INVESTIGATION OF VIBRATION CHARACTERISTICS FOR SIMPLY SUPPORTED PIPE CONVEYING FLUID BYMECHANICAL SPRING

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ABSTRACT

Finite element analysis was used in this study to analyse dynamically the stability of a pipe which is stiffened by linear spring and conveying an internal flow of fluid. Several effective parameters play an important role in stabilizes the system, such as stiffness addition. The effect of stiffness addition (linear spring) and effect of spring location with different diameter ratio were studied. Also, effect of the velocity of flowing on the dynamic stability of the system was taken into the consideration. There is a spring constant at which the dynamic behaviour becomes more sensitive and the spring offers best results for frequency of the system. Results show that the best spring's locations depend on the spring's constant and velocity of flowing.

Keywords: stability, finite element method, pipe conveyed fluid, linear spring stiffness.

1. INTRODUCTION

Piping systems are very important and main content in a wide range of industrial applications especially in chemical plants. Unfortunately, vibrations of piping system represent a challenge that must be studied in order to overcome this problem or at least reduce it in many applications [1]. Researchers found that straight pipes are stable if the velocity of fluids that they convey inside is low. Such pipes will be going back to stability after a period of time after exciting with an external effect. They found also that there is a critical flow velocity for each system after which the system can't go back to stability and the entire system will be referred to as unstable system even if the external exciter is small [2].

Nabeel *et al* [3] studied effects of the residual stresses caused by welding on the natural frequencies of a piping system conveying fluid flow. They found that for a mid-span welded pipe, natural frequencies will be reduced for clamped-pinned pipe and also for clamped-clamped one. Mohamed [4] investigated the vibration analysis and stability of curved pipe with linear spring stiffener for fixed-fixed ends conveying fluid consisting from several of straight pipe elements by using finite element model. He found that the location of spring stiffener would influence the critical flow velocity and the natural frequency of the system. Results are compared with the numerical approach and gives good agreement of used technique.

A semi-circular piping system conveying fluid within plane and also out of plane was studied by Jung [5]. He investigated the dynamic characteristics of such pipes after obtaining the linearized equations by applying the Galerkin method around the equilibrium position. Sugiyama, *et al*, [6] studied the dynamic behaviour of the pipe conveyed fluid with spring support. They stated that the support location has a great effect on the dynamic characteristics of the structure.

Lumijarvi investigated designs of cantilever pipes that conveying fluid flow [7]. An optimization was made to increase the critical speed of fluid flow by adding additional masses, springs, and dampers along piped with specific characterizing and locations. Mediano and Garcia presented a mathematical model for dynamic behaviour of a clamped-pinned pipe conveying fluid flow [8]. According to their model, materials that pipes made from play a role in the stability of the entire piping systems.

Paidoussis and Semler focused on the chaotic regions and their locations and hence they studied the effect of adding additional springs to overcome the instability problem and hence to increase the fluid flow velocity [9]. Veerapandi *et al*. produced an experimental study and analysis of flow induced vibration by study the effect of turbulence of gas flow in the piping system during the course of sub cooling of flight static fire test or flow trial [10]. The critical fluid velocity had been calculated by analytical approach. Also, modal Analysis in the pipe flow configuration was studied. The dynamic fluid behaviours in the valve annular region were studied and obtained results in CFD and Fluid Flow analysis to be extended full pipe configuration. Flow Induced Vibration of Coupled analysis was done by ANSYS CFX.

This paper, aims to study the stability analysis, by additional spring element and its location, on the dynamic behaviour of a simply supported pipe conveying fluid using finite element technique. Furthermore, it determines whether the critical flow velocity affected by the magnitude of the spring stiffness. Also, the mass, stiffness, damping (Coriolis) matrices are derived and the eigenvalue analysis will be performed.

2. DERIVATION OF GOVERNING DIFFERENTIAL EQUATION

The straight pipe is very important in the engineering application especially that of simply supported. The main reason of study the simply supported system is that one of the ways transpose any type of fluid, the pipe is not conservative system, Sugiyama, et al, 1985 [11]. Figure-1 presents a simply supported ends pipe conveyed fluid with fluid velocity of U m/s, length L, linear spring of stiffness (K), and pipe mass per unit length m.

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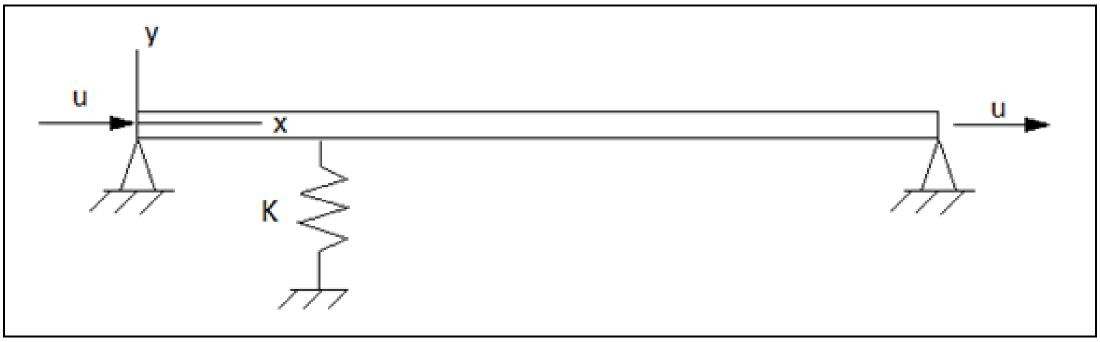

Figure-1. A simply supported pipe system with linear spring.

The reaction forces and moments of fluid and pipe elements are shown in Figure-2 where (M) represents the mass of fluid per unit length, (EI) is the flexural rigidity,(A) represents the cross-sectional flow area, (S) is the inner perimeter, (p) is the fluid pressure. **Fδx** represents there action forces of the pipe on the fluid normal to the fluid element. Consider then elements **δx** of the fluid and the pipe.

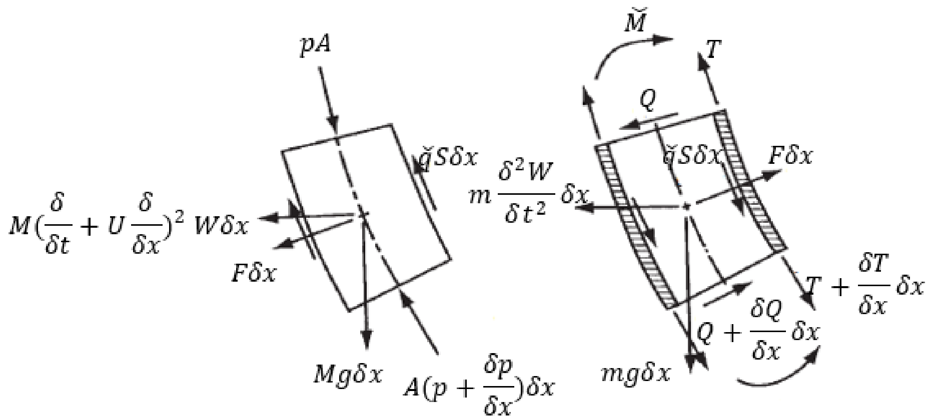

Figure-2. Fluid and pipe elements and reaction forces and moments [1].

The overall equilibrium equation of the element shown in (Figure-2) can be written as follows [1]:

$$EI\frac{\partial^4 W}{\partial x^4}+MU^2\frac{\partial^2 W}{\partial x^2}+2MU\frac{\partial^2 W}{\partial x\partial t}+(m+M)\frac{\partial^2 W}{\partial t^2}=0 \tag{1}$$

The term $(EI\frac{\partial^4 W}{\partial x^4})$ is the force component ~~acti~~ng on the pipe caused by pipe bending while the expression $(MU^2\frac{\partial^2 W}{\partial x^2})$ represents the force component acting on the caused by the fluid flow around a deflected pipe. Some researchers refer to this term as a centrifugal force of the fluid element due to its instantaneous velocity and instantaneous curvature of the deflected pipe. This term has a great effect on the pipe stability and accelerates the pipe to be unstable. The term $(2MU\frac{\partial^2 W}{\partial x\partial t})$representsthe inertial force which is associated with Coriolis acceleration caused by fluid flows velocity U. The inertia force for both fluid and pipe density is referred in term $((m+M)\frac{\partial^2 W}{\partial t^2})$ As a final sight, it was noted that, the dynamic behaviour of the ∂t^2 system is largely depend on the elastic stiffness of the pipe, flow velocity and lateral displacement (which mean the boundary conditions). So any change in the system stiffness should change the dynamic behaviour. Thereby, addition of spring (stiffness) is of majority importance [4]. The boundary conditions for the simply supported pipe are:

$$w|_{x=0,L}=0 \qquad EI\frac{\partial^2 W}{\partial x^2}|_{x=0,L}=0 \tag{2}$$

3. FINITE ELEMENT FORMULATION

Form the literature and the highlight of the previous section, it was noted that there were some difficulties in the analytical methods for study the dynamic behaviour. FEM was adopted to obtain the dynamic behaviour as a numerical technique due to the complexity in solving the differential equation of pipe conveyed fluid (equation 1) since it is a higher partial and higher order differential equation.

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In its local coordinates, there are two DOFs at nodes of planar beam elements. Additionally, there is a rotation Ɵz around Z-axis and a deflection in Y-direction and therefore, the overall DOFs for each beam element will be four. For a (2a) length beam element shown in (Figure-3) with originate the centre of the element, X-axis can be considered in the axial direction of the element.

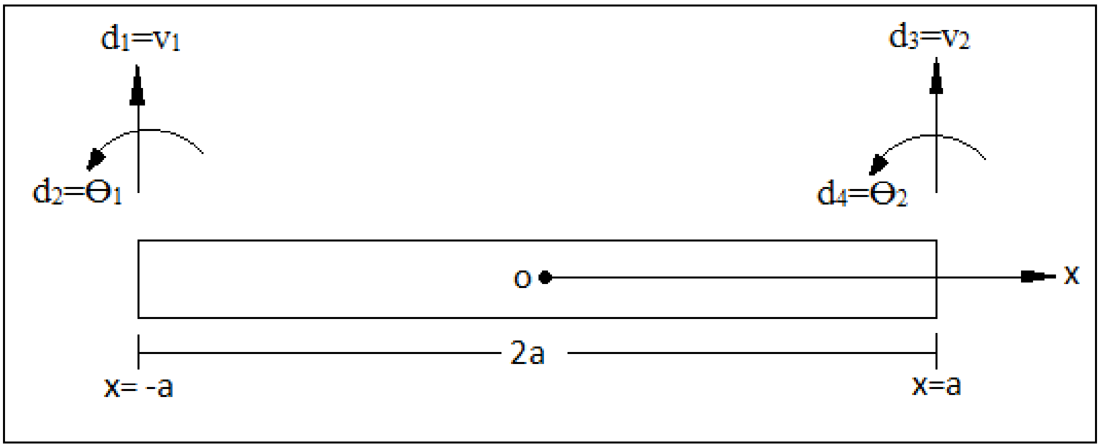

Figure-3. Local coordinate system of a beam element.

To derive the four shape functions, it can be considered that the equation which describe the displacement occurs in an element is of a third order with four unknown constants. The displacement filed in element direction can be therefore written as follows [12]:

$$W(x) = \sum_{i=1}^{4} N_i(x) q_i \tag{3}$$

Where q_i are the generalized coordinates. The shape functions Ni are equal to:

$$N_1 = \frac{1}{l^3}(2x^3 - 3lx^2 + l^3) \tag{4-a}$$

$$N_2 = \frac{1}{l^2}(x^3 - 2lx^2 + l^2 x) \tag{4-b}$$

$$N_3 = \frac{1}{l^3}(3lx^2 - 2x^3) \tag{4-c}$$

$$N_4 = \frac{1}{l^2}(x^3 - lx^2) \tag{4-d}$$

Where, l is the element length. The kinetic and potential energies of the pipe element can be expressed by

$$T = \frac{1}{2}\int_0^l (M+m)\left(\frac{\partial W}{\partial t}\right)^2 dx = \frac{1}{2}\sum_e \dot{q}^T (M+m) \int_0^l N^T N \, dx \, \dot{q} \quad ...(5)$$

$$V_1 = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 W}{\partial x^2}\right)^2 dx = \frac{1}{2}\sum_e q^T \, EI \int_0^l \bar{\bar{N}}^T \bar{\bar{N}} \, dx \, q \quad ...(6)$$

Each prime sign that appear above the shape function symbol, i.e. "N", represents one-time derivative with respect to x- coordinate. Thus, mass ($\hat{m}$) and stiffness ($\hat{k}1$) matrices are equal to [12].

$$[\hat{m}] = \frac{(m+M)l}{420}\begin{bmatrix} 156 & 22l & -54 & -13l \\ 22l & 4l^2 & -13l & -3l^2 \\ 54 & 13l & -156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \tag{7}$$

$$[\hat{k}_1] = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \tag{8}$$

The term ($MU^2 \frac{\partial^2 W}{\partial x^2}$) has a potential energy that can be represented in terms of displacement shape function derived for
the pipe as;

$$V_2 = \frac{1}{2}\int_0^l M\,U^2\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial x}\right) dx = \frac{1}{2}\sum_e q^T M U^2 \int_0^l \bar{N}^T \bar{N} \, dx \, q \tag{9}$$

The stiffness matrix that comes from flow around the deflected pipe is [12]:

$$[\hat{k}_2] = \frac{MU^2}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \tag{10}$$

It is important to clear that stiffness matrix [$\hat{k}2$] leads to weaken the overall stiffness of the pipe system [13].

The dissipation energy can be represented in term of the Coriolis force expression ($2MU \frac{\partial^2 W}{\partial x \, \partial t}$) as

$$\mathcal{R} = \frac{1}{2}\int_0^l 2MU\left(\frac{\partial W}{\partial x}\right)\left(\frac{\partial W}{\partial t}\right) dx = \frac{1}{2}\sum_e q^T \; 2MU \int_0^l \bar{N}^T N \, dx \, \dot{q} \tag{11}$$

This gives the unsymmetrical damping matrix is [12]:

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$$[\hat{C}] = \frac{MU}{30}\begin{bmatrix} -30 & -6l & -30 & 6l \\ 6l & 0 & -6l & l^2 \\ 30 & 6l & 30 & -6l \\ -6l & -l^2 & 6l & 0 \end{bmatrix} \quad (12)$$

In general, the above stiffness matrices are classified according to their category. Then each category element matrices are assembled to represent the whole pipe length model. These overall matrices will be arranged in a proper form to get the dynamic characteristics of the structure.

4. DYNAMIC ANALYSIS

The standard equation of motion in the finite element form is:

$$[m + M]\{\ddot{q}\} + [\hat{C}]\{\dot{q}\} + [k_{total}]\{q\} = \{0\} \quad (13)$$

Where $k_{total} = \hat{k}1 - \hat{k}2$. Since the above equation has a damping term with skew-symmetric characteristic, thus the solution of eigenvalues problem should be executed to the characteristic matrix [Ω], which is equal to[14]:

$$[\Omega] = \begin{bmatrix} [0] & [I] \\ -[m+M]^{-1}[k_{total}] & -[m+M]^{-1}[C] \end{bmatrix} \quad (14)$$

The solution of eigenvalue problem yields complex roots. The imaginary part of these roots represents the fundamental natural frequencies of damped system, i.e. when the fluid flows through the pipe, while the real part indicates the rate of decay of the free vibration.

The natural frequency of the simply supported pipe conveying fluid and the critical velocity of fluid flow through the pipe can be expressed as follows [15]:

$$wn = \left(\frac{\pi^2}{L^2}\right)\sqrt{\frac{E*I}{m+M}} \quad (15)$$

$$Uc = \left(\frac{\pi}{L}\right)\sqrt{\frac{E*I}{\rho*A}} \quad (16)$$

Where E is the pipe modulus of elasticity, I is the pipe area moment of inertia, ρ is the density of the fluid, and A is the inner area of the pipe.

For equations (15) and (16) above, these equations showed the additional spring does not effect on the dynamic characteristics of pipe line system

5. RESULTS AND DISCUSSIONS

Number of elements represents the major parameter for accuracy of the results and the time required to solve the problem. Some types of errors such rounds off error are increase or decrease with increasing or decreasing elements number. Therefore, it was observed that there is a critical elements number at which the result become converges. Table-1 presents the best numbers of elements. It is noted that, twelve elements gave a convergence in the results and then they will be used to discretise the pipe system.

Table-1. Effect number of elements along the pipe with properties of (U=0m/s, t=1mm, OD=10mm, L=2m and p=100kPa).

Number of elements	**Frequency** (rad/s)
2	3.5768
4	3.5637
8	3.5628
12	3.5628

Figures (4-6) present effect of spring location and dimensionless spring constants (K=kl3/EI) on the natural frequency of simply supported pipe conveying fluid. These figures are remarked as data points for easier comparison. As a general view, it was noted that the frequency is increased with increasing the dimensionless spring constant. For the same value of spring, the frequency is seemed to be increased with increase x/L ratio (spring location measured from the supported end). This behaviour was dominated for a specific spring constant started from K=100 to 1000 so the maximum frequency occurred at the centre length of pipe. With increasing the dimensionless spring constant, a different behaviour was observed. Where an inflection point, at which the frequency has a maximum value then after it will decrease, is brought to light. Up to now, the inflection point of frequency can be called as a critical spring location and the corresponding spring constant is critical value of the stiffness. One can observe that the critical spring location (x/L) moved from the supported end and directed to the centre length of pipe with increasing spring constant above its critical value.

For the aforementioned, two interested features were observed. The first; there was a critical stiffness value in which the frequency response is started to changed. The second; a critical spring location at which the frequency attain its maximum value.

Figures (4-6) present the relationship of the frequency and spring location for different spring values at 0, 3 and 6 m/s of fluid flow, respectively. The general sight is that; the frequency was decreased with increase the flow velocity. The frequency of the system is largely depending on the structural stiffness of the pipe. With increasing the flow velocity, the hydrodynamic stiffness resulted from increasing the flow velocity (as mentioned in Eq. (10) is increased also due to the direct relationship between them. The hydrodynamic stiffness leads to weaken the structure stiffness of the pipe. Then increasing the flow velocity means increasing in hydrodynamic stiffness and thereby decreasing in the pipe stiffness which leads to decreasing in the frequency.

The more distinctive Figures (7-9) shown that all spring constants have an effect of the frequency at values of x/L with different value of diameter ratio, where the frequency tend to increase with the flow velocity at closed the ratio x/L to the centre length of pipe. Also, these Figures show the effect of spring constants on the frequency of the pipe with increase the diameter ratio.

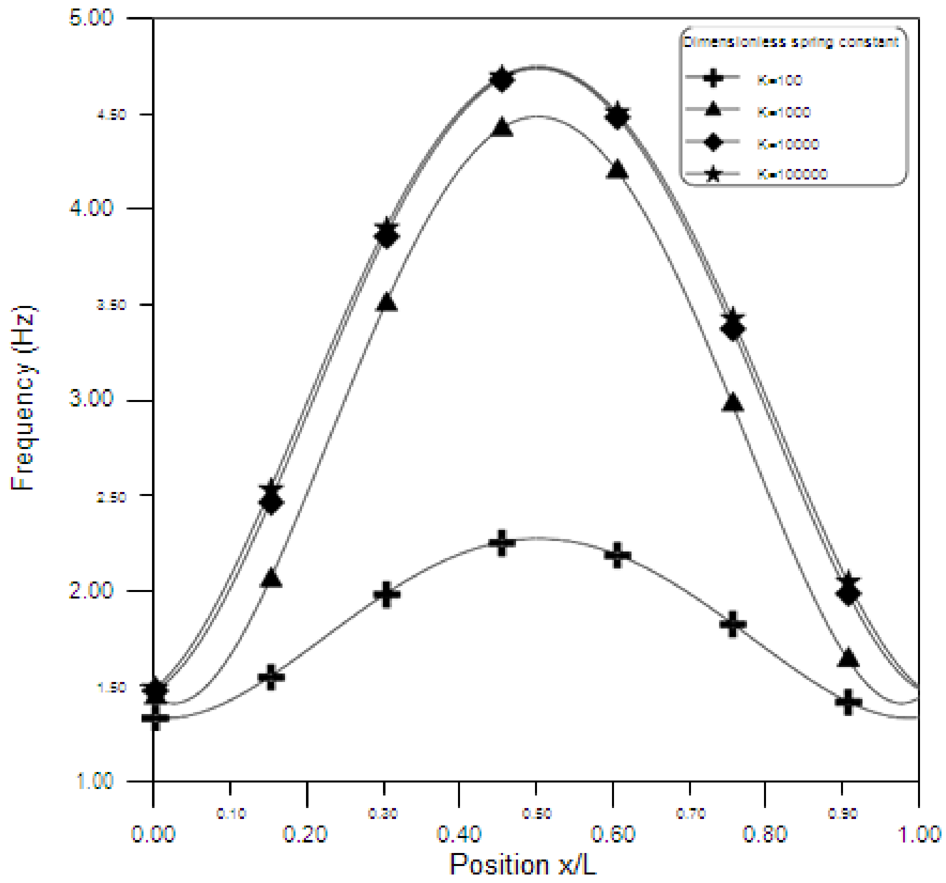

Figure-4. Effect of dimensionless spring constant and spring location on the frequency with no fluid velocity (U=0).

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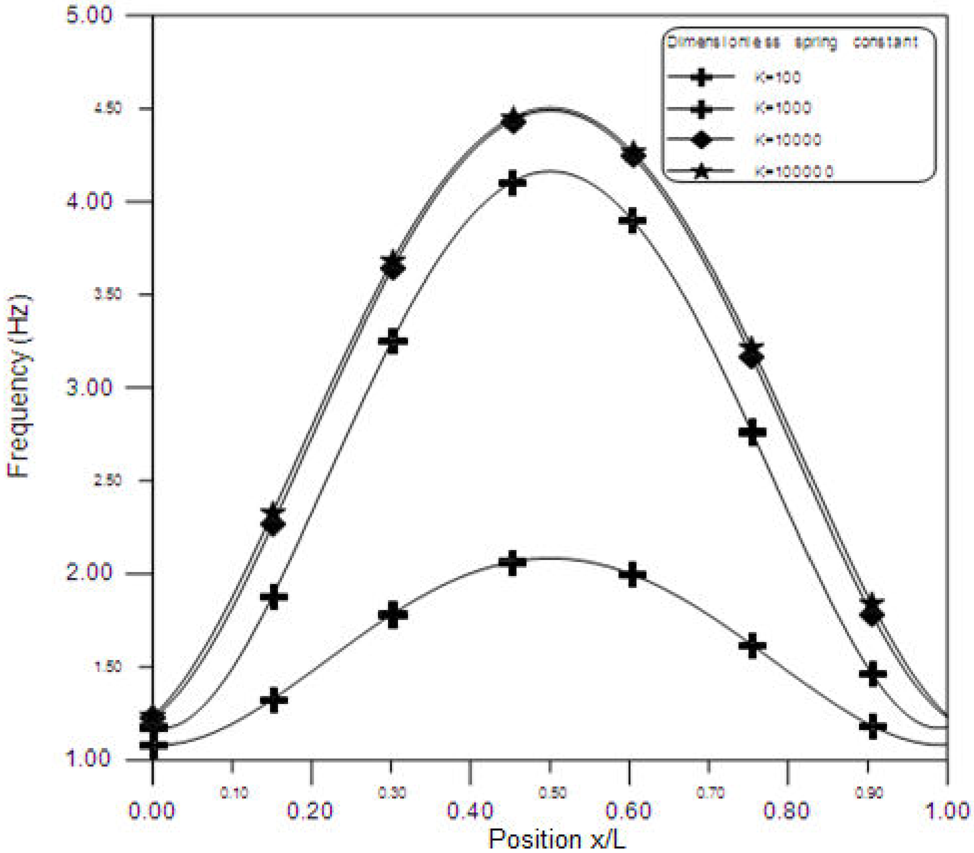

Figure-5. Effect of dimensionless spring constant and spring location on the frequency with fluid velocity (U=3 m/sec).

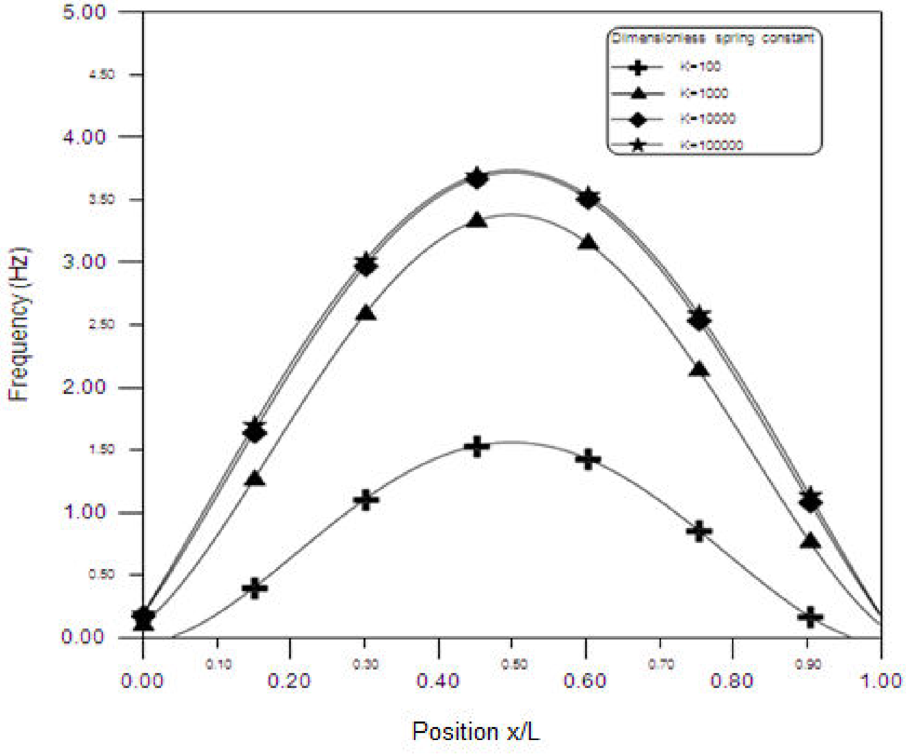

Figure-6. Effect of dimensionless spring constant and spring location on the frequency with fluid velocity (U=6 m/sec).

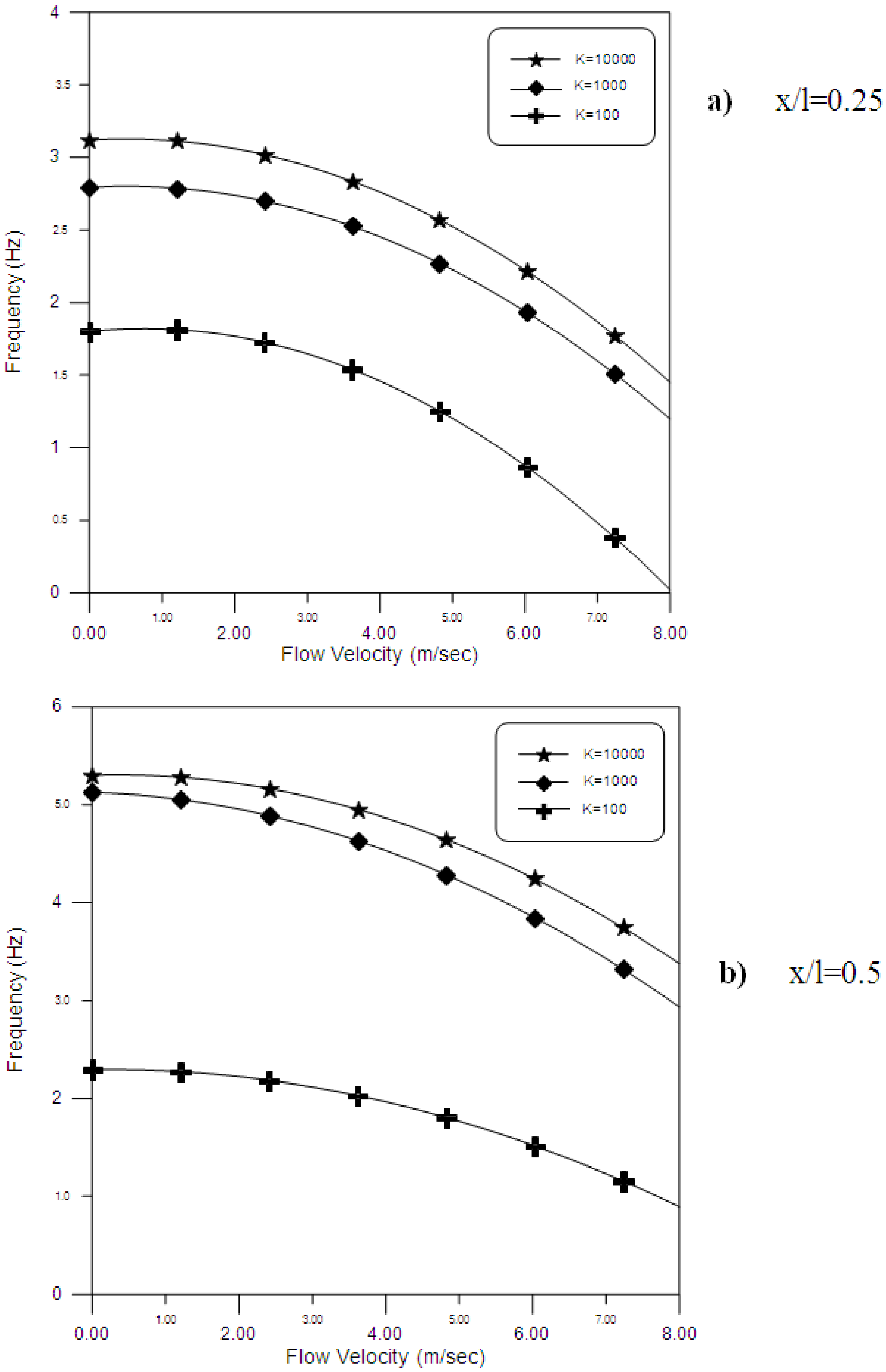

Figure-7. Effect of the flow velocity on the frequency for different dimensionless spring constants and diameter ratio OD/ID=1.25 for (a) spring location x/l=0.25 and (b) spring location x/l=0.5.

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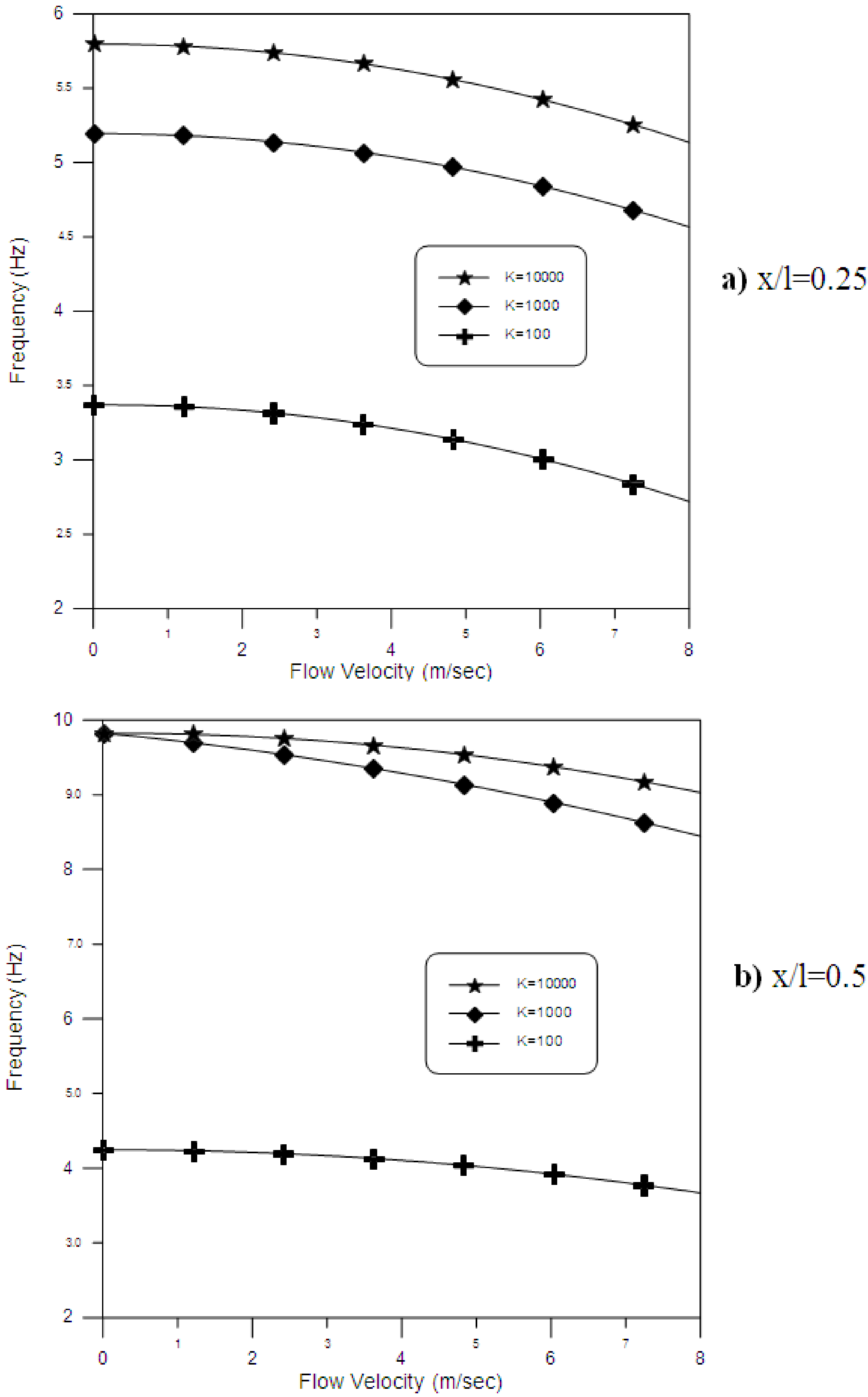

Figure-8. Effect of the flow velocity on the frequency for different dimensionless spring constants and diameter ratio OD/ID=2 for a) spring location x/l=0.25 and b) spring location x/l=0.5.

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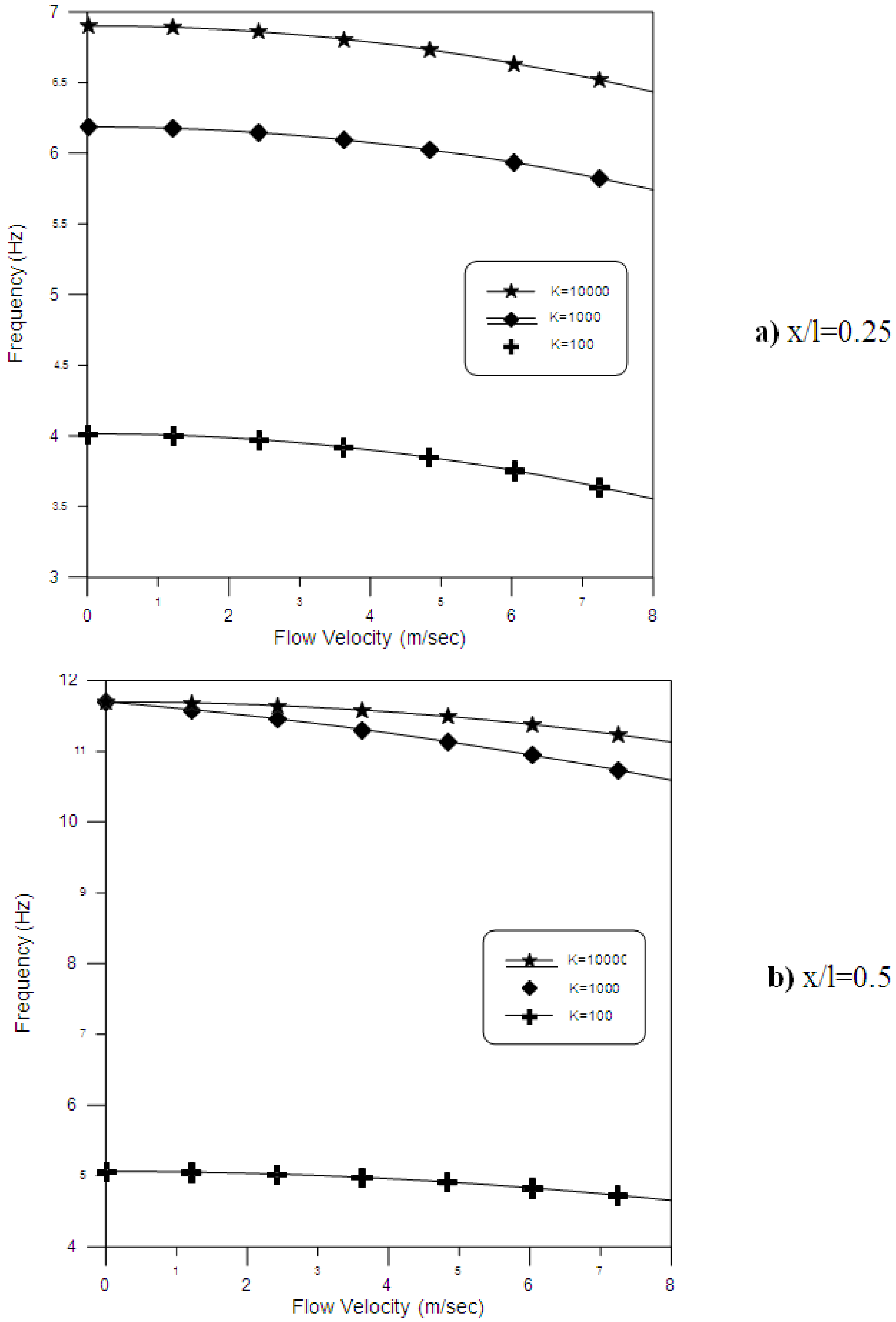

Figure-9. Effect of the flow velocity on the frequency for different dimensionless spring constants and diameter ratio OD/ID=2.5 for a) spring location x/l=0.25 and b) spring location x/l=0.5.

CONCLUSIONS

From results, some conclusions can be listed as followings:

a) The dynamic behaviour of the simply supported pipe conveying fluid will significantly change when using a spring as an additional support.

b) The frequency of the pipe is largely depends on the spring constant and flow velocity. In addition, spring location has a great effect on the results of the system stability.

c) There is a critical spring value at which the frequency attains its maximum value.

d) The frequency of the pipeline system increases with increase the diameter ratio.

e) There is a good agreement between the numerical approach and the Finite Element Method.

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