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LEAST COST ROUTING ALGORITHM WITH THE STATE SPACE RELAXATION IN A CENTRALIZED NETWORK

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ABSTRACT

This paper aims to present the least cost algorithm finding loop routes between a central node and a large number of end-nodes in the centralized network. End-nodes request service on the loop route originated from a central node. Demands at end-node are deterministic and distance matrix is given. We find a set of minimum cost loops to link end-nodes to a central node satisfying the service requirements at end-nodes. In this paper, we propose a heuristic algorithm which first generates possible routes by applying the space relaxation, and then finds a set of loop routes through matching. The proposed algorithm is easy to implement and enable us to obtain the solution regardless of network configuration in the short computation time for the small network. The computation comparison with previous algorithms is also presented. Our algorithm can be applied find the broadcasting loops in centralized network.

Keywords: centralized network, routing algorithm, loop route, broadcasting.

1. INTRODUCTION

Routing is one of important issues in the network problem. Network is classified into the centralized and distributed network. Centralized network is composed of a central node and a large number of end-nodes which require service from central node.

Shortest route handling all the end-nodes originated from central node is affected by the capacity of service facility which is maintained by the central node. If the capacity is larger than sum of demands at end-nodes, an optimal loop route can be obtained by TSP (Travelling Salesman Problem).

On the other hand, if the capacity of service facility at the central node is smaller than sum of demands at end--nodes, several loop routes should be obtained. This problem is also called vehicle routing problem [1]. In order to solve the problem, we should build an algorithm finding a set of loop routes which minimizes distance costs while satisfying the demands at end-nodes by the capacity of facility at the central node.

As examples of this problem in the real situation, we can find the delivery of items between a shop and customers, commuter bus from a company or school to a large number of employees or students, and ring topology in the centralized communication network.

Since the problem is NP-hard [2], most previous researches are based on the heuristic algorithm. Many heuristic techniques are presented. Clustering first route second, route first cluster second, saving insertion, improvement exchange, mathematical programming based, and interactive optimization methods are proposed [3]. Haimovich and Kan [4] found the bound for the problem, but there is the limitation which represents the capacity of facility of centre node as the number of nodes. Altinkemer and Gavish [5] improve this limitation.

Network is classified into undirected, directed, and mixed in accordance with the configuration. Most of previous researches assumed that distance matrix is undirected. However, Integer programming (IP) [6], TSP segment (TSPS) [7], NNR (nearest neighbour rule), and Clarke and Write [8] model can find solution regardless of configuration. Eulerian tour [9] partition method can be applied to the directed network.

In this study, we propose two-phase least cost algorithm. In the first phase, we enumerate the possible routes by applying dynamic programming with the state space relaxation. In the second phase, we find least cost loop routes by applying matching procedure.

Since our proposed algorithm can deal with all types of networks regardless of configuration, we compare our algorithm with IP, TSPS, NNR, and Eulerian tour partition algorithms in terms of distance cost. Comparison result shows that our algorithm produces the least cost routes for typical examples.

Our algorithm can be applied to finding the least cost loop routes in a centralized network and the topological network design [10,11,12] including the communication network.

The rest of the paper is organized as follows. Section 2 presents least cost loop algorithm with the state space relaxation, and section 3 presents comparison with previous algorithms according to the network configuration, followed by concluding remarks in section 4.

2. LEAST COST ROUTING ALGORITHM

In the problem formulation, a set of assumptions should be made:

a) The demand at the end-node is deterministic.
b) There is only one centre node, with unlimited capacity.
c) The demand at the end-node is not splintered.
d) The demand at a given end-node (W_i) cannot exceed the capacity of facility ($\max(W_i) \leq T$, i=1,..,n).
e) The total demand exceeds the capacity of facility ($\Sigma_{i=1}^{n} W_i > T$).

With these assumptions, our problem formulation can find a set of loop routes with minimum total distance cost, which satisfies that the demand capacity limit is less than a given value (T) in each loop. Figure-1 shows an example of the problem definition. A centre node is

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indexed by 1. The sum of demands (W_i) on one loop route is less than T and d_{ij} represents the distance cost between node i and j.

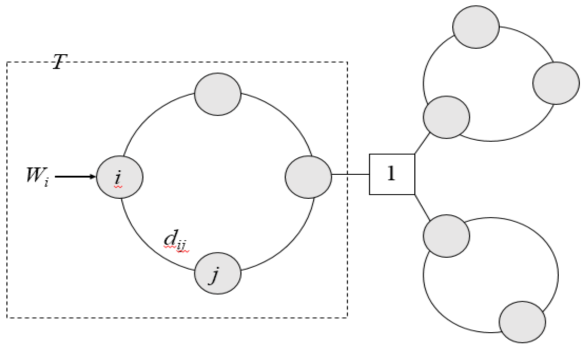

Figure-1. An example of the VRP.

We can obtain the following recurrence relation by using the principle of optimality. In Equation (1), $f_k(j,S)$ represents the minimum cost among the connecting costs of k intermediate nodes set (S) from centre node to node j. If the sum of demands on the S is larger than the capacity of facility (T), then $f_k(j,S)$ is set to ∞. N_j is the set of node number excluding centre node, that is $N_j = \{2,3,4,..,n\}$. S is the subset of N_j.

if $\Sigma_{q\in S} W_q + W_i \le T$,

$$F_k(j,S) = \min_{q\in S, q\neq j}[F_{k-1}(q, S-\{q\}) + d_{qj}] \quad (1)$$

else

$$F_k(j,S) = \infty$$

Boundary condition represents the direct connection cost from centre node to node j without intermediate nodes and defined as

$$F_0(j,-) = d_{1j} \quad (2)$$

When computing the recurrence relation in (1) and (2), we can reduce the amount of computations by the following state space relaxation procedure.

We let $j\cup S$ be τ. Φ_t represents the ascending order sorted set of τ which has (k+1) elements at each stage k. $\Phi_t(k)$ represents the extendable set satisfying the constraints among $\Phi'_t(t=1,2..)$. Φ''_t represents the extendable set not satisfying the constraints among $\Phi_t's(t=1,2..)$. π shows the set which has $\Phi_t(k)$ as elements. $\max(\Phi_t(k))$ is the maximum value among $\Phi_t(k)$.

We compute route costs by adding the return cost to the center node to $F_k(j,S)$ which is composed of the same node number set with the different sequence order, and then find minimum cost.

$$F_0(P) = F_0(j,-) + d_{j1} \qquad k = 0 \quad (3)$$

$$F_k(P) = \min_{(j,S)\in P}[F_k(j,S) + d_{j1}] \quad k = 1,2\,\ldots, L-1$$

Costs computed in (3) represent the partial route costs without duplication and the corresponding node set is represented as R.

Theorem 2.1. In the finite set $S = \{1,2,\ldots,N\}$, two finite sets such that $S_i \cap S_j = \Phi\ (i\neq j)$ are divided into $S_1, S_2,..,S_k$, that is, $\cup_{i=1}^{k} S_i = S$. Then, if $S_p = \cup_{i=1, j\neq i}^{k} S_i$,

$$\min_{i\in S,\ p\in\{S-S_i\}}(S_i + S_p) = \min(S_i) + \min(S_p)$$

Proof: Let $m = \min(S_i) + \min(S_p)$ for two finite sets, $S_i+S_p = \{a+b \mid a\in S_i,\ b\in S_p\}$. In order to prove the theorem, we should verify $m \le \min(S_i+S_p)$. Assume that $m > \min(S_i+S_p)$. Since S_i+S_p is equal to S, thus that is finite set. Assume that a′ and b′ exists such that $a'+b' = \min(S_i+S_p)$, $a'\in S_i$, $b'\in S_p$. For two elements, a and b such that $a_0 + b_0 = m$ and $a_0 \le a'$, $b_0 \le b'$, $(a_0 + b_0)$ becomes m and is less than $(a'+b')$, it is contradiction. Therefore, $m \le \min(S_i+S_p)$. Let $S = S_p = \bigcup_{j=1, j\neq i}^{k} S_j$, by repeating the above procedure for all cases, we can prove theorem.

By theorem 2.1, N-1 nodes are included in the route, so, least cost routes can be found in (4).

$$G = \min[F(R) + F(\tilde{R})] \quad \forall R \cup \tilde{R} = N_j \quad (4)$$

Considering network configuration, we can describe the least cost routing algorithm with the state space relaxation as the follows:

STEP 1: Check the network configuration
  if the network != undirected,
    by using the shortest path algorithm [13], convert the distance matrix to the shortest connected distance matrix
  else initialization (for k = 0),
  $\Phi_t(k) = j,\ \forall j\in N_j$
  $\pi = \Phi_t(k)$;
  $F_0(j, -) = d_{1j}$;
  $F_0(P) = F_0(j, -) + d_{j1},\ p\in \Phi_t(k)$;
  Find R corresponding to $F_0(P)$;
  endif
STEP 2: Compute the recurrence relation
  for k = 1,2,…
    if $\pi == \Phi$ *Let L=k, then* **goto** STEP 3;
    for all $\Phi_t(k)$
      $\Phi_t(k) = j \cup \Phi_t(k)$ with $j > \max(\Phi_t(k))$
      satisfying the constraint, $\Sigma W_{q\in\Phi_t} \le T$
      $\pi = \Phi_t(k)$;
  end for
  for τ such that $\Phi_t == \Phi_t(k)$
      $F_k(j,S) = \min_{q\in S, q\neq j}[F_{k-1}(q, S-\{q\}) + d_{qj}]$;
      $F_k(P) = \min_{(j,S)\in P}[F_k(j,S) + d_{j1}]$;
      Find R corresponding to $F_k(P)$;
  end for
  for τ such that $\Phi_t == \Phi''_t$
    $F_k(j, S) = \infty$;
  end for

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STEP 3: Match and compute the least cost

$$G = \min[F(R) + F(\tilde{R})] \quad \forall R \cup \tilde{R} = N_j$$

STEP 4: Find the least cost routes corresponding to G

Figure-2. Least cost routing algorithm with the state space relaxation.

In the above algorithm, the number of elements included in the set (π) is equal to k+1. If the constraint is not satisfied, $\Phi_t(k\text{-}1)$ element is deleted from π. If all $F_k(j,S)$ is equal to ∞, we cannot extend the route any more, feasible route sets are equal to those obtained at previous stage k (=1,2,.., L-1).

Example 2.1. Let us consider the following example. The capacity of facility at the center node (T) is 8 units. Demands at end-node (W_i) and non-symmetric distance matrix (d_{ij}) in undirected network are given by Table-1.

Table-1. Demands and non-symmetric distance for undirected network.

Demand		**Non-symmetric**				
W_i	node	1	2	3	4	5
-	1	0	3	1	5	4
1	2	1	0	5	4	3
2	3	5	4	0	2	1
3	4	3	1	3	0	3
4	5	5	2	4	1	0

The computation procedure by our least cost algorithm in Figure-2 is shown in Figure-3.

Step	Description
1	π = {2, 3, 4, 5} $F_0(2,\text{-}) = 3$, $F_0(3,\text{-}) = 1$, $F_0(4,\text{-}) = 5$, $F_0(5,\text{-}) = 4$
	$F_0(2) = F_0(2,\text{-}) + d_{21} = 3 + 1 = 4$ ($\boldsymbol{R}$ = 2), $F_0(3) = 6$ ($\boldsymbol{R}$ = 3), $F_0(4) = 8$ ($\boldsymbol{R}$ = 4), $F_0(5) = 9$ ($\boldsymbol{R}$ = 5)
2	π = {(2, 3), (2,4), (2,5), (3,4), (3,5), (4, 5)} $F_1(2,\{3\}) = F_0(3,\text{-}) + d_{32} = 5$, $F_1(2,\{4\}) = 6$, $F_1(2,\{5\}) = 6$, $F_1(3,\{2\}) = 8$, $F_1(3,\{4\}) = 8$, $F_1(3,\{5\}) = 8$, $F_1(4,\{2\}) = 7$, $F_1(4,\{3\}) = 3$, $F_1(4,\{5\}) = 5$, $F_1(5,\{2\}) = 6$, $F_1(5,\{3\}) = 2$, $F_1(5,\{4\}) = 8$
	$F_1(2,3) = \min[F_1(2,\{3\}) + d_{21}, F_1(3,\{2\}) + d_{31}]$ $= \min[6, 13] = 6$ ($\boldsymbol{R}$ = 3,2), $F_1(2,4) = 7$ ($\boldsymbol{R}$ = 4,2), $F_1(2,5) = 7$ ($\boldsymbol{R}$ = 5,2), $F_1(3,4) = 6$ ($\boldsymbol{R}$ = 4,3), $F_1(3,5) = 7$ ($\boldsymbol{R}$ = 3,5), $F_1(4,5) = 8$ ($\boldsymbol{R}$ = 5,4)
	π = {(2, 3, 4), (2, 3, 5), (2, 4, 5)} $F_2(2,\{3,4\}) = \min[F_1(3,4) + d_{32}, F_1(4,\{3\}) + d_{42}]$ $= \min[12, 4] = 4$ $F_2(2,\{3,5\}) = 4$, $F_2(2,\{4,5\}) = 6$, $F_2(3,\{2,4\}) = 10$, $F_2(3,\{2,5\}) = 10$, $F_2(4,\{2,3\}) = 9$, $F_2(4,\{2,5\}) = 7$, $F_2(5,\{2,3\}) = 8$, $F_2(5,\{2,4\}) = 9$
	$F_2(2,3,4) = \min[F_2(2,\{3,4\}) + d_{21}$, $F_2(3,\{2,4\}) + d_{31}$, $F_2(4,\{2,3\}) + d_{41}] = 5$ ($\boldsymbol{R}$ = 3,4,2) $F_2(2,3,5) = 5$ ($\boldsymbol{R}$ = 3,5,2), $F_2(2,4,5) = 7$ ($\boldsymbol{R}$ = 5,4,2)
3	$G = \min[F(3,4,2) + F(5) = 14$, $F(3,5,2) + F(4) = 13$, $F(5,4,2) + F(3) = 13$, $F(3,2) + F(5,4) = 14$, $F(4,2) + F(3,5) = 14$, $F(5,2) + F(3,4) = 13] = 13$
4	Least cost route: 1→5→4→2→1, 1→3→1

Figure-3. Computation procedure of least cost routing algorithm with the state space relaxation.

3. PERFORMANCE EVALUATION

In this section, we compare the route costs between our algorithm and well known benchmark algorithms in the accordance with the network configuration.

3.1. Undirected network

Table-2 shows demands at end-node and symmetric distance network. The capacity of facility at the centre node (T) is 8 units.

Table-2. Demands and symmetric distance for undirected network.

Demand		**Symmetric**				
W_i	node	1	2	3	4	5
-	1	0	1	2	2	2.5
1	2		0	1.2	∞	∞
2	3			0	1	2
3	4				0	2.2
4	5					0

Routing costs for Table-1 and Table-2 are given by Table-3 and Table-4, respectively. TSP partition algorithm builds routes by partitioning TSP tour. Clarke & Write [8] is the benchmark algorithm for this problem. NNR is the routing cost by nearest neighbour rule.

Table-3. Routing costs for non-symmetric network.

Algorithm	**Non-symmetric**	**Cost**
	Routes	
TSP partition	1→3→5→1, 1→4→2→1	14
IP based	1→3→5→2→1, 1→4→1	13
Least cost	1→5→4→2→1, 1→3→1	13*

Table-4. Routing costs for symmetric network.

Algorithm	**Non-symmetric**	**Cost**
	Routes	
Clarke &	1→5→3→4→1,	9.5

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Write	1→2→1	
NNR	1→2→3→4→5→1	7.9
Least cost	1→2→3→4→5→1	7.9*

3.2. Directed network

In the directed network, distance matrix should be converted to the connected distance matrix by the algorithm [13] before applying the least cost routing algorithm. The capacity of facility at the centre node (T) is 8 units. Demands at end-node, distance matrix and the shortest path matrix for directed network is given by Table-5.

Table-5. Demands, distance and shortest path matrix for directed network.

Demand		Distance					Shortest path				
W_i	node	1	2	3	4	5	1	2	3	4	5
-	1	0	2	-	-	4	0	2	3	6	4
1	2	-	0	1	-	-	9	0	1	4	6
2	3	-	-	0	3	5	8	∞	0	3	5
3	4	5	-	-	0	-	5	∞	∞	0	∞
4	5	-	-	-	6	0	11	∞	∞	6	0

Routing costs for Table-5 are given by Table-6. Eulerian tour partition and TSP partition algorithm build routes by partitioning Eulerian tour and TSP tour, respectively.

Table-6. Routing costs for directed network.

Algorithm	Routes	Cost
Eulerian tour partition	1→2→3→4→1, 1→5→1	26
TSP partition	1→2→3→5→1, 1→4→1	30
NNR	1→2→3→4→5, 1→5→1	26
Least cost	1→2→3→4→1, 1→5→1	26*

3.3. Mixed network

In the mixed network, undirected arcs and directed arcs are combined. Like in the directed network, we need the connected distance matrix in order to applying the least cost routing algorithm. The capacity of facility at the centre node (T) is 8 units. Demands at end-node, distance matrix and the shortest path matrix for mixed network are given by Table-7.

Table-7. Demands, distance and shortest path matrix for mixed network.

Demand		Distance						Shortest path					
W_i	node	1	2	3	4	5	6	1	2	3	4	5	6
-	1	0	8	-	2	-	-	0	8	10	2	19	27
1	2	-	0	-	8	-	-	10	0	16	8	25	33
2	3	8	-	0	-	-	-	8	∞	0	∞	∞	∞
3	4	2	-	8	0	17	-	2	∞	8	∞	17	25
4	5	-	-	-	-	0	8	18	∞	∞	24	16	8
5	6	-	-	-	8	8	0	10	∞	16	8	8	∞

Routing costs for Table-7 is given by Table-8. Among three algorithms, our least cost routing algorithm produces the minimum cost routes.

Table-8. Routing costs for mixed network.

Algorithm	Routes	Cost
TSP partition	1→2→5→1, 1→6→4→1, 1→3→1	106
NNR	1→4→3→1, 1→6→1 1→2→1, 1→5→1	110
Least cost	1→2→5→3→1, 1→6→4→1	102*

4. CONCLUSIONS

In this study, we propose the least cost routing algorithm which can find the loop routes regardless of network configuration when demands at end-nodes are deterministic in a centralized network. The proposed algorithm is composed of enumeration phase which reduces the amount of computations by the state space relaxation and matching phase which find the exact solution for small network.

Performance evaluation shows that our least cost routing algorithm produces better solution than previous benchmark algorithms. Our algorithm can be applied to the communication network design including vehicle routing problem. In the future works, more efficient algorithm with the improved space relaxations and optimal algorithm in the problem with dynamic demands is expected.

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