PRESSURE DERIVATIVE ANALYSIS FOR HORIZONTAL WELLS IN SHALE RESERVOIRS UNDER TRILINEAR FLOW CONDITIONS

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ABSTRACT
Unconventional shale reservoirs appear as a solution to the depletion of conventional reserves, however, their ultra low permeability, requires hydraulic fracturing that helps improving the fluid flow towards the well. The design and creation of these fractures is complex. Knowing their properties, and the reservoir’s, as well, is of great importance for field management. This study presents a practical methodology for well test interpretation in shale reservoirs using the analytical trilinear flow model, which describes a system consisting of a horizontal well with multiple fractures in extremely low permeability reservoirs. Analytical expressions were developed based upon unique features found on the pressure and pressure derivative curves for the determination of fracture conductivity (extremely low permeability reservoirs). The design and field management. This study presents a practical methodology for well test interpretation in shale reservoirs using the trilinear flow model. The design and field management. This study presents a practical methodology for well test interpretation in shale reservoirs using the trilinear flow model.

Keywords: unconventional resources, shale reservoirs, hydraulic fractures, trilinear flow model, TDS technique.

1. INTRODUCTION
In the continuous search to increase hydrocarbon production under the high global energy demand, the world has currently turned its attention to unconventional deposits. There several types of unconventional resources; one of them refers to shales. What makes shales their unconventional character is their ultra-low permeability, that can oscillate between $10^{-6}$ and $10^{-12}$ darcies, but generally is given in the order of nanodarcies ($10^{-9}$ darcies) and possess small pore diameters that oscillate between 1 and 10 µm, explaining why they commonly are considered as impermeable. Exploitation of these types of reservoirs requires long horizontal wells and the implementation of a good hydraulic fracturing job, with the solid purpose of creating extensive networks of artificial fractures, to increase the well productivity index and to optimize the recovery of reserves.


The trilinear flow model, Brown et al. (2009), divides the reservoir into three zones with average properties: internal reservoir, external reservoir and hydraulic fractures. Their solution of the diffusivity equations was obtained in the Laplace domain space separately for each zone and the calculated results in the Laplace domain were reversed again in the time domain using the Stehfest algorithm (1970). Al-Hussainy and Ramey (1966) and Al-Hussainy, Ramey and Crawford (1966) introduced the idea of gas pseudopressure in order to give an analytical solution to gas deposits. Because the internal reservoir is a naturally fractured medium, the dual porosity model of Kazemi (1969), De-Swaan (1976) and Serra, Reynolds and Raghavan (1983) form the basis of the trilinear flow model. In addition, the model assumes that the productive life of a fractured horizontal well depends on the volume of hydraulic fractures, Raghavan, Chen and Agarwal (1997).

The behavior of fluids within a reservoir composed of very tight matrix, hydraulic fractures and natural fractures (usually induced at the time of hydraulic fracturing), in a horizontal well, can be interpreted using an analytical model known as trilinear flow model, formulated and verified by Brown (2009) and Brown et al. (2009). As implied by its name, the trilinear flow model assumes three linear flows during the productive life of the well and its interpretation was based on a series of previous work on the behavior of fluids in porous media, as well as in vertical and horizontal wells in naturally and/or hydraulically fractured reservoirs.

This research presents an extension of the TDS Technique, Tiab (1995), for well pressure test interpretation based upon the trilinear flow model presented by Brown et al. (2009), with which new equations were developed to characterize formations with multiple fractures in horizontal wells for both oil and gas reservoirs. The equations have been formulated for three different cases depending of the flow regime types. The
developed equations were successfully tested with synthetic examples.

Figure-1. Schematic representation of the trilinear flow model with three continuous flow regions for a horizontal multi-fractured well, Brown, M. (2009).

2. MATHEMATICAL MODEL

The trilinear flow model formulated by Brown (2009) and Brown et al. (2009) assumes linear flow in three different reservoir zones: internal deposit, external deposit and hydraulic fractures, as shown in Figure-1. The remaining model assumptions can be found in Brown et al. (2009).

The analytical dimensionless pressure solution for a horizontal multi-fractured well, Brown et al. (2009), is given by:

$$P_D = (P_{FD})_{x=0} = \frac{\pi}{s C_D D_F} \tanh\left(\sqrt{\alpha_F}\right)$$

Other model-related parameters are provided in Appendix A. The dimensionless oil pressure and pressure derivative are given by:

$$P_D = \frac{k_f h}{141.2 q_{Fi,\omega} B \mu} (P_i - P_{wf})$$

$$t_D \cdot P_D' = \frac{k_f h}{141.2 q_{Fi,\omega} B \mu} (t^* \Delta P')$$

For gas Wells,

$$m(P) = \frac{k_f h}{1422 q_{Fi,\omega} B \mu} [m(P) - m(P_{wf})]$$

$$t_D \cdot m(P)' = \frac{k_f h}{1422.52 q_{Fi,\omega} B \mu} (t^* \Delta m(P')$$

$m(P)$ is a gas pseudopressure function, Al-Hussainy et al. (1966); Al-Hussainy and Ramey (1966); given by:

$$m(P) = 2 \int_0^P \frac{dP'}{\mu Z}$$

The dimensionless time is:

$$t_D = \frac{0.0002637 k_f t}{(\phi c_i)_{F} \mu X_f}$$

Some dimensionless variables related to reservoir geometry and fractures are:

$$x_D = \frac{x}{x_f}$$

$$y_D = \frac{y}{x_f}$$

$$w_D = \frac{w}{x_f}$$

3. WELL PRESSURE BEHAVIOR

As mentioned before, three cases can take place:

Case 1 considers three characteristic flow regimes (Figure-2). A bilinear flow regime is observed at early time. This occurs once the transient wave detects two simultaneous linear flow regimes in the fractures and inner reservoir, respectively. A second linear flow (first linear) appears in response the fluid flow from the natural fracture network to the hydraulic fractures. Finally, a (first) pseudosteady-state period is because the matrixes do not feed the natural fractures. Generally, the first linear flow can be seen when the $\lambda \omega$ product is les or equal to 1, or simply when $\lambda$ and $\omega$ take very low values which ensures the flow incapability from matrix to induced natural fractures. Because $\lambda$ is matrix permeability independent, this case normally takes place when the mentioned permeability value is ultra-low enough (<1x10^-8) so the pressure in the test changes linearly with time.

Case 2 involves the acting of five flow regimes (Figure-2): Bilinear, first linear and first pseudosteady-state period of case 1; then, second linear and second pseudosteady-state. Followed by the first pseudosteady-state period, and after a considerable response time, the pressure changes linearly with time and the second linear flow can be observed; once the transient wave reaches the hydraulic fracture limits or the internal reservoir boundary, the second pseudosteady-state period is developed. There exists a certain relationship between $\omega - \lambda$ and the second linear flow regime. Frequently, when $\omega$ has large values, the second linear flow becomes visible in the pressure derivative versus dimensionless time log-log plot; when $\omega$ grows, the duration of the second linear flow also does so, and this, in turn, occurs late, as $\lambda$ becomes small.
In Case 3 is possible to differentiate three flow regimes (Figure-2): at early times, a bilinear flow is observed, later a linear flow, the same that we have called the second linear flow in case 2, followed by a pseudosteady or second pseudosteady state. This case is given due to the simultaneous movement of matrix fluids to natural fracture and natural fracture to hydraulic fracture, under conditions of large $\lambda$ and $\omega$ values.

![Figure-2. Dimensionless pressure derivative behavior for a reservoir under trilinear flow conditions with case 1, case 2 or case 3.](image)

3.1. TDS TECHNIQUE FOR OIL SHALE RESERVOIRS

Bilinear flow regime. The governing equation developed for this flow regime is:

$$ (t_D * P_D \text{'} )_{BL} = 1.6289 \left( \frac{x_F h_f \rho_f \mu}{k_f w_f} \right)^{0.5} t_{D,1}^{0.25} $$

Replacing the dimensionless quantities, Equations 3 and 7 into Equation 11 and solving for the fracture conductivity, $k_{FW}$:

$$ k_{FW} = 122.0157 h_f \rho_f \mu \left[ \frac{t_{D,1}}{h_f(t * \Delta P)_{BL}} \right]^{0.5} $$

First linear flow regime: The governing equation for the first linear flow is:

$$ (t_D * P_D \text{'} )_{L1} = \frac{1}{1.1360} t_{D,1}^{0.5} $$

Replacing dimensionless variables and solving for the internal reservoir permeability, $k_i$:

$$ k_i = 4.0739 \left[ \frac{q_{F,\infty} B}{x_F h_f (t * \Delta P)_{L1}} \right]^{2} $$

First pseudosteady-state period: The governing equation for the first pseudosteady-state is:

$$ (t_D * P_D \text{'} )_{pss1} = \frac{\pi x_F}{2 y_e} t_{D,1}^{0.5} $$

Again replacing the dimensionless parameter and solving for half-fracture length, $x_F$:

$$ x_F = \frac{1}{17.0966} \frac{q_{F,\infty} B t_{D,1}}{y_e h_f (t * \Delta P')_{pss1}} $$

Second linear flow: Its governing equation is:

$$ (t_D * P_D \text{'} )_{L2} = \frac{1}{2.2783} \frac{h_f}{y_e h_f (t * \Delta P')_{L2}} k_i (\phi_c) \left[ \frac{k_m (\phi_c)}{k_i (\phi_c)} \right]^{0.5} t_{D,2}^{0.5} $$

The matrix permeability, $k_m$, is solved for once the dimensionless quantities are replaced in the above expression:

$$ k_m = 1.0128 \left( \frac{q_{F,\infty} B h_f}{x_F y_e h_f (t * \Delta P')_{L2}} \right)^{0.5} $$

The Second pseudosteady-state period has the following governing Equation:

$$ (t_D * P_D \text{'} )_{pss2} = 1.5511 \frac{x_F h_f (t * \Delta P')_{pss}}{y_e h_m (\phi_c) (t * \Delta P')_{pss}} t_{D,1}^{0.5} $$

Which allows finding an expression for determining the half-fracture length, $x_F$, once the dimensionless variables are used:

$$ x_F = \frac{1}{17.3145} \frac{q_{F,\infty} B h_f t_{D,2}}{y_e h_m (\phi_c) (t * \Delta P')_{pss2}} $$

3.2 TDS TECHNIQUE FOR GAS SHALE RESERVOIRS

The Bilinear flow regime governing equation is:

$$ (t_D * m(P_D \text{'} )_{BL} = \frac{1}{1.6289} \left( \frac{x_F h_f \rho_f \mu}{k_f w_f} \right)^{0.5} t_{D,1}^{0.25} $$

Replacing the dimensionless quantities given by Equations 5 and 7 in Equation 21 and solving and solving for the fracture conductivity, $k_{FW}$:
The governing equation for the First linear flow regime is:

\[
(t_D * m(P)_D)_{1,1} = \frac{1}{1.1360} t_{D,1,1}^{0.5}
\]  

After replacing the dimensionless variables leads to find the internal reservoir permeability, \(k_I\):

\[
k_I = 413.4878 \left( \frac{t_{D,1,1}^{0.5} \mu \phi_c I}{x_I h (t * \Delta m(P)')} \right)^{0.5}
\]  

First pseudosteady-state period: The governing equation for the first pseudosteady-state is:

\[
(t_D * m(P)_{pss})_{1,1} = \frac{\pi x_F}{2} \frac{t_{D,1,1}}{x_I h (t * \Delta m(P)')}_{pss}
\]  

Replacing the dimensionless parameter and solving for half-fracture length, \(x_F\):

\[
x_F = 1.6971 \left( \frac{t_{D,1,1}^{0.5} \mu \phi_c I}{x_I h (t * \Delta m(P)')} \right)^{0.5}
\]  

The second linear flow has the following governing equation:

\[
(t_D * m(P)_D)_{2,2} = \frac{1}{2.2783} \frac{t_{D,2,2}}{y_F \mu h (t * \Delta m(P)')}_{pss}^{0.5}
\]  

Replacing the dimensionless terms in the above expression leads to solve for the matrix permeability, \(k_m\):

\[
k_m = 102.8034 \left( \frac{t_{D,2,2}^{0.5} \mu \phi_c I}{x_F h (t * \Delta m(P)')} \right)^{0.5}
\]  

The governing equation of the Second pseudosteady-state period is given by:

\[
(t_D * m(P)_D)_{pss}^2 = 1.5511 \frac{x_F h (\phi_c)}{y_F h (\phi_c)} t_{D,2,2}^{0.5}
\]  

Replacing the dimensionless terms and, then, solving for the half-fracture length, \(x_F\), will result:

\[
x_F = \frac{1}{1.7187} \frac{q_{F,M,T} t_{pss}^2}{y_F h (\phi_c)} \left( \frac{h_I \rho_f}{k_F w_F} \right)^2 \left( \frac{\mu (\phi_c) I}{t_{pss,1,1,1}} \right)^{0.5}
\]  

Intersection points

The equations resulting from the points of intersection of the different flows are the same for both gas and oil reservoirs.

Intersection of first pseudosteady-state and bilinear flow:

\[
k_i = 1.2719 \times 10^9 \left( x_F y_F \right)^4 \left( \frac{h_I \rho_f}{k_F w_F} \right)^2 \left( \frac{\mu (\phi_c) I}{t_{pss,1,1,1}} \right)^{0.5}
\]  

Intersection of first pseudosteady state and linear flow:

\[
y_F = \frac{1}{34.5077} \left( \frac{k_i t_{pss,1,1,1} \mu (\phi_c) I}{h_I \rho_f} \right)^{0.5}
\]  

Intersection of bilinear-first linear:

\[
k_i = 0.34 \times 10^9 \left( \frac{h_I \rho_f}{k_F w_F} \right)^2 \left( \frac{\mu (\phi_c) I}{t_{pss,1,1,1}} \right)^{0.5}
\]  

Intersection of second pseudosteady-state and bilinear flow:

\[
k_i = 303.6558 \frac{\mu h^2 (\phi_c)}{t_{pss,2,2,2}}
\]  

Intersection of bilinear-second linear:

\[
k_i = 120.4702 \frac{y_F^2 x_F^2 k_i (\phi_c) F}{h_I \rho_f} \left( \frac{\mu \phi_c I}{h_i \rho_f} \right)^{0.5}
\]  

Intersection of second pseudosteady state and first linear:

\[
y_F = \frac{1}{34.9485} \frac{h_F (\phi_c)}{h_F (\phi_c)} \left( \frac{k_i t_{pss,2,2,2} \mu}{h_I \rho_f} \right)^{0.5}
\]
Intersection of second linear with first pseudosteady state

\[ k_u = \frac{296.0935 \mu}{(\phi c_m)_{t_{12-pms}}} \left[ h_f (\phi c_m)_{t_{12-pms}} \right]^2 \]  

(38)

4. SYNTHETIC EXAMPLES

The worked examples used data from Bowen et al. (2009).

4.1. Example 1

A pressure test was simulated for an oil reservoir drained by a horizontally fractured hydraulic well (Figure-3) and data presented in Table-1. Estimate: the half-fracture length, hydraulic fracture conductivity and permeability of the natural fractures.

Solution: The following computations are performed using Equations A-9, A-11 and A-12:

\[ \lambda = 2.0758 \times 10^{-10}, \omega = 0.00333333, \eta_o = 6.2786 \times 10^{-12} \text{ md-psi/cp}, \eta = 10.4642857 \text{ md-psi/cp} \text{ and } \eta_P = 5806026.32 \text{ md-psi/cp}. \]

Case 1 is confirmed from Figure-3 from which the following characteristic points were read:

\[ t_{BL} = 0.0119 \text{ hr}, \Delta P_{BL} = 4.82 \times 10^{-3} \text{ psi}, \]

\[ t_{pss}^1 = 4760 \text{ hr}, \Delta P_{pss}^1 = 0.887 \text{ psi}, \]

\[ t_L^1 = 13.1 \text{ hr}, \Delta P_L^1 = 0.033 \text{ psi}, \]

\[ tpss_{1-L}^1 = 360.7915 \text{ hr}, \]

\[ \eta_P = 1.20 \times 10^{-3} \text{ psi}. \]

Find the fracture conductivity, \( k_{wF} \), with Equation 12, natural fractured network permeability, \( k_I \), for Equation 14 and half-fracture length, \( x_F \), with Equation 16, so that:

\[ x_F = \frac{1}{17.0966} \frac{34.83 \times 1.35 \times 4.76 \times 10^5}{108.5 \times 380 \times 0.7 \times 0.006 \times 8.12 \times 10^{-1}} \]

Find natural fracture permeability, \( k_I \), with Equation 31, reservoir length, \( y_c \), using Equation 32 and fracture conductivity, \( k_{wF} \), with Equation 33.

\[ y_c = 93.1035 \text{ ft} \]

\[ k_f = 4.0739 \times 1.31 \times 10^1 \times 0.3 \times 34.83 \times 1.35 \times 0.7 \times 0.006 \times 93 \times 380 \times 1.20 \times 10^{-2} \]

\[ x_F = 93.1035 \text{ ft} \]

\[ k_f = 46.8633 \text{ md} \]

\[ k_f w_P = 122.0157 \times 0.005 \times 1.35 \times 34.83 \times 1.35 \times 0.7 \times 0.006 \times 50 \times 0.7 \times 0.006 \times 1.19 \times 10^{-2} \]

\[ x_F = 93.1035 \text{ ft} \]

\[ k_f = 46.8633 \text{ md} \]

\[ k_f w_P = 249.5560 \text{ md-ft} \]

\[ y_c = 93.1035 \text{ ft} \]

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\[ x_F = 93.1035 \text{ ft} \]

\[ k_f = 46.8633 \text{ md} \]

\[ k_f w_P = 249.5560 \text{ md-ft} \]

\[ y_c = 93.1035 \text{ ft} \]

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\[ y_c = 93.1035 \text{ ft} \]

\[ y_c = 93.1035 \text{ ft} \]
\[ k_f = 1.2719 \times 10^9 \times 93 \times 108.5^2 \times \left(\frac{0.005 \times 1}{251}\right)^2 \]

\[ k_g = \frac{0.3 \times 0.7 \times 0.006}{60.8027} = 46.5643 \text{ md} \]

\[ k_f w_f = 29.9503 \times 93^2 \times 0.005 \times 1 \times 1.6240^{0.5} \]

\[ k_f w_f = 255.1012 \text{ md-ft} \]

**Table-2.** Comparison of results for example 1.

<table>
<thead>
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<th>Equation</th>
<th>Parameter</th>
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<th>Error, %</th>
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<td>( k_{fFWF} ), md-ft</td>
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<td>( k_{fFWF} ), md-ft</td>
<td>251</td>
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**Table-3.** Comparison of results for example 2.

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**5. CONCLUSIONS**

a) New equations are presented to characterize systems consisting of a horizontal well with multiple fractures in ultra-low permeability reservoirs using characteristic points found on the pressure derivative so natural fracture permeability, matrix permeability, half-fracture length and hydraulic fracture conductivity can be estimate and verified.

b) The trilinear model allows three possible cases according the developed flow regimes. In case 1: bilinear flow, first linear flow and pseudosteady-state period are presented. For case 2, the developed flow
regimes are: bilinear, first linear, first pseudosteady-state, second linear and second pseudosteady-state. Finally, for case 3, the observed flow regimes are: bilinear, followed by a linear and pseudosteady-state - same called as second linear and second pseudosteady-state in case 2, which can be identified from such properties as matrix permeability and the resulting values of the interporosity flow parameter and the dimensionless storativity ratio.

Nomenclature

\[ \begin{align*}
B & \quad \text{Formation volume factor, rb/STB, ft}^3/\text{SCF} \\
C_{FD} & \quad \text{Hydraulic fracture conductivity, dimensionless} \\
C_{RD} & \quad \text{Reservoir conductivity, dimensionless} \\
c_i & \quad \text{Total compressibility, 1/psi} \\
d_f & \quad \text{Distance between two adjacent fractures, ft} \\
h & \quad \text{Reservoir thickness, ft} \\
h_f & \quad \text{Thickness of natural fractures, ft} \\
h_{ft} & \quad \text{Total thickness of natural fractures, ft} \\
h_m & \quad \text{Thickness of matrix slabs, ft} \\
k & \quad \text{Permeability, md} \\
k_f & \quad \text{Hydraulic fracture intrinsic permeability, md} \\
k_F & \quad \text{Hydraulic fracture permeability, md} \\
k_I & \quad \text{Permeability of the inner reservoir, md} \\
k_O & \quad \text{Permeability of the outer reservoir, md} \\
k_m & \quad \text{Matrix intrinsic permeability, md} \\
m(P) & \quad \text{Pseudopressure, psi}^2/\text{cp} \\
n_F & \quad \text{Number of hydraulic fractures} \\
n_f & \quad \text{Number of natural fractures} \\
P & \quad \text{Pressure, psia} \\
P_D & \quad \text{Dimensionless pressure} \\
P_i & \quad \text{Initial reservoir pressure, psi} \\
q & \quad \text{Flow rate, STB/d} \\
q_{FD} & \quad \text{Flow rate for a hydraulic fracture, oil STB/d, gas Mscf/d} \\
S & \quad \text{Laplace parameter} \\
t & \quad \text{Time, hr} \\
T & \quad \text{Reservoir Temperature, °R} \\
t_D & \quad \text{Dimensionless time} \\
t_0 & \quad \text{Dimensionless pressure derivative} \\
t_0*P_D & \quad \text{Dimensionless pseudopressure derivative} \\
t*Am(P) & \quad \text{Pseudopressure derivative, d/Mscf} \\
w_F & \quad \text{Hydraulic fracture width, ft} \\
x_e & \quad \text{Reservoir size, x-direction, ft} \\
x_F & \quad \text{Hydraulic half-fracture length, ft} \\
y_e & \quad \text{Reservoir size, y-direction, ft} \\
E & \quad \text{Type of medium: L,F,O} \\
\alpha & \quad \text{Parameter defined in trilinear flow model} \\
\beta & \quad \text{Parameter defined in trilinear flow model} \\
\Delta & \quad \text{Difference operator} \\
\phi & \quad \text{Porosity, fraction} \\
\eta & \quad \text{Diffusivity, ft}^2/\text{hr} \\
\lambda & \quad \text{Transmissivity ratio, transient dual porosity model} \\
\mu & \quad \text{Viscosity, cp} \\
\pi & \quad \text{Pi Constant} \\
\rho_f & \quad \text{Density of natural fractures, fractures/ft} \\
\omega & \quad \text{Storativity ratio, transient dual porosity model}
\end{align*} \]

Subscripts

\[ \begin{align*}
BL & \quad \text{Bilinear flow} \\
D & \quad \text{Dimensionless} \\
e & \quad \text{External boundary} \\
f & \quad \text{Natural fracture} \\
F & \quad \text{Hydraulic fracture} \\
i & \quad \text{Initial} \\
I & \quad \text{Inner Reservoir} \\
m & \quad \text{Matrix} \\
L1 & \quad \text{First linear flow} \\
L2 & \quad \text{Second linear flow} \\
O & \quad \text{Outer Reservoir} \\
pss1 & \quad \text{First pseudosteady state} \\
pss2 & \quad \text{Second pseudosteady state} \\
R & \quad \text{Reservoir} \\
Sc & \quad \text{Standard conditions} \\
t & \quad \text{Total} \\
w & \quad \text{Flowing wellbore} \\
\xi & \quad \text{Type of medium: L,F,O}
\end{align*} \]

REFERENCES


SI metric conversion factor

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<td>9.290 204 x 10⁻²</td>
<td>m²</td>
</tr>
<tr>
<td>psi</td>
<td>6.894 757 x 10⁰</td>
<td>kPa</td>
</tr>
</tbody>
</table>

Appendix A

Additional equations for the trilinear flow model, (Ozkan et al. 2011) are given below:

\[ C_{FD} = \frac{k_f w_f}{k_i x_f} \quad (A-1) \]

\[ \alpha_f = \frac{2 \beta_f}{C_{FD}} + \frac{S}{\eta_{FD}} \quad (A-2) \]

\[ \eta_{FD} = \frac{\eta_f}{\eta_i} \quad (A-3) \]

\[ \beta_f = \sqrt{\alpha_f} \tanh\left[ \sqrt{\alpha_f} \left( y_{so} - \frac{w_{FD}}{2} \right) \right] \quad (A-4) \]

\[ \alpha_s = \frac{\beta_s}{C_{so} y_{so}} + u \quad (A-5) \]

\[ C_{so} = \frac{k_s x_s}{k_o y_o} \quad (A-6) \]
\[ \beta_R = \frac{S}{\eta_{RD}} \tanh \left[ \frac{S}{\eta_{RD}} (x_{o} - 1) \right] \]  \hspace{1cm} (A-7)

\[ \eta_{RD} = \frac{\eta_o}{\eta_i} \]  \hspace{1cm} (A-8)

In Equations A-3 and A-8, \( \xi = I, F, O \).

\[ \eta_\xi = \frac{k_\xi}{(\phi c_\xi)_\xi \mu} \]  \hspace{1cm} (A-9)

In Equation A-5,

\( u = sf(S) \)

Internal naturally fractured reservoir:

\[ f(S) = 1 + \frac{2\omega}{3S} \tanh \left( \frac{3S\omega}{\lambda} \right) \]  \hspace{1cm} (A-10)

Dual porosity parameter for the internal naturally fractured reservoir:

\[ \omega = \frac{(\phi c, h_m)}{(\phi c, h_f)} \]  \hspace{1cm} (A-11)

\[ \lambda = \frac{12x_f^2k_m}{h_m h_f k_f} \]  \hspace{1cm} (A-12)