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# SOLUTION OF AN UNSTEADY FLOW THROUGH POROUS MEDIA PAST ON MOVING VERTICAL PLATE WITH VARIABLE TEMPERATURE AND HEAT SOURCE IN THE PRESENCE OF INCLINED MAGNETIC FIELD IN THE PRESENCE OF VISCOUS DISSIPATION

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## ABSTRACT

In this paper we analyze and study the problem of an unsteady flow through porous media past on moving vertical plate with variable temperature and heat source in the presence of inclined magnetic field and viscous dissipation is studied. The governing equations are discritized by Galerkin finite-element method and are solved by Crank-Nicolson method using C-Program.The effects of variable parameters to the velocity and temperature profiles are discussed graphically and the numerical values obtained for skin-friction have been tabulated.

**Keywords:** inclined magnetic field, viscous dissipation, heat source, porous media, finite-element method.

## INTRODUCTION

The problem of free convection flows past a vertical surface has been studied extensively because of its wide application in many branches of engineering and sciences. Power generation systems, cooling of nuclear reactors, Geophysics, agriculture and thermal insulation. Rajput and Kumar [1] have analyzed MHD flow past an impulsively started moving vertical plate with variable temperature and mass diffusion. Seth and Ghosh [2] studied an unsteady hydro-magnetic flow in a rotating channel in the presence of inclined magnetic field. The effect of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous plate has been studied by Seddek and Salama [3]. Chamkha AJ *et al* [4] presented unsteady MHD free convection flow past an exponentially accelerated vertical plate with mass transfer, chemical reaction and thermal radiation. R.N. Barik [5] studied free convection heat and mass transfer MHD Flow in a vertical channel in the presence of the chemical reaction. Hossain M. A. [6] analyzed the effect on of hall current on unsteady hydromagnetic free-convection flow near an infinite vertical porous plate. Mohmed Abd Ei-Aziz [7] presented unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. U.S Rajput and S. Kumar [8] have studied radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. A. Ogulu [9] studied the influence of radiation absorption on unsteady free convection and mass transfer flow in the presence of a uniform magnetic field. T. Arunkumar and L Anand Babu [10] have analyzed the study of Radiation effect of MHD flow past an impulsive started vertical plate with variable temperature and uniform mass diffusion through A finite element method

The objective of this paper is to study about the unsteady flow through porous media past on moving vertical plate with variable temperature and heat source in the presence of inclined magnetic field and viscous dissipation.

## FORMULATION OF THE PROBLEM

Consider an unsteady MHD flow of a viscous incompressible radiating fluid by a vertical non conducting plate and the plate impulsively started moving with velocity $u_0$ taking into account in the presence of viscous dissipation and heat source. A uniform magnetic field $B_0$ is assumed to be applied on the plate with angle $\alpha$. Initially the plate and the fluid are at same temperature $T_\infty^*$. At time $t^* > 0$, temperature of the plate is increased to $T_w^*$. The governing equations of flow field are as under follows:

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} = n\frac{\partial^2 u^*}{\partial y^{*2}} + gb\left(T^* - T_\infty^*\right) + \left(\frac{s B_0^2 \sin^2 a}{r} - \frac{v}{k^*}\right)u^* \quad (1)$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{rC_p}\frac{\partial T^{*2}}{\partial y^{*2}} + \frac{m}{rC_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{Q_0}{rC_p}\left(T^* - T_\infty^*\right) \quad (2)$$

where $u^*$ is the velocity components in, $g$ is the gravitational acceleration, $b$ is the thermal expansion coefficient, $T^*$ is the thermal temperature inside the thermal boundary layer, $k$ is the thermal conductivity, $r$ is the fluid density, $\mu -$ is the coefficient of viscosity, $K$ is the permeability of the medium $C_p$ is the specific heat at constant pressure, $s$ is the electric conductivity, $T$ is

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the temperature of the fluid far away from the plate, $t^*$ - is time.

The boundary conditions are:

$$\begin{cases} t^* \le 0 : u^* = 0, T^* = T_\infty^* & \forall y^* \\ t^* > 0 : u^* = u_0,\ T^* = T_\infty^* + \left(T_w^* - T_\infty^*\right)\dfrac{u_0^2 t^*}{\nu} & at\ y^* = 0 \\ u^* \to 0 \quad T^* \to T_\infty^* & as\ y^* \to \infty \end{cases} \tag{3}$$

Where $u_o$ is the velocity of the fluid, $T_w^*$ and $C_w^*$ are the temperature and concentration of the wall respectively, $t^*$ is the time.

Introducing the following non- dimensional quantities,

$$\begin{aligned} & u = \frac{u^*}{u_0}, y = \frac{u_0 y^*}{\nu}, n = \frac{n^*}{u_0}, \Pr = \frac{\nu r C_p}{k}, K = \frac{k^* u_0^2}{\nu^2}, t = \frac{t^* u_0^2}{\nu}, Q = \frac{\nu Q_0}{r C_p u_0^2}, \nu = \frac{m}{r} \\ & q = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, Gr = \frac{g b \nu \left(T_w^* - T_\infty^*\right)}{u_0^3}, Ha^2 = \frac{s B_0^2 l^2}{m}, l^2 = \frac{\nu^2}{u_0^2}, Ec = \frac{u_0^2}{c_p (T_w^* - T_\infty^*)}, \end{aligned} \tag{4}$$

where $Gr$, $\Pr$, $Ha$, $Ec, l$ and $Q$ are the thermal Grashof number, Prandtl number, Hartmann number, Eckert number, characteristic length scale and heat absorption parameter respectively.

With the help of non- dimensional quantities, equations (1) and (2) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - \left(M + \frac{1}{K}\right)u \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Q\theta \tag{6}$$

Here $M = Ha^2 * \sin^2 \alpha$

The corresponding dimensions less boundary condition are

$$\begin{cases} t \le 0 : u = 0, q = 0 & \forall y \\ t > 0 : u = 1, q = t & at\ y = 0 \\ u \to 0 \quad q \to 0 & as\ y \to \infty \end{cases} \tag{7}$$

## METHOD OF SOLUTION

On solving the equations (5) and (6). By applying Galerkin finite element method for equation (5) over the element $(e)$, $\left(y_j \le y \le y_k\right)$ is:

$$\int_{y_j}^{y_k} \left\{ N^{(e)T}\left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - N u^{(e)} + P\right]\right\} dy = 0 \tag{8}$$

Where $P = (Gr)\theta$, $N = M + \dfrac{1}{K}$ integrating the first term in equation (8) by parts and neglecting the first term weget

$$\int_{y_j}^{y_k} \left\{ \frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y} + N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t} + N u^{(e)} - P\right)\right\} dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)} =$ be the linear piecewise approximation solution over the element $(e)$ $(y_j \le y \le y_k)$ where $N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix}$, $\phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T$ and $N_j = \dfrac{y_k - y}{y_k - y_j}, N_k = \dfrac{y - y_j}{y_k - y_j}$ are the basis functions.

On simplifying we get

$$\frac{1}{l^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} + \frac{N l^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{P l^{(e)}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where dot are denotes differentiation with respect to $y$. Assembling the element equations for two consecutive elements $\left(y_{i-1} \le y \le y_i\right)$ and $\left(y_i \le y \le y_{i+1}\right)$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_{i-1} \\ \dot{u}_i \\ \dot{u}_{i+1} \end{bmatrix} + \frac{N}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{P}{2}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{9}$$

Now put row corresponding to the node $i$ to zero, from equation (10) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{6}\left[\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right] + \frac{1}{h^2}\left[-u_{i-1} + 2u_i - u_{i+1}\right] + \frac{N}{6}\left[u_{i-1} + 4u_i + u_{i+1}\right] = P$$

Applying Crank - Nicholson method to the above equation, we get

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^{j} + A_5 u_i^{j} + A_6 u_{i+1}^{j} + P^* \quad (10)$$

where

$A_1 = A_3 = 2 - 6r + Nk \qquad A_4 = A_6 = 2 + 6r - Nk$

$A_2 = 8 + 12r + 4Nk \qquad A_5 = 8 - 12r - 4Nk, \quad P^* = 12Pk = 12kGr\theta_i^j$

Now from equation (6) following equations obtained:

$$B_1\theta_{i-1}^{j+1} + B_2\theta_i^{j+1} + B_3\theta_{i+1}^{j+1} = B_4\theta_{i-1}^{j} + B_5\theta_i^{j} + B_6\theta_{i+1}^{j} + P^{**} \quad (11)$$

Where

$B_1 = B_3 = 2\Pr - 6r - kQ\Pr$, $B_4 = B_6 = 2\Pr + 6r + kQ\Pr$, $P^{**} = 12P_1k = 12k\Pr Ec\left(\frac{\partial u_i}{\partial y_i}\right)^2$

$B_2 = 4\Pr + 12r - 4Qk\Pr$, $\quad B_5 = 4\Pr - 12r + 4Qk\Pr$,

Here, $r = \dfrac{k}{h^2}$ and $h, k$ are mesh size along the $y$ direction and the time direction respectively. Index $i$ refers to the space, and $j$ refers to the time. In the Equations (10) - (11), taking $i = 1,\ldots,n$ and using boundary conditions (7), the following system of equations are obtained:

$$A_i X_i = B_i, \qquad i = 1,\ldots,n \quad (12)$$

Where $A_i$'s are matrix of order $n$ and $X_i, B_i$'s column matrices having $n$ components. The solutions of above systems of equations are obtained by using the Thomas algorithm for velocity, temperature and concentration. Also the numerical solutions are obtained by executing the C-program with the smaller values of $h$ and $k$. No significant change was observed in $u$, and $\theta$ then the Galerkin finite element method is stable and convergent. The dimension less skin friction obtained as $\tau = \left(\dfrac{\partial u}{\partial y}\right)_{y=0}$

## RESULTS AND DISCUSSIONS

The numerical values were computed with respect to the physical parameters, likethermal Grashofnumber $Gr$, permeability $K$, Eckert number $Ec$, Prandtl number $\Pr$, time $t$, angle of inclination of magnetic field α, Hartmann number $Ha$, and heat source parameter as shown in figures.

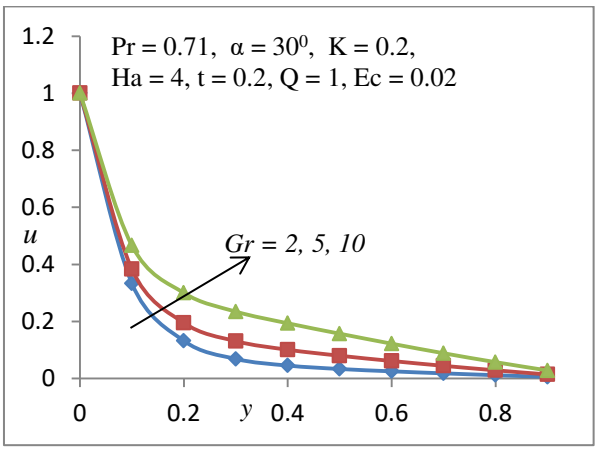


**Figure-1.** Effect of Gr on velocity profiles.

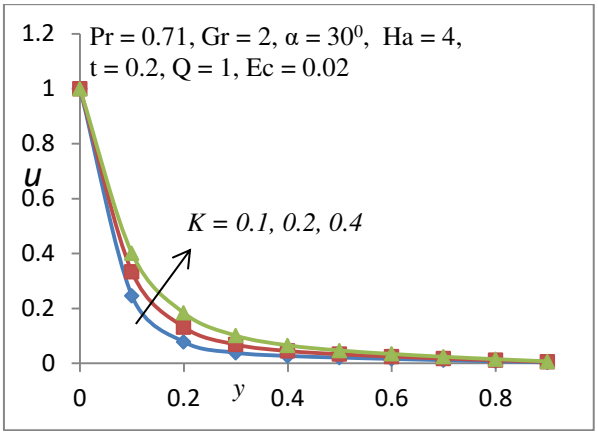


**Figure-2.** Effect of K on velocity profiles.

Figures 1 and 2 illustrate the behavior of the velocity for different values of the *Gr* and *K*. The numerical results show that the effect of increasing values of Grashof number and permeability results in a decreasing velocity. From figure 3 it is observed that an increasing in the Eckert number $Ec$ leads to rise in the values of velocity. Figures 4 and 5 represent the velocity and temperature profiles for different values of the Prandtlnumber $\Pr$. It is noticed that the velocity and temperature decrease as there increase in the Pr values

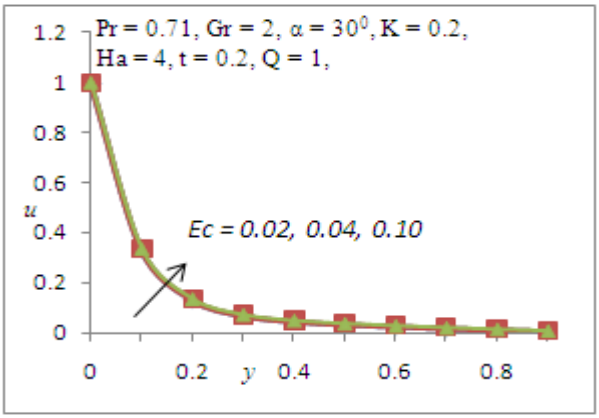


**Figure-3.** Effect of Ec on velocity profiles.

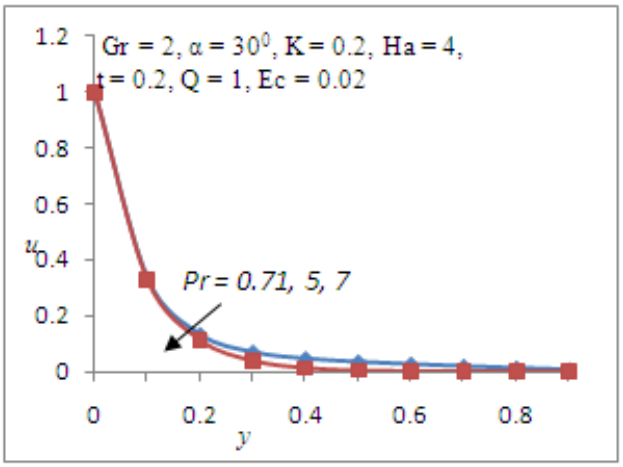


**Figure-4.** Effect of Pr on velocity profiles.

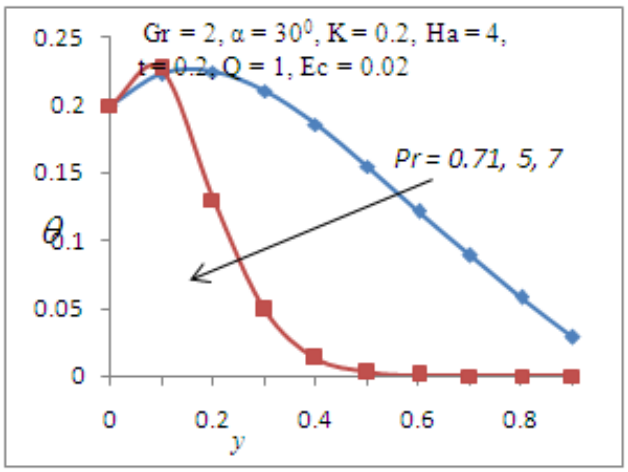


**Figure-5.** Effect of Pr on temperature profiles.

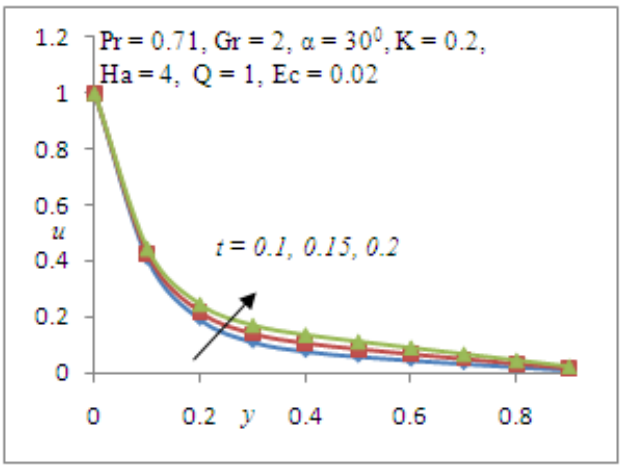


**Figure-6.** Effect of *t* on velocity profiles.

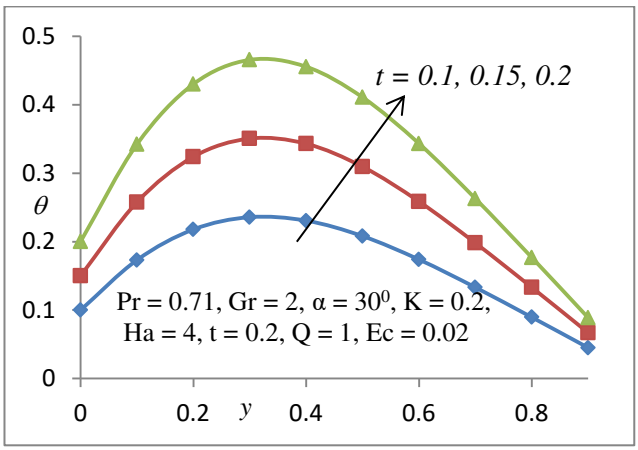


**Figure-7.** Effect of *t* on temperature profiles.

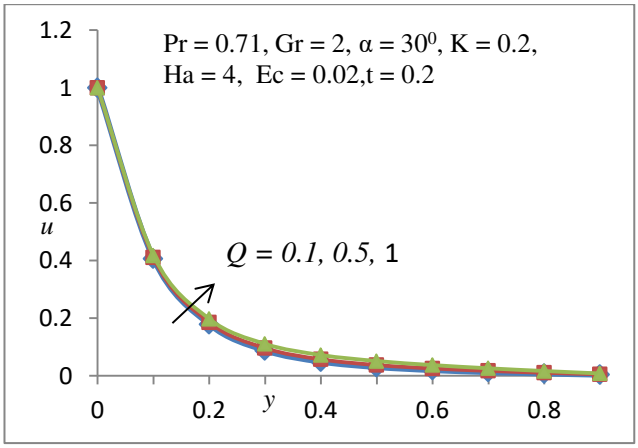


**Figure-8.** Effect of *Q* on velocity profiles.

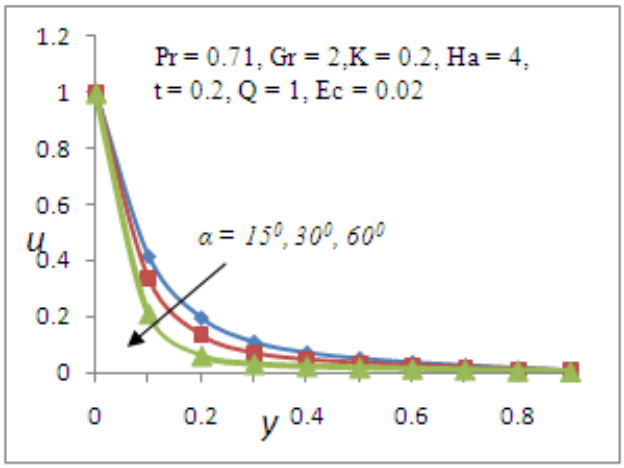


**Figure-9.** Effect of $\alpha$ on velocity profile.

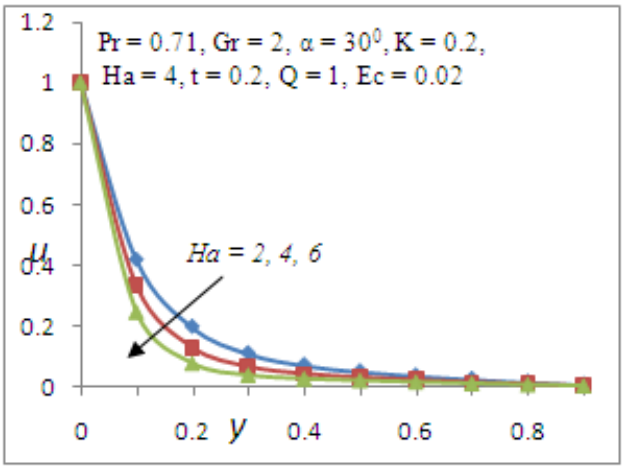


**Figure-10.** Effect of $Ha$ on velocity profiles.

Figures 6 and 7 represent the velocity and temperature profiles for different values of *t*. As the time increases, the velocity and temperature increase.

The effect of the heat source parameter on the velocity and temperature are shown in Figures 8 and 11. As heat source parameter increases, velocity and temperature increases.

Figures 9 and 10 illustrate the behavior of the velocity and temperature for different values of angle of inclination of magnetic field α and Hartmann number (Ha). It is observed that an increase in α and *Ha*leads to a decrease in the values of velocity and temperature.

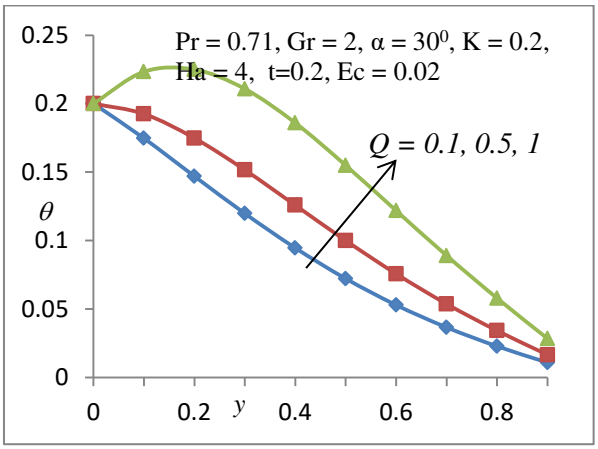


**Figure-11.** Effect of *Q* on temperature profiles.

**Table values of Skin friction.**

| Pr | Gr | Ha | K | α | Q | Ec | t | $\tau$ |
|---|---|---|---|---|---|---|---|---|
| 0.71 | 2 | 2 | 0.2 | 15 | 1 | 0.02 | 0.2 | -2.30513 |
| 0.71 | 2 | 2 | 0.2 | 30 | 1 | 0.02 | 0.2 | -2.27401 |
| 7 | 2 | 2 | 0.2 | 30 | 1 | 0.02 | 0.2 | -2.30063 |
| 0.71 | 4 | 2 | 0.2 | 30 | 1 | 0.02 | 0.2 | -2.10432 |
| 0.71 | 2 | 4 | 0.2 | 30 | 1 | 0.02 | 0.2 | -2.85542 |
| 0.71 | 2 | 2 | 0.4 | 30 | 1 | 0.02 | 0.2 | -1.65822 |
| 0.71 | 2 | 2 | 0.2 | 30 | 0.5 | 0.02 | 0.2 | -2.27401 |
| 0.71 | 2 | 2 | 0.2 | 30 | 1 | 0.04 | 0.2 | -2.27548 |
| 0.71 | 2 | 2 | 0.2 | 30 | 1 | 0.02 | 0.1 | -2.36712 |

## CONCLUSIONS

In this paper a numerical study has been done to study an unsteady flow through porous media past on moving vertical plate with variable temperature and heat source in the presence of inclined magnetic field and viscous dissipation. Results for the model have been derived by Galarkin finite element technique. Some conclusions of the study are as below:

- Velocity increases with the increase in thermal Grashofnumber $Gr$, permeability $K$, Eckert number $Ec$, heat source parameter Q and time $t$.
- Velocity decreases with the increase in Prandtlnumber $\Pr$, an angle of inclination of magnetic field α, Hartmann number $Ha$.
- Temperature decreases with the increase in Prandtlnumber $\Pr$.
- Temperature increases with the increase in time *t* and heat source parameter Q.
- Skin friction decreases with the increase Hartmann number *Ha*.

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