ANALYSIS OF POSITION EFFECTS OF CPVA MECHANISM IN REDUCING VIBRATION OF MULTI DOF DAMPED SYSTEM AND GENERATING ELECTRICAL ENERGY

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ABSTRACT
CPVA (Cantilever piezoelectric vibration absorber) is a mechanism consisting of masses, springs, dampers and piezoelectric cantilever. In this study; CPVA acts as a vibration absorber and can generate electrical energy. CPVA is able to reduce the vibration of the damped system that vibrates translations and rotations due to the shift in position of excitation force on the main mass. At the first natural frequency the damped system is 7.56 rad/s, the CPVA mechanism reduction percentage are 3.52E-7 watt and 20, 36% at natural frequency. CPVA position in reducing vibration of the system that generates the maximum reduction is Dynamic Vibration Absorber (DVA) using a simulation method. The results of the simulation show that the generated power and the highest CPVA reduction percentage are 3.52E-7 watt and 20, 36% at natural frequency. Wiwiek Hendrowati [9] also investigated the mechanisms that use Multilayer Piezoelectric to harvest the kinetic energy of shock absorber. By pairing the Multilayer Piezoelectric Vibration Energy Harvesting (ML PZT VEH) mechanism to shock absorber, the shock absorber performance is undisturbed and the wasted energy can be utilized into electrical energy, i.e. 6.23 volts and 1.6 m Watt.

INTRODUCTION
This paper discusses the multi-DOF vibration reduction analysis caused by the excitation force applied not on the center of gravity of the main mass or the unbalanced stiffness characteristic that can produce translational and rotational vibration. The vibration reduction analysis of the damped system is modeled and simulated. The effect of Cantilever Piezoelectric Vibration Absorber (CPVA) position in reducing vibration of the damped system and the electrical energy produced by Cantilever Piezoelectric is also discussed. CPVA is a mechanism that combines the working principles of DVA and energy harvesting to reduce vibration while generating electrical energy.

METHODOLOGY
Damped system without CPVA
Multi-DOF vibration occurs because the excitation force applied to the main mass is not at the center of gravity. The damped system comprising the main mass \(M_1\), spring \(k_1, k_2\), and damper \(c_1, c_2\), which is excited by a harmonic excitation force as shown in Figure-1. The excitation force applied to the main mass is the result of the base motion connected with the spring \(k_0\). The shift in position of the excitation force with distance \(a\) from the center of gravity of the main mass produce a multi DOF translational and rotational vibration of the main system. Figure-1 is the dynamic model of a damped system without CPVA.
in which

\[ M_1 : \text{main mass (5 kg)} \]
\[ Y : \text{amplitude of excitation displacement (0.01 m)} \]
\[ k_0 : \text{spring coefficients 1 and 2(2N/m)} \]
\[ k_1, k_2 : \text{spring coefficients 1 and 2(3561.6N/m)} \]
\[ c_1, c_2 : \text{damping coefficients 1 and 2(2 Ns/m)} \]
\[ l_1, l_2 : \text{length distance 1 and 2(0.4 m)} \]
\[ \alpha : \text{excitation force distance to the center of gravity of the main mass (0.2 m)} \]

From the dynamic model of the system seen in Figure-1. The dynamic equation of the system can be derived as follows:

\[
\begin{align*}
M_2 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2 + k_3)x_1 + (k_1l_1 - k_2l_2 - k_3a)\dot{\theta}_1 + (c_1l_1 - c_2l_2)\dot{\theta}_1 = k_0y \\
J_1\ddot{\theta}_1 + (c_1l_1 - c_2l_2)\dot{x}_1 + (k_1l_1 - k_2l_2 - k_3a)x_1 + (k_1l_1^2 - k_2l_2^2 - k_3a^2)\dot{\theta}_1 + (c_1l_1^2 - c_2l_2^2)\dot{\theta}_1 = -k_0y. \alpha
\end{align*}
\]

Equations (1) and (2) can be written in matrix form as (3),

\[
[M]\ddot{x} + [C]\dot{x} + [K]x = \{F\}
\]

where \([M]\), \([C]\), and \([K]\) are called the mass, damping, and stiffness matrices, respectively, and are given by

\[
[M] = \begin{bmatrix} M_1 & 0 \\ 0 & \frac{J_1}{l_1} \end{bmatrix}, \quad [C] = \begin{bmatrix} (c_1 + c_2) & (c_1l_1 - c_2l_2) \\ (c_1l_1 - c_2l_2) & (c_1l_1^2 - c_2l_2^2) \end{bmatrix}, \quad [K] = \begin{bmatrix} (k_1 + k_2 + k_3) & (k_1l_1 - k_2l_2 - k_3a) \\ (k_1l_1 - k_2l_2 - k_3a) & (k_1l_1^2 - k_2l_2^2 - k_3a^2) \end{bmatrix}
\]

The motion equation of dynamic model in matrix form as

\[
\begin{bmatrix} M_1 & 0 \\ 0 & \frac{J_1}{l_1} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} (c_1 + c_2) & (c_1l_1 - c_2l_2) \\ (c_1l_1 - c_2l_2) & (c_1l_1^2 - c_2l_2^2) \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} (k_1 + k_2 + k_3) & (k_1l_1 - k_2l_2 - k_3a) \\ (k_1l_1 - k_2l_2 - k_3a) & (k_1l_1^2 - k_2l_2^2 - k_3a^2) \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} K_1y \\ 0 \end{bmatrix}
\]

To determine the natural frequency of the damped system can be determined by a fundamental analysis. Equation (4) can be arranged as follows:

\[
M_1J_1\omega^4 - (J_1(k_1 + k_2 + k_3) + M_1(k_1l_1^2 - k_2l_2^2 - k_3a^2))\omega^2 + (k_1l_1^2 - k_2l_2^2 - k_3a^2) \dot{\theta}_1 + (c_1l_1^2 - c_2l_2^2)\dot{\theta}_1 = -k_0y. \alpha
\]

Equation (5) is called the frequency or characteristic equation because the solution will produce the value of the natural frequency of the system \(\omega_1\) and \(\omega_2\).

**Damped system with CPVA**

In this case, a damped system is added to the CPVA (Cantilever Piezoelectric Vibration Absorber) mechanism which acts as a vibration absorber and can generate electrical energy. Figure-2 shows a damped system with an excitation force at \(\alpha = 0.2\) m and the position of CPVA is shifted as far as \(\beta\) from the center of gravity of the main mass.

From the dynamic model of the damped system with CPVA as seen in Figure-2. The dynamic equation can be derived as follows:

The equation of motion of the main mass can be written as equation (6) for translational motion and as equation (7) for rotational motion.

\[
M_2 \ddot{x}_1 + (c_1 + c_2 + c_3) \dot{x}_1 + (k_1 + k_2 + k_3 + k_0)x_1 + (k_1l_1 - k_2l_2 - k_3a)\dot{\theta}_1 + (c_1l_1 - c_2l_2 + c_3\beta)\dot{\theta}_1 = k_0y
\]

\[
J_1\ddot{\theta}_1 + (c_1l_1 - c_2l_2 + c_3\beta)\dot{x}_1 + (k_1l_1 - k_2l_2 - k_3a)\dot{\theta}_1 + (c_1l_1^2 - c_2l_2^2 + c_3\beta^2)\dot{\theta}_1 = -k_0y. \alpha
\]
The equation of motion of the absorber mass is written in equation (8)

\[ M_2 \ddot{x}_2 - c_3 \dot{x}_2 + c_5 \dot{x}_2 - k_3 x_3 + (k_3 + k_{4eq}) x_2 - k_{4eq} x_3 - k_4 \ddot{\theta}_1 \beta - c_5 \dot{\theta}_1 \beta + \Gamma. n. V_p = 0 \]  

(8)

The equation of motion of the cantilever piezoelectric mass is written in equation (9)

\[ M_{3eq} \ddot{x}_3 - k_{4eq} x_2 + k_{4eq} x_3 - \Gamma. n. V_p = 0 \]  

(9)

where

\[ M_{3eq} : \text{the equivalent mass of the cantilever piezoelectric} \]

\[ k_3 : \text{spring coefficient 3 (400 N/m)} \]

\[ k_{4eq} : \text{the equivalent spring coefficient of the cantilever piezoelectric (805 N/m)} \]

\[ c_3 : \text{damping coefficient of the cantilever piezoelectric mass} \]

\[ n : \text{quantity of piezoelectric cantilever} \]

\[ r : \text{the coupling factor of the cantilever piezoelectric} \]

The specification of piezoelectric material used in this study can be seen in Table-1.

### Table-1. Parameters of piezoelectric materials.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric Mass</td>
<td>( M_{pzt} )</td>
<td>3 x 10^{-4}</td>
<td>kg</td>
</tr>
<tr>
<td>Thickness of Piezoelectric</td>
<td>( t )</td>
<td>1 x 10^{-3}</td>
<td>m</td>
</tr>
<tr>
<td>Width of Piezoelectric</td>
<td>( w_{pzt} )</td>
<td>6 x 10^{-3}</td>
<td>m</td>
</tr>
<tr>
<td>Length of Piezoelectric</td>
<td>( L_{pzt} )</td>
<td>17.8 x 10^{-3}</td>
<td>m</td>
</tr>
<tr>
<td>Capacitance</td>
<td>( C_{pzt} )</td>
<td>244 x 10^{-12}</td>
<td>F</td>
</tr>
<tr>
<td>Strain coefficient of Piezoelectric</td>
<td>( d_{31} )</td>
<td>110 x 10^{-12}</td>
<td>C/N</td>
</tr>
<tr>
<td>Electromechanical coupling factor</td>
<td>( k_{31} )</td>
<td>12</td>
<td>%</td>
</tr>
<tr>
<td>Stiffness constant of Piezoelectric</td>
<td>( k_{pzt} )</td>
<td>5.75 x 10^{-1}</td>
<td>N/m</td>
</tr>
<tr>
<td>Modulus Young</td>
<td>( E )</td>
<td>3 x 10^{-9}</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

The matrices \([M]\), \([C]\) and \([K]\) are formulated according to equation (10)

\[
[M] = \begin{bmatrix}
M_1 & 0 & 0 & 0 \\
0 & l_1 & 0 & 0 \\
0 & 0 & M_2 & 0 \\
0 & 0 & 0 & M_3 \\
\end{bmatrix}
\]

\[
[K] = \begin{bmatrix}
(K_1 + K_2 + K_3 + K_0) & (K_1 l_1 - K_2 l_2 + K_3 l_3 - K_0 \alpha) & -K_3 & 0 \\
(K_1 l_2 - K_2 l_1 + K_3 l_3 - K_0 \alpha) & (K_1 l_1^2 - K_2 l_2^2 + K_3 l_3^2 + K_0 \alpha^2) & -K_3 \beta & 0 \\
-K_3 & -K_3 \beta & 0 & 0 \\
-K_4 & K_4 & -K_4 & 0 \\
\end{bmatrix}
\]

(10)

Based on equation (6) to (10), the characteristic equation of the damped system with CPVA will produce four natural frequencies.

### RESULT AND DISCUSSIONS

From the dynamic equation of motion of the damped system without CPVA, the bode diagram graph in Figure-3 is obtained. The figure shows that the shift in location of the excitation force of the center of gravity of the main mass affects the natural frequency of the system.

Figure-3 shows the graph of the system natural frequency due to the shifting of the excitation force as far as \( \alpha \) from the center of gravity of the main mass. At the distance of the excitation force of \( \alpha = 0 \), the system will experience translational vibration with natural frequency of 38.14 rad/s. Furthermore, shifting the distance of excitation force between \( \alpha = 0.2 \) m and \( \alpha = 0.4 \) m will cause the system to have 2DOF, i.e. translation and rotation, so that the damped system has 2 natural frequencies.
Where, the farer the distance between the excitation forces to the center of gravity of the main mass, increase the first natural frequency of the system. Furthermore, the second natural frequency of the damped system has a magnitude similar to the first natural frequency in the 1DOF damped system. In addition, the displacement amplitude occurring at its natural frequency will be smaller, if the excitation force is farer from the center of gravity of the main mass.

Figure-4. Bode diagram of the damped system with CPVA.

Installation of the CPVA mechanism on the main mass causes the reduction of vibration response in the main mass. Figure-5 shows the main mass translational vibration response without CPVA and with CPVA, which is placed as far as β from the main mass center. The excitation force is given at α = 0.2m from the main mass center.

Figure-5. The translational displacement and reduction of the main system.

Analysis of the main mass vibration reduction

The reduction analysis is done at the first natural frequency of 7.56 rad/s from the main system because the design of CPVA is used to damped the first natural frequency of the main system. Installation of CPVA at distance β = 0m from the center of gravity of main mass causes the amplitude of the translational vibration response to coincide with the amplitude of the main system response without CPVA.

Whereas in the position of CPVA away from the center of gravity of main mass, the amplitude of the main mass vibration response at the first natural frequency is low. At the first natural frequency, the maximum vibration response reduction at β = 0.4 m is about 98%. However, at β = 0m, the reduction value is -28%. Its means at β = 0m, the translational vibration of the system is not reduced.

The position of CPVA with distance β from the center of gravity of main mass also produces a rotational vibration response of the main mass. In Figure-6, an angular vibration response with β = 0.2m coincides with the angular vibration response of the main mass without CPVA. Likewise, at the distance β that far from the center of gravity of main mass, the angular vibration response at the first natural frequency of the main system is lower.
The maximum value of the angular vibration reduction occurs at $\beta = 0.4\ m$ approximately 67%.

Figure-6. The angular displacement and reduction of the main system.

Generated electrical energy from the CPVA mechanism

1. 400 piezoelectric cantilevers are also installed in the CPVA mechanism. When the mass of the absorber vibrates, the piezoelectric mass also moves, causing a deflection on cantilever and generating electricity. The power generated by the piezoelectric cantilever is shown in Figure-7.

Figure-7. Electrical power generation.

The Figure-8 shows the effect of the position of CPVA from the center of gravity of main mass to the generated electrical energy. At a distance $\beta = 0\ m$, CPVA is unable to damp the translational vibration of the main mass, so that the cantilever piezoelectric deflection is getting bigger and the electrical energy generated is also large. While at $\beta = 0.4\ m$, CPVA is able to reduce translational vibration by 98% and rotation by 67%, so the cantilever piezoelectric deflection is small. This causes the generated electrical energy is also small.

The number of piezoelectric cantilever also affects the generated electrical energy. Figure 9 shows that a growing number of cantilever piezoelectric cause greater stiffness and smaller deflections. While the number of cantilever piezoelectric as much as 1400 pieces produce electrical energy for 8.6 $e^{-05}$ W.

Figure-8. Electrical energy of CPVA mechanism.

CONCLUSIONS

In this study, the position of CPVA at the center of gravity of the main mass will affect the first and second natural frequency ranges. Installation of CPVA on the main mass is designed to reduce vibration at the first natural frequency. At the first natural frequency of 7.56 rad/s and the CPVA position at $\beta = 0.4\ m$, causing a
reduction in the maximum translational vibration response of about 98% and a rotational vibration of about 67%. However, the position of CPVA at $\beta = 0$ m, the system is not reduced its translational vibration. While the number of cantilever piezoelectric as many as 1,400 pieces generate electrical energy for 8.6 e-05 W.

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REFERENCES


