



ANALYSIS OF POSITION EFFECTS OF CPVA MECHANISM IN REDUCING VIBRATION OF MULTI DOF DAMPED SYSTEM AND GENERATING ELECTRICAL ENERGY

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ABSTRACT

CPVA (Cantilever piezoelectric vibration absorber) is a mechanism consisting of masses, springs, dampers and piezoelectric cantilever. In this study; CPVA acts as a vibration absorber and can generate electrical energy. CPVA is able to reduce the vibration of the damped system that vibrates translations and rotations due to the shift in position of excitation force on the main mass. At the first natural frequency the damped system is 7.56 rad/s, the CPVA mechanism reduction value with distance $\beta = 0.4\text{m}$ from the center of main mass, capable of damping the maximum translational vibration by 98% and the maximum rotational vibration of 67%. Furthermore, the number of piezoelectric cantilever as many as 1400 pieces produce electrical energy for $8.6 \times 10^{-5} \text{ W}$.

Keywords: cantilever piezoelectric, CPVA mechanism, vibration absorber.

INTRODUCTION

Research on Dynamic Vibration Absorber (DVA) was done by Pachpute [1] and Dobry [2] to find out the response of the system without DVA and with damped DVA. From the experimental results, the system that produces the maximum reduction is Dynamic Vibration Absorber (DVA) with the optimum ratio of mass ratio (μ) = 0.2, the damping ratio (ζ) = 0.125 and the tuning ratio (f) = 0.7 to 0.8. Similarly, research conducted by Aida AAD [3], about the simulation and testing methods of cantilever absorber Dynamic Vibration Absorber with the ratio of the distance (r/L) variation. The results stated that the optimum reduction occurred at $r/L = 0.4$.

Patel's research [4] investigated the use of cantilever piezoelectric to determine the effect of piezoelectric length and mass on natural frequency and its deflection, as the deflection affects the generated electrical energy. Similarly, Mineto's research [5], de Silva [6] which states that the greatest electrical energy occurs at a natural frequency.

G.A. Hassaan [7] conducted a DVA system research to reduce vibration in the main system and serve as a harvester of electrical energy. It is found that the optimum natural frequency of absorber-harvester with mass ratio between 0.05 to 0.45 and damping ratio between 0.1 to 0.4. W. Rahma Efendy [8] investigated the mechanism of CPVA (Cantilever Piezoelectric Vibration Absorber) using a simulation method. The results of the simulation show that the generated power and the highest CPVA reduction percentage are $3.52 \times 10^{-7} \text{ watt}$ and 20, 36% at natural frequency. Wiwiek Hendrowati [9] also investigated the mechanisms that use Multilayer Piezoelectric to harvest the kinetic energy of shock absorber. By pairing the Multilayer Piezoelectric Vibration Energy Harvesting (ML PZT VEH) mechanism to shock absorber, the shock absorber performance is undisturbed and the wasted energy can be utilized into electrical energy, i.e. 6.23 volts and 1.6 m Watt.

This paper discusses the multi-DOF vibration reduction analysis caused by the excitation force applied not on the center of gravity of the main mass or the unbalanced stiffness characteristic that can produce translational and rotational vibration. The vibration reduction analysis of the damped system is modeled and simulated. The effect of Cantilever Piezoelectric Vibration Absorber (CPVA) position in reducing vibration of the damped system and the electrical energy produced by Cantilever Piezoelectric is also discussed. CPVA is a mechanism that combines the working principles of DVA and energy harvesting to reduce vibration while generating electrical energy.

METHODOLOGY

Damped system without CPVA

Multi-DOF vibration occurs because the excitation force applied to the main mass is not at the center of gravity. The damped system comprising the main mass (M_1), spring (k_1, k_2), and damper (c_1, c_2), which is excited by a harmonic excitation force as shown in Figure-1. The excitation force applied to the main mass is the result of the base motion connected with the spring k_0 . The shift in position of the excitation force with distance α from the center of gravity of the main mass produce a multi DOF translational and rotational vibration of the main system. Figure-1 is the dynamic model of a damped system without CPVA.

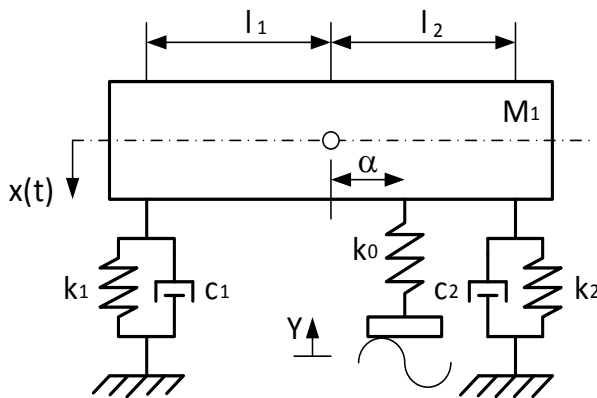


Figure-1. The dynamic model of damped system without CPVA.

in which

- M_1 : main mass (5 kg)
 Y : amplitude of excitation displacement (0.01 m)
 k_0 : spring coefficient of excitation force (152.2 N/m)
 k_1, k_2 : spring coefficients 1 and 2 (3561.6 N/m)
 c_1, c_2 : damping coefficients 1 and 2 (2 Ns/m)
 l_1, l_2 : length distance 1 and 2 (0.4 m)
 α : excitation force distance to the center of gravity of the main mass (0.2 m)

From the dynamic model of the system seen in Figure-1. The dynamic equation of the system can be derived as follows:

$$M_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_0 + k_2) x_1 + (k_1 l_1 - k_2 l_2 - k_0 \alpha) \theta_1 + (c_1 l_1 - c_2 l_2) \dot{\theta}_1 = k_0 y \quad (1)$$

$$J_1 \ddot{\theta}_1 + (c_1 l_1 - c_2 l_2) \dot{x}_1 + (k_1 l_1 - k_2 l_2 - k_0 \alpha) x_1 + (k_1 l_1^2 - k_2 l_2^2 - k_0 \alpha^2) \theta_1 + (c_1 l_1^2 - c_2 l_2^2) \dot{\theta}_1 = -k_0 y \cdot \alpha \quad (2)$$

Equations (1) and (2) can be written in matrix form as (3),

$$[M] \ddot{x} + [C] \dot{x} + [K] x = \{F\} \quad (3)$$

where $[M]$, $[C]$, and $[K]$ are called the mass, damping, and stiffness matrices, respectively, and are given by

$$[M] = \begin{bmatrix} M_1 & 0 \\ 0 & J_1 \end{bmatrix}$$

$$[C] = \begin{bmatrix} (C_1 + C_2) & (C_1 l_1 - C_2 l_2) \\ (C_1 l_1 - C_2 l_2) & (c_1 l_1^2 - c_2 l_2^2) \end{bmatrix}$$

$$[K] = \begin{bmatrix} (K_1 + K_2 + K_0) & (K_1 l_1 - K_2 l_2 - K_0 \alpha) \\ (K_1 l_1 - K_2 l_2 - K_0 \alpha) & (k_1 l_1^2 - k_2 l_2^2 - k_0 \alpha^2) \end{bmatrix}$$

The motion equation of dynamic model in matrix form as

$$\begin{bmatrix} M_1 & 0 \\ 0 & J_1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \end{Bmatrix} + \begin{bmatrix} (C_1 + C_2) & (C_1 l_1 - C_2 l_2) \\ (C_1 l_1 - C_2 l_2) & (c_1 l_1^2 - c_2 l_2^2) \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{\theta}_1 \end{Bmatrix} + \begin{bmatrix} (K_1 + K_2 + K_0) & (K_1 l_1 - K_2 l_2 - K_0 \alpha) \\ (K_1 l_1 - K_2 l_2 - K_0 \alpha) & (k_1 l_1^2 - k_2 l_2^2 - K_0 \alpha^2) \end{bmatrix} \begin{Bmatrix} x_1 \\ \theta_1 \end{Bmatrix} = \begin{Bmatrix} k_0 y \\ 0 \end{Bmatrix} \quad (4)$$

To determine the natural frequency of the damped system can be determined by a fundamental analysis. Equation (4) can be arranged as follows:

$$M_1 J_1 \omega^4 - (J_1 (K_1 + K_2 + K_0) + M_1 (K_1 l_1^2 - K_2 l_2^2 - K_0 \alpha^2)) \omega^2 + (K_1 + K_2 + K_0) (K_1 l_1^2 - K_2 l_2^2 - K_0 \alpha^2) + (K_1 l_1 - K_2 l_2 - K_0 \alpha) (K_1 l_1 - K_2 l_2 - K_0 \alpha) = 0 \quad (5)$$

Equation (5) is called the frequency or characteristic equation because the solution will produce the value of the natural frequency of the system ω_1 and ω_2 .

Damped system with CPVA

In this case, a damped system is added to the CPVA (Cantilever Piezoelectric Vibration Absorber) mechanism which acts as a vibration absorber and can generate electrical energy. Figure-3 shows a damped system with an excitation force at $\alpha = 0.2$ m and the position of CPVA is shifted as far as β from the center of gravity of the main mass.

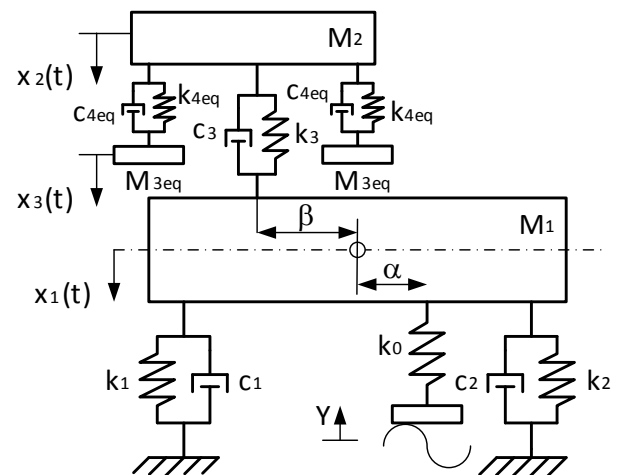


Figure-2. The dynamic model of damped system with CPVA.

From the dynamic model of the damped system with CPVA as seen in Figure-2. The dynamic equation can be derived as follows:

The equation of motion of the main mass can be written as equation (6) for translational motion and as equation (7) for rotational motion.

$$M_1 \ddot{x}_1 + (c_1 + c_2 + c_3) \dot{x}_1 + (k_1 + k_2 + k_3 + k_0) x_1 + (k_1 l_1 - k_2 l_2 + k_3 \beta - k_0 \alpha) \theta_1 + (c_1 l_1 - c_2 l_2 + c_3 \beta) \dot{\theta}_1 - k_3 x_2 - c_3 \dot{x}_2 = k_0 y \quad (6)$$

$$J_1 \ddot{\theta}_1 + (c_1 l_1 - c_2 l_2 + c_3 \beta) \dot{x}_1 + (k_1 l_1 - k_2 l_2 - k_0 \alpha + k_3 \beta) x_1 + (k_1 l_1^2 - k_2 l_2^2 + k_0 \alpha^2 + k_3 \beta^2) \theta_1 + (c_1 l_1^2 - c_2 l_2^2 + c_3 \beta^2) \dot{\theta}_1 - K_3 \beta \cdot x_2 - c_3 \beta \cdot \dot{x}_2 = -k_0 y \cdot \alpha \quad (7)$$



The equation of motion of the absorber mass written in equation (8)

$$M_2\ddot{X}_2 - c_3\dot{x}_1 + c_3\dot{x}_2 - k_3x_1 + (k_3 + k_{4eq})x_2 - k_{4eq}x_3 - k_3\theta_1\beta - c_3\dot{\theta}_1\beta + \Gamma \cdot n \cdot V_p = 0 \quad (8)$$

The equation of motion of the cantilever piezoelectric mass written in equation (9)

$$M_{3eq}\ddot{X}_3 - k_{4eq}x_2 + k_{4eq}x_3 - \Gamma \cdot n \cdot V_p = 0 \quad (9)$$

where

M_{3eq} : the equivalent mass of the cantilever piezoelectric

k_3 : spring coefficient 3 (400 N/m)

k_{4eq} : the equivalent spring coefficient of the cantilever piezoelectric (805 N/m)

c_3 : damping coefficient 3 (2 Ns/m)

n : quantity of piezoelectric cantilever

Γ : the coupling factor of the cantilever piezoelectric

The specification of piezoelectric material used in this study can be seen in Table-1.

Table-1. Parameters of piezoelectric materials.

Parameters	Symbol	Value	Unit
Piezoelectric Mass	M_{pzt}	3×10^{-4}	kg
Thickness of Piezoelectric	t	1×10^{-3}	m
Width of Piezoelectric	w_{pzt}	6×10^{-3}	m
Length of Piezoelectric	L_{pzt}	17.8×10^{-3}	m
Capacitance	C_{pzt}	244×10^{-12}	F
Strain coefficient of Piezoelectric	d_{31}	110×10^{-12}	C/N
Electromechanical coupling factor	k_{31}	12	%
Stiffness constant of Piezoelectric	k_{pzt}	5.75×10^{-1}	N/m
Modulus Young	E	3×10^{-9}	N/m^2

The matrices $[M]$, $[C]$ and $[K]$ are formulated according to equation (10)

$$[M] = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & M_3 \end{bmatrix}$$

$$[C] = \begin{bmatrix} (C_1 + C_2 + C_3) & (C_1l_1 - Cl_2 + C_3\beta) & -C_3 & 0 \\ (C_1l_1 - Cl_2 + C_3\beta) & (C_1l_1^2 - C_2l_2^2 + C_3\beta^2) & -C_3\beta & 0 \\ -C_3 & -C_3\beta & C_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K] = \begin{bmatrix} (K_1 + K_2 + K_3 + K_0) & (K_1l_1 - K_2l_2 + K_3l_3 - K_0\alpha) & -K_3 & 0 \\ (K_1l_1 - K_2l_2 + K_3l_3 - K_0\alpha) & (K_1l_1^2 - K_2l_2^2 + K_3l_3^2 + K_0\alpha^2) & -K_3\beta & 0 \\ -K_3 & -K_3\beta & (K_3 + K_4) & -K_4 \\ 0 & 0 & -K_4 & K_4 \end{bmatrix} \quad (10)$$

Based on equation (6) to (10), the characteristic equation of the damped system with CPVA will produce four natural frequencies.

RESULT AND DISCUSSIONS

From the dynamic equation of motion of the damped system without CPVA, the bode diagram graph in Figure-3 is obtained. The figure shows that the shift in location of the excitation force of the center of gravity of the main mass affects the natural frequency of the system.

Figure-3 shows the graph of the system natural frequency due to the shifting of the excitation force as far as α from the center of gravity of the main mass. At the distance of the excitation force of $\alpha = 0$, the system will

experience translational vibration with natural frequency of 38.14 rad/s. Furthermore, shifting the distance of excitation force between $\alpha = 0.2$ m and $\alpha = 0.4$ m will cause the system to have 2DOF, i.e. translation and rotation, so that the damped system has 2 natural frequencies.

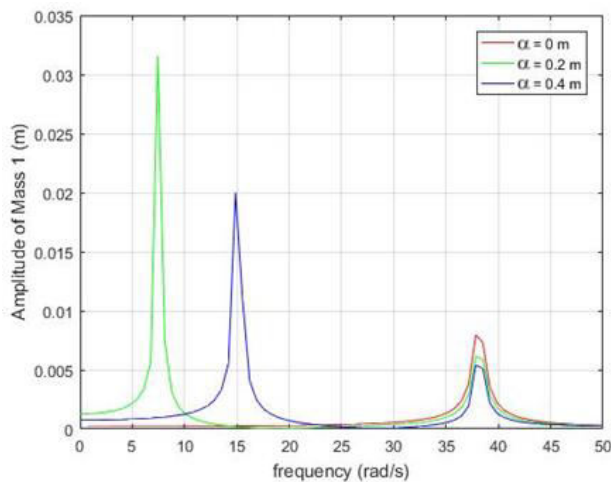


Figure-3. Bode diagram of the damped system without CPVA.

Where, the farther the distance between the excitation forces to the center of gravity of the main mass, increase the first natural frequency of the system. Furthermore, the second natural frequency of the damped system has a magnitude similar to the first natural frequency in the 1DOF damped system. In addition, the displacement amplitude occurring at its natural frequency will be smaller, if the excitation force is farther from the center of gravity of the main mass.

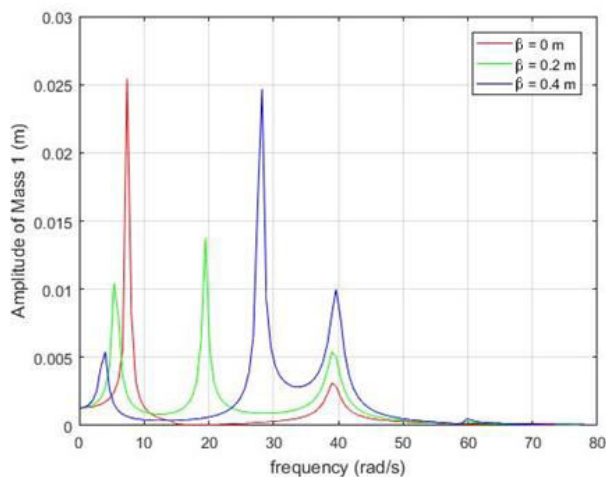


Figure-4. Bode diagram of the damped system with CPVA.

Figure-4 shows that the position of CPVA with a certain distance from the center of the main mass will accelerate the achievement of the first natural frequency. But the position of CPVA away from the center of the main mass will make the first and second natural frequency ranges larger. While at third and fourth natural frequency, the effect of the position of CPVA is not very significant, at all distances β .

Analysis of the main mass vibration reduction

Installation of the CPVA mechanism on the main mass causes the reduction of vibration response in the main mass. Figure-5 shows the main mass translational vibration response without CPVA and with CPVA, which is placed as far as β from the main mass center. The excitation force is given at $\alpha = 0.2m$ from the main mass center.

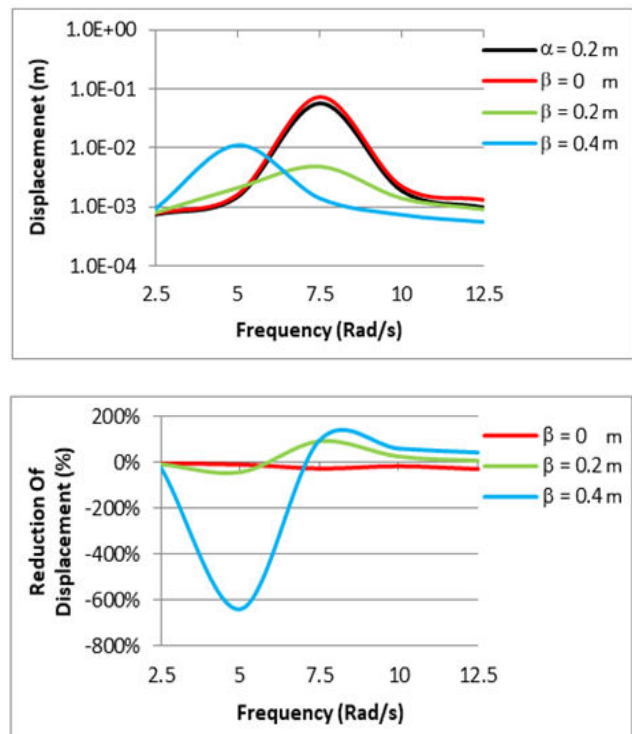


Figure-5. The translational displacement and reduction of the main system.

The reduction analysis is done at the first natural frequency of 7.56 rad/s from the main system because the design of CPVA is used to damped the first natural frequency of the main system. Installation of CPVA at distance $\beta = 0m$ from the center of gravity of main mass causes the amplitude of the translational vibration response to coincide with the amplitude of the main system response without CPVA.

Whereas in the position of CPVA away from the center of gravity of main mass, the amplitude of the main mass vibration response at the first natural frequency is low. At the first natural frequency, the maximum vibration response reduction at $\beta = 0.4 m$ is about 98%. However, at $\beta = 0m$, the reduction value is -28%. Its means at $\beta = 0m$, the translational vibration of the system is not reduced.

The position of CPVA with distance β from the center of gravity of main mass also produces a rotational vibration response of the main mass. In Figure-6, an angular vibration response with $\beta = 0.2m$ coincides with the angular vibration response of the main mass without CPVA. Likewise, at the distance β that far from the center of gravity of main mass, the angular vibration response at the first natural frequency of the main system is lower.



The maximum value of the angular vibration reduction occurs at $\beta = 0.4$ m approximately 67%.

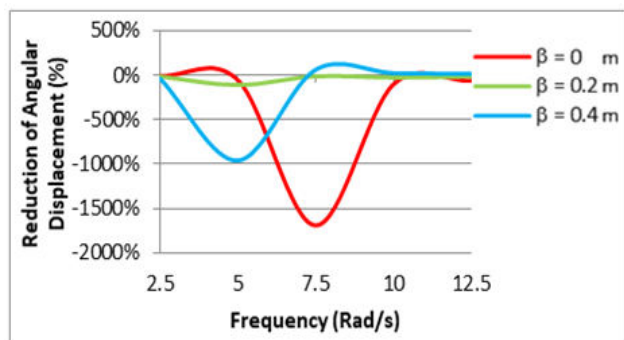
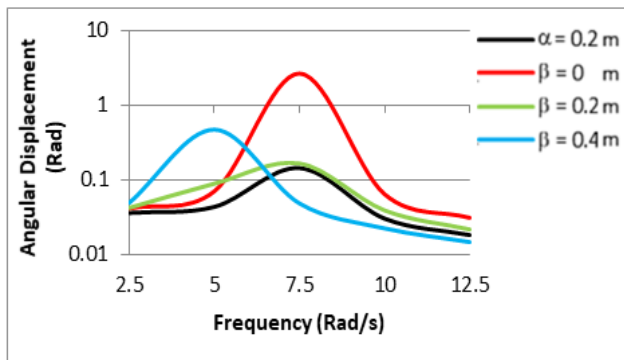


Figure-6. The angular displacement and reduction of the main system.

Generated electrical energy from the CPVA mechanism

1, 400 piezoelectric cantilevers are also installed in the CPVA mechanism. When the mass of the absorber vibrates, the piezoelectric mass also moves, causing a deflection on cantilever and generating electricity. The power generated by the piezoelectric cantilever is shown in Figure-7.

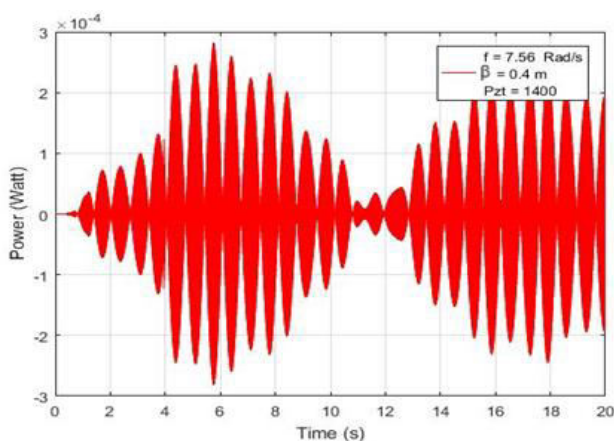


Figure-7. Electrical power generation.

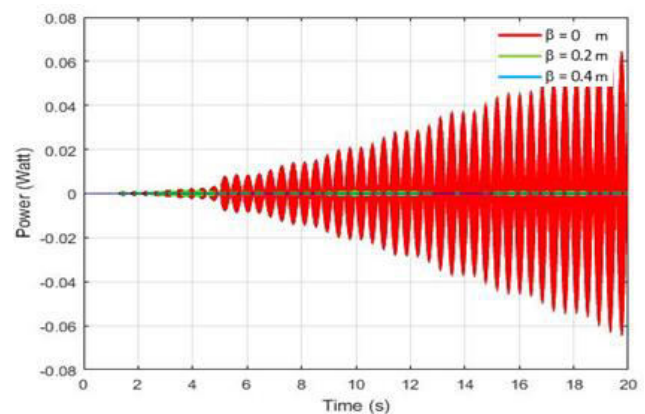


Figure-8. Electrical energy of CPVA mechanism.

The Figure-8 shows the effect of the position of CPVA from the center of gravity of main mass to the generated electrical energy. At a distance $\beta = 0$ m, CPVA is unable to damp our the translational vibration of the main mass, so that the cantilever piezoelectric deflection is getting bigger and the electrical energy generated is also large. While at $\beta = 0.4$ m, CPVA is able to reduce translational vibration by 98% and rotation by 67%, so the cantilever piezoelectric deflection is small. This causes the generated electrical energy is also small.

The number of piezoelectric cantilever also affects the generated electrical energy. Figure 9 shows that a growing number of cantilever piezoelectric cause greater stiffness and smaller deflections. While the number of cantilever piezoelectric as much as 1400 pieces produce electrical energy for 8.6×10^{-5} W.

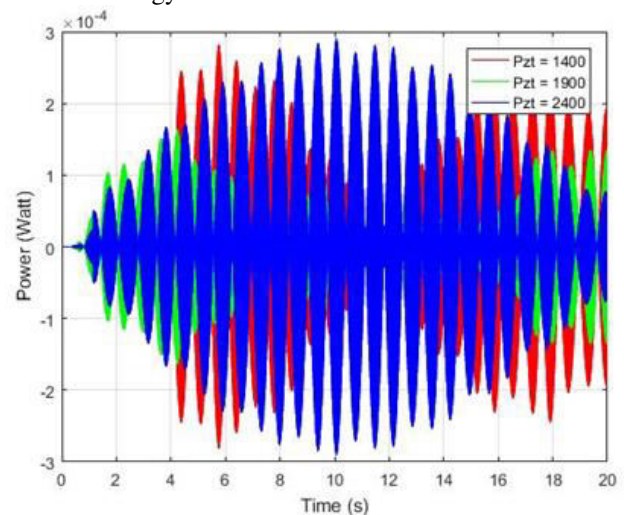


Figure-9. The influence of the number of piezoelectric cantilever on the generated electrical energy.

CONCLUSIONS

In this study, the position of CPVA at the center of gravity of the main mass will affect the first and second natural frequency ranges. Installation of CPVA on the main mass is designed to reduce vibration at the first natural frequency. At the first natural frequency of 7.56 rad/s and the CPVA position at $\beta = 0.4$ m, causing a



reduction in the maximum translational vibration response of about 98% and a rotational vibration of about 67%. However, the position of CPVA at $\beta = 0$ m, the system is not reduced its translational vibration. While the number of cantilever piezoelectric as many as 1,400 pieces generate electrical energy for 8.6 e-05 W.

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