformation have. It was the first step taken by the authors in an arbitrary way, however, later, one can see the consequences of this decision. The times will be: $[\tau_f]_1 = 1$, $[\tau_f]_2 = 3$, $[\tau_f]_3 = 5$, $[\tau_f]_4 = 20$, $[\tau_f]_5 = 1E^{+302}$ is necessary to reproduce the asymptotic fluency).

c) Evaluating the Matrix of Exponential Functions and its Transpose: The exponential matrix in table 2 is displayed and its transpose is not placed since it is a simple operation to change rows to columns.

Table-2. Matrix of exponentials using equation (9).

	τ				
t (s)	**1**	**3**	**5**	**20**	**1E+302**
1	3,68E-01	7,17E-01	8,19E-01	9,51E-01	1,00
3	4,98E-02	3,68E-01	5,49E-01	8,61E-01	1,00
5	6,74E-03	1,89E-01	3,68E-01	7,79E-01	1,00
10	4,54E-05	3,57E-02	1,35E-01	6,07E-01	1,00
15	3,06E-07	6,74E-03	4,98E-02	4,72E-01	1,00
20	2,06E-09	1,27E-03	1,83E-02	3,68E-01	1,00
30	9,36E-14	4,54E-05	2,48E-03	2,23E-01	1,00
40	4,25E-18	1,62E-06	3,35E-04	1,35E-01	1,00
50	1,93E-22	5,78E-08	4,54E-05	8,21E-02	1,00
60	8,76E-27	2,06E-09	6,14E-06	4,98E-02	1,00
70	3,98E-31	7,35E-11	8,32E-07	3,02E-02	1,00
80	1,80E-35	2,62E-12	1,13E-07	1,83E-02	1,00
90	8,19E-40	9,36E-14	1,52E-08	1,11E-02	1,00
100	3,72E-44	3,34E-15	2,06E-09	6,74E-03	1,00

d) Calculation of the square matrix and vector of the coefficients according to equation (9): Having the exponential matrix of Table-2 and its transpose, calculate the square matrix and vector of the independent terms, using equation (9).
e) Calculation of the inverse matrix and its product with the vector of independent terms: Determination of the coefficients FD_i by the solution of the matrix Equation (9).
f) Determination of parameters of the function of the formal fluency using the coefficients FD_i of the series Dirichlet: Equations (7), (8) and (2).

Table-3. Parameters of the formal fluency function.

Módulo [E]	**Valor**	**Coeficiente** [η]	**Valor**
E1	54321,472	η1	54321,472
E2	34794,912	η2	104384,736
E3	-48267,495	η3	-241337,474
E4	61380,530	η4	1227610,604
E0	101166,008	---	---

As we can see a negative modulus E3 and a negative coefficient η3 that has no physical sense, next to the data graph and the Dirichlet series curve show remarkable differences, as can be seen in Figure-4. As the added value the next step consists of an optimization scheme proposed by the authors for the calibration of the series, in order to obtain an ideal fit without negative parameters.

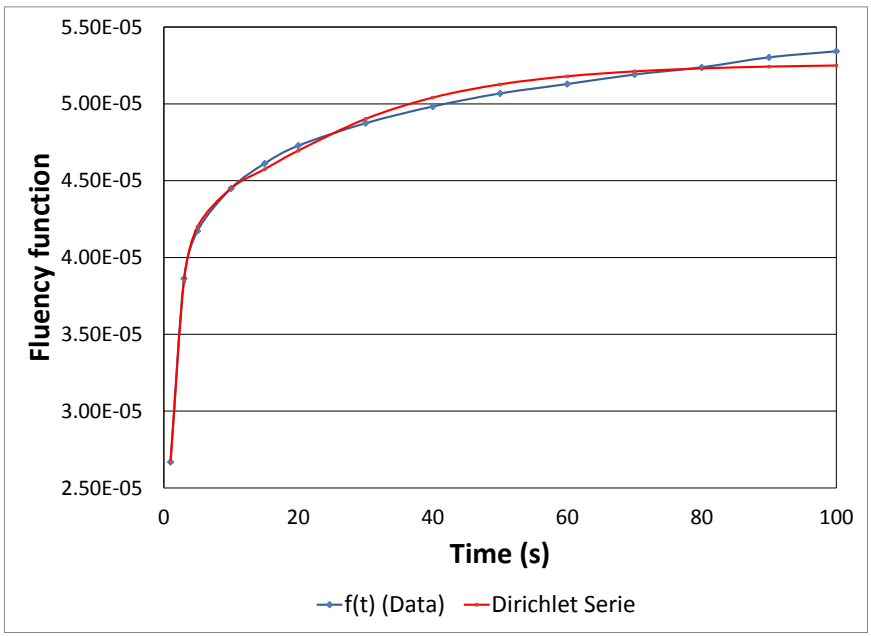

Figure-4. Generalized Kelvin model with oscillations.

3.3 Optimization scheme adapted for the calibration of the Dirichlet series and obtaining the parameters of the generalized Kelvin model

To obtain the ideal TAUs the optimization scheme appears that minimizes the distance between the data and the model using the Euclidean norm of the vector difference between the CREEP test data and the model is presented in the following lines of expression(10),

Minimizing: $\sqrt{\sum_{i=1}^{i=n}\left(fl_i - fd(t_i)\right)^2}$

Decision variables: $[\tau_f]_2, [\tau_f]_3, [\tau_f]_4$.
$(Fixedvalues [\tau_f]_1 = 1, [\tau_f]_5 = 1E302)$
Subjected to:
$[\tau_f]_2 > [\tau_f]_1$
$[\tau_f]_3 > [\tau_f]_2$
$[\tau_f]_4 > [\tau_f]_3$
$1 < [\tau_f]_4, [\tau_f]_3, [\tau_f]_3, [\tau_f]_3 < 99$
$FD_1, FD_2, FD_3, FD_4 < 0$
Where:
fl_i is the value of the laboratory creep function for point i.
$fd(t_i)$ is the value of Dirichlet Series for point i (evaluated for t_i).
$[\tau_f]_i$ is the characteristic creep time i.
FD_i is the Dirichlet Series Coefficient i. (10)

For implementation, additional calculations are performed to determine the Euclidean norm of the vector difference between the data and the prediction of the series (objective function to be minimized), as well as the statistical r2 to know the goodness of fit (The table corresponds to the calculation under initial TAUs Listed below).

Figure-5 shows the optimization scheme implemented in solver, where the evolutionary algorithm module is used as a non-linear problem solver. It is taken as TAUs (remembering that TAUs 1 and 5 are fixed, since they correspond to the ends) of starting: The times will be: $[\tau_f]_1 = 1$, $[\tau_f]_2 = 3$, $[\tau_f]_3 = 5$, $[\tau_f]_4 = 20$, $[\tau_f]_5 = 1E^{+302}$ (infinity - this time is necessary to reproduce the asymptotic fluency).

Table-4 below shows the data of optimization scheme and Table-5 presents the adjustment obtained.

From the results of Table-5 we can see that medium value of fluency function is: fl_{media}=4,687E-05 and its Euclidean Norm is [fl(t)-fd(t)]=1,860E-07. Also we can see that the model represents 99.995% of the variability of the data, for an excellent fit that confirms the benefits found in the literature on the Dirichlet or Prony series as a more versatile alternative for the representation of fluency in asphalt mixtures according to with the coefficient of determination.

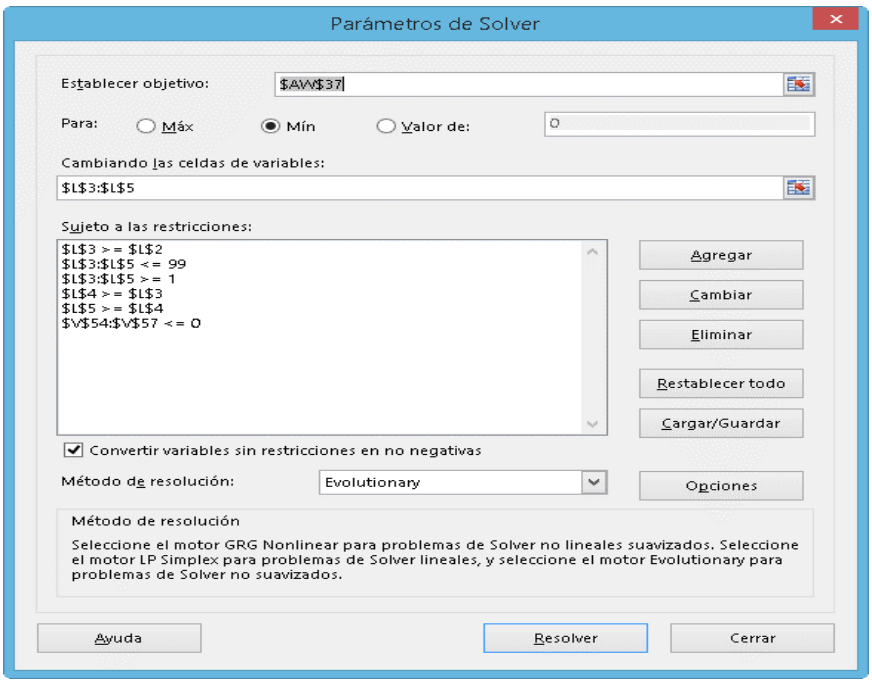

Figure-5. Optimization scheme in Excel Solver (Interface in Spanish language).

Table-4. Optimization scheme data.

Módulo [E]	**Valor**	**Coeficiente** [η]	**Valor**	**TAUs** [τ]	**Valor**	**FDi**	**Valor**
E1	32270,742	η1	32270,742	τ1	1,0000000	FD1	-3,10E-05
E2	209626,416	η2	1068191,396	τ2	5,09569079	FD2	-4,77E-06
E3	220348,377	η3	2695069,634	τ3	12,2309484	FD3	-4,54E-06
E4	90992,278	η4	8293885,953	τ4	91,1493385	FD4	-1,10E-05
E0	173301,708	---	---	τ5	101166,008	FD5	5,71E-05

Table-5. Optimization scheme data.

fl(t)	**t(s)**	**fd(t)**	**fl(t)-fd(t)**	**[fl(t)-fd(t)]^2**	**[f(t)-fl$_{media}$]^2**
2,669E-05	1	2,66845E-05	5,452E-09	2,973E-17	4,07348E-10
3,863E-05	3	3,8681E-05	-5,095E-08	2,596E-15	6,79447E-11
4,172E-05	5	4,16409E-05	7,905E-08	6,249E-15	2,65519E-11
4,450E-05	10	4,45334E-05	-3,337E-08	1,113E-15	5,63045E-12
4,612E-05	15	4,61518E-05	-3,178E-08	1,010E-15	5,66794E-13
4,728E-05	20	4,72532E-05	2,681E-08	7,185E-16	1,65765E-13
4,874E-05	30	4,87451E-05	-5,136E-09	2,638E-17	3,48622E-12
4,983E-05	40	4,97963E-05	3,370E-08	1,136E-15	8,74469E-12
5,068E-05	50	5,06305E-05	4,952E-08	2,452E-15	1,44943E-11
5,129E-05	60	5,1333E-05	-4,301E-08	1,850E-15	1,95112E-11
5,191E-05	70	5,1943E-05	-3,304E-08	1,092E-15	2,53728E-11
5,238E-05	80	5,24811E-05	-1,011E-07	1,023E-14	3,03286E-11
5,303E-05	90	5,29595E-05	7,046E-08	4,965E-15	3,79104E-11
5,342E-05	100	5,33866E-05	3,343E-08	1,117E-15	4,28651E-11

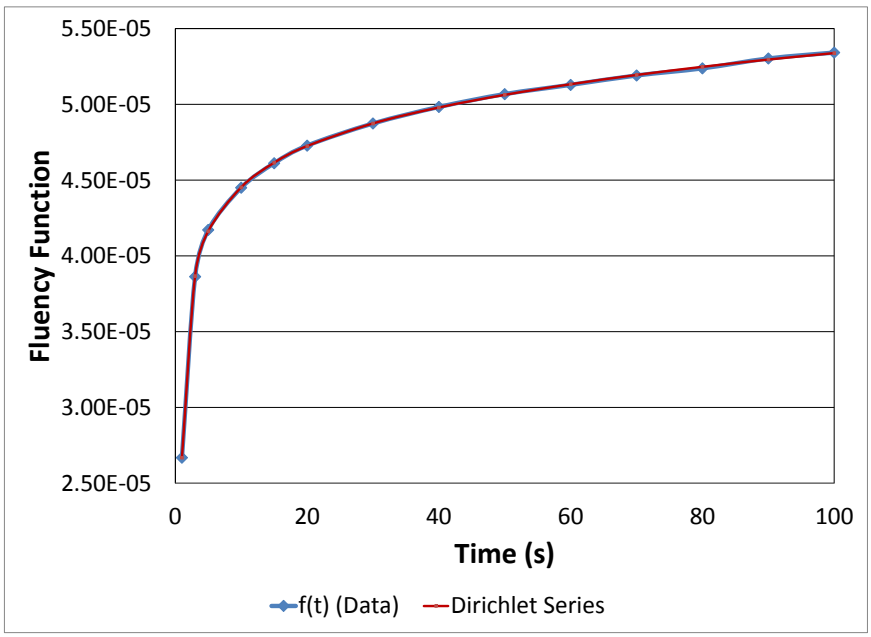

Figure-6. Dirichlet Series curve calibrated by optimization.

Figure-6 graphically illustrates the excellent final fit of the series.

4. CONCLUSIONS

The physical-mechanical tests that are currently made for asphalt and asphalt mixtures cover basic and advanced aspects of their behavior against monotonic loads and dynamic loads under the viscoelastic rheological model. It can be said that, both in theory and in experimentation, there are enough elements for the rigorous study of the behavior of asphalt and asphalt mixtures in different states of application and in different environmental conditions and age.

In the study of the formal mathematical model interesting aspects such as the historical circumstances of the difficulties for the calibration of the parameters of the generalized Kelvin mathematical model and its homologous Dirichlet series were found. The article has a parameter calibration approach inspired by its implementation in three elements: the Shapery Placement Method (1961), the General Theory of Restricted Optimization and Evolutionary Algorithms. The calibration scheme is successful and convergent for the case study and shows to be easy to use in its implementation using a simple, high-availability tool such as the Excel spreadsheet.

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