SORET EFFECT ON UNSTEADY MHD FREE CONVECTIVE TWO IMMISCIBLE FLUID FLOW THROUGH A HORIZONTAL CHANNEL WITH HEAT AND MASS TRANSFER

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ABSTRACT

Unsteady MHD Free convective two immiscible fluid flow through a Horizontal channel with Heat and mass transfer have been studied in this paper. The effect of Soret is also taken into account. The equations are solved analytically and numerically with the appropriate boundary conditions for each fluid and the solutions have been studied. The governing equations of the flow were transformed into ordinary differential equations by a regular perturbation method and the expression for the velocity; temperature and concentration field have been presented graphically.

Keywords: MHD, heat transfer, mass transfer, immiscible fluid, soret effect.

INTRODUCTION

MHD is the study of the magnetic properties of electrically conducting fluids. MHD was first initiated by the Swedish electrical engineer Hannes Alfvén (1942), Shercliff (1956), Sparrow and Cess (1961), Singh and Ram (1978), Abdulla (1986), Singh (1993) among others have studied several motions of these electrically conducting fluids. MHD flows have applications in solar system based physics, cosmic fluid dynamics. Most of the problem relating to the petroleum industry, Plasma physics, magnetic field dynamics etc involved in multi fluid flow situations. The immiscible fluid flow through a porous medium and heat transfer is significant in the problem of petroleum extraction and transport. Examining the wide range of applications of such flow, some authors and scholars have made their contribution due to the importance of Soret effects for the fluids with very light molecular weights many investigators have studied and reported the results for these flows.

Chamka [1] who considered the flow of two immiscible fluids in porous and non porous channels. Also he studied about unsteady MHD free convective heat and mass transfer effect. Umavathi [2, 3] reported on unsteady MHD two fluid flow and heat transfer in a Horizontal channel and Vafai and Kim [1990] was one of the earlier who studied the exact solution for the fluid mechanics of the interface region between a porous medium and a fluid layer. Malashetty and Umavathi [1997] studied two phase MHD flow and heat transfer in an inclined channel. Reddy and N. B. Reddy [2009] analyzed the radiation and mass transfer effect on an unsteady MHD Free Concentration flow past a heated vertical porous plate with viscous dissipation. Kumar et el., [2009] discussed the unsteady MHD and heat transfer of two viscous immiscible fluids through a porous medium in Horizontal channel. B. K. Sharma and Kailash Yadav Dufour and soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium with heat source and chemical reaction and radiation effect. Simon [2013] studied the convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient.

In the above investigation the effect of soret is neglected in most of the studies on multiple phase flows. This present study hereby investigates the effect of soret on unsteady MHD Free convective immiscible fluid flow through a horizontal channel with Heat and Mass Transfer. The equations of continuity, linear momentum, energy equations and diffusion equations, which governs the flow field are solved by using a regular perturbation method. Results and discussion are presented graphically by using MATLAB.

Formulation of the problem

The geometry considered here consists of two immiscible fluids having heat at constant pressure Cp with porous upper channel and non-porous lower channel bounded by two infinite horizontal parallel plates extending in the X and Z directions with the Y-direction normal to the plates. The regions 0≤y≤h and -h≤y≤0 are denoted as Region-I and Region-II respectively. The fluid flowing through Region-I is having density ρ1, dynamic viscosity µ1, thermal conductivity k1, thermal diffusivity D1. Similarly the fluid flowing through Region-II is having density ρ2, dynamic viscosity µ2, thermal conductivity k2, thermal diffusivity D2.

All the variables are functions of y’ and t’ only, due to the bounding surface being infinitely long along the x’ axis. The flow is assumed to be fully developed and that all fluid properties are constants. The magnetic field Reynolds number is assumed very small. Hence the governing equations of the fluid flow for the different regions are as follows:
\[ \frac{\partial U_1}{\partial y} = 0 \]  

Assuming that the boundary and interface conditions on velocity are no slip, given that at the boundary and interface, the fluid particles are at rest, \( x \) component of the velocity vanish at the wall.

The boundary and interface conditions on the velocity for both fluids are:

\[ U_1'(h) = 0, \quad U_2'(-h) = 0, \quad U_1(h) = U_2(h), \quad \mu_1 \frac{\partial U_1}{\partial y} = \mu_2 \frac{\partial U_2}{\partial y} \text{ at } y' = 0 \]

The boundary and interface conditions on the temperature for both fluids are:

\[ T_1'(h) = T_{w1}', T_2'(-h) = T_{w2}', T_1'(0) = T_2'(0), \quad k_1 \frac{\partial T_1}{\partial y} = k_2 \frac{\partial T_2}{\partial y} \text{ at } y' = 0 \]

The boundary and interface conditions on the concentration for both fluids are:

\[ C_1(h) = C_{w1}, C_1'(-h) = C_{w2}, C_1'(0) = C_2'(0), D_1 \frac{\partial C_1}{\partial y} = D_2 \frac{\partial C_2}{\partial y} \text{ when } y' = 0 \]

The equations (1) and (5) implies that \( V_1' \) and \( V_2' \) are independent of \( y' \), they are functions of time alone.

Hence \( V' = V_0(1 + \epsilon A e^{i\omega t}) \)  

\( \frac{\partial q}{\partial y} = \frac{\partial U_1}{\partial y} + \frac{\partial U_2}{\partial y} = \frac{\partial T_1}{\partial y} = \frac{\partial T_2}{\partial y} = \frac{\partial C_1}{\partial y} = \frac{\partial C_2}{\partial y} = \frac{\partial \rho}{\partial y} = \frac{\partial \theta}{\partial y} = \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y} \text{ at } y = 0 \)  

\( \theta_1(1) = 1, \quad \theta_2(-1) = 0 \)  

\( \theta_1(0) = \theta_2(0), \frac{\partial q}{\partial y} = \frac{\partial U_1}{\partial y} = \frac{\partial U_2}{\partial y} = \frac{\partial T_1}{\partial y} = \frac{\partial T_2}{\partial y} = \frac{\partial C_1}{\partial y} = \frac{\partial C_2}{\partial y} = \frac{\partial \rho}{\partial y} = \frac{\partial \theta}{\partial y} = \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y} \text{ when } y = 0 \)

**SOLUTION PROCEDURE OF THE PROBLEM**

To solve the governing equations (13) to (18) under the boundary and interface conditions (19) to (21), one expand \( U_1(y,t), \theta_1(y,t), C_1(y,t), U_2(y,t), \theta_2(y,t) \).
\( C_2(y,t) \), as a power series on the perturbative parameter \( \epsilon \). Here, let \( \epsilon \leq 1 \). Thus

\[
\begin{align*}
U_1(y,t) &= U_{10}(y) + \epsilon e^{i\omega t} U_{11}(y) \\
\theta_1(y,t) &= \Theta_{10}(y) + \epsilon e^{i\omega t} \Theta_{11}(y) \\
C_1(y,t) &= C_{10}(y) + \epsilon e^{i\omega t} C_{11}(y) \\
U_2(y,t) &= U_{20}(y) + \epsilon e^{i\omega t} U_{21}(y) \\
\theta_2(y,t) &= \Theta_{20}(y) + \epsilon e^{i\omega t} \Theta_{21}(y) \\
C_2(y,t) &= C_{20}(y) + \epsilon e^{i\omega t} C_{21}(y)
\end{align*}
\]

Substitute the above set of equations into the equations (13) to (18) and equate the periodic and non-periodic terms, and neglect the terms containing \( \epsilon^2 \), one can get the following set of ordinary differential equations:

**REGION-I:**

Non – Periodic Terms:

\[
\frac{\partial^2 U_{10}}{\partial y^2} + \frac{\partial U_{10}}{\partial y} - M^2 U_{10} = - P - Gr \Theta_{10} + Gc C_{10}
\quad (22)
\]

\[
\frac{\partial^2 \Theta_{10}}{\partial y^2} - Pr \frac{\partial \Theta_{10}}{\partial y} - F \Theta_{10} = 0
\quad (23)
\]

\[
\frac{\partial^2 C_{10}}{\partial y^2} - Sc \frac{\partial C_{10}}{\partial y} = Sr \frac{\partial^2 \Theta_{10}}{\partial y^2}
\quad (24)
\]

Periodic Terms:

\[
\frac{\partial^2 U_{11}}{\partial y^2} + \frac{\partial U_{11}}{\partial y} - (M^2 + i\omega) U_{11} = \frac{\partial U_{10}}{\partial y} - Gr \Theta_{11} - Gc C_{11}
\quad (25)
\]

\[
\frac{\partial^2 \Theta_{11}}{\partial y^2} - Pr \frac{\partial \Theta_{11}}{\partial y} - (F + i\omega Pr) \Theta_{11} = Pr \frac{\partial \Theta_{10}}{\partial y}
\quad (26)
\]

\[
\frac{\partial^2 C_{11}}{\partial y^2} - Sc \frac{\partial C_{11}}{\partial y} = Sc \frac{\partial C_{10}}{\partial y} - Sr \frac{\partial^2 \Theta_{11}}{\partial y^2}
\quad (27)
\]

**REGION-II:**

Non-periodic terms:

\[
\frac{\partial^2 U_{20}}{\partial y^2} - \frac{1}{a_1} \frac{\partial U_{20}}{\partial y} - \left( \frac{m_2 M^2 + a_1^2 k_2^2}{a_1 c_1^2} \right) U_{20} = 0
\quad (28)
\]

\[
\frac{\partial^2 \Theta_{20}}{\partial y^2} - \frac{Pr}{a_1 c_1^2} \frac{\partial \Theta_{20}}{\partial y} = 0
\quad (29)
\]

\[
\frac{\partial^2 C_{20}}{\partial y^2} - \frac{Sc}{\gamma_1} \frac{\partial C_{20}}{\partial y} = 0
\quad (30)
\]

Periodic terms:

\[
\frac{\partial^2 U_{21}}{\partial y^2} - \frac{1}{a_1} \frac{\partial U_{21}}{\partial y} - \left( \frac{m_2 M^2 + a_1^2 k_2^2 + i\omega}{a_1 c_1^2} \right) U_{21} = 0
\quad (31)
\]

The equations (22) to (33) are ordinary linear differential equations with constant coefficients. The following are the boundary and interface conditions:

**Non- periodic terms**

\[
U_{10}(1) = 0 , U_{20}(-1) = 0 , U_{10}(0) = U_{20}(0) , \frac{\partial U_{10}}{\partial y} = \theta_{10}(1) = 1 , \theta_{20}(-1) = 0 , \theta_{10}(0) = \theta_{20}(0) , \frac{\partial \theta_{10}}{\partial y} = 0
\quad (34)
\]

**Periodic terms**

\[
\theta_{11}(1) = 0 , \theta_{21}(-1) = 0 , \theta_{11}(0) = \theta_{21}(0) , \frac{\partial \theta_{11}}{\partial y} = 0
\quad (35)
\]

The solutions of the differential equations (22) to (33) using the above boundary conditions (34) to (39) are

\[
U_{10}(y) = C_1 e^{m_2 y} + C_2 e^{m_3 y} + K_1 + K_2 e^{m_1 y} + K_3 e^{m_2 y} + K_4 e^{m_3 y}
\quad (40)
\]

\[
U_{20}(y) = C_{17} e^{m_{17} y} + C_{18} e^{m_{18} y} + K_{20} + K_{21} e^{m_{13} y} + K_{22} e^{m_{14} y} + K_{23} e^{m_{15} y} + K_{24} e^{m_{16} y}
\quad (41)
\]

\[
\frac{\partial \theta_{10}}{\partial y} = C_1 e^{m_1 y} + C_2 e^{m_2 y}
\quad (42)
\]

\[
\frac{\partial \theta_{20}}{\partial y} = C_{13} e^{m_{13} y} + C_{14} e^{m_{14} y}
\quad (43)
\]

\[
C_{10}(y) = C_3 e^{m_3 y} + C_4 e^{m_4 y} + K_{39} e^{m_{15} y} + K_{40} e^{m_{16} y}
\quad (44)
\]

\[
C_{20}(y) = C_{19} e^{m_{15} y} + C_{16} e^{m_{16} y} + K_{47} e^{m_{17} y} + K_{48} e^{m_{18} y}
\quad (45)
\]

\[
U_{11}(y) = C_{11} e^{m_{11} y} + C_{12} e^{m_{12} y} + K_{10} e^{m_{10} y} + K_{11} e^{m_{12} y} + K_{12} e^{m_{13} y} + K_{13} e^{m_{14} y} + K_{14} e^{m_{15} y} + K_{15} e^{m_{16} y}
\quad (46)
\]

\[
\frac{\partial^2 \theta_{11}}{\partial y^2} - \frac{Pr}{\beta_1 c_1^2} \frac{\partial \theta_{11}}{\partial y} - \left( \frac{m_2 M^2 + a_1^2 k_2^2 + i\omega}{a_1 c_1^2} \right) \theta_{21} = \frac{Pr}{\beta_1 c_1^2} \frac{\partial \theta_{20}}{\partial y}
\quad (32)
\]

\[
\frac{\partial^2 \theta_{21}}{\partial y^2} - \frac{Sc}{\gamma_1} \frac{\partial \theta_{21}}{\partial y} - \left( \frac{m_2 M^2 + a_1^2 k_2^2 + i\omega}{a_1 c_1^2} \right) \Theta_{21} + \frac{Sr}{\gamma_1} \frac{\partial^2 \theta_{21}}{\partial y^2} = 0
\quad (33)
\]
\[ K_{16} \text{e}^{m_1 y} + K_{17} \text{e}^{m_2 y} + K_{18} \text{e}^{m_3 y} + K_{19} \text{e}^{m_4 y} \] (46)

\[ U_{21}(y) = C_{23} \text{e}^{m_3 y} + C_{24} \text{e}^{m_4 y} + K_{25} \text{e}^{m_5 y} + K_{26} \text{e}^{m_6 y} + K_{30} \text{e}^{m_1 y} + K_{31} \text{e}^{m_2 y} + K_{32} \text{e}^{m_3 y} + K_{33} \text{e}^{m_4 y} + K_{34} \text{e}^{m_5 y} + K_{35} \text{e}^{m_6 y} + K_{36} \text{e}^{m_7 y} + K_{37} \text{e}^{m_8 y} + K_{38} \text{e}^{m_9 y} + K_{39} \text{e}^{m_{10} y} \] (47)

\[ \theta_{11}(y) = C_{7} \text{e}^{m_1 y} + C_{8} \text{e}^{m_2 y} + K_{5} \text{e}^{m_3 y} + K_{6} \text{e}^{m_4 y} \] (48)

\[ \theta_{21}(y) = C_{19} \text{e}^{m_3 y} + C_{20} \text{e}^{m_4 y} + K_{25} \text{e}^{m_5 y} + K_{26} \text{e}^{m_6 y} \] (49)

\[ C_{21}(y) = C_{6} \text{e}^{m_3 y} + C_{10} \text{e}^{m_4 y} + K_{6} \text{e}^{m_3 y} + K_{8} \text{e}^{m_4 y} + K_{41} \text{e}^{m_5 y} + K_{42} \text{e}^{m_6 y} + K_{43} \text{e}^{m_7 y} + K_{44} \text{e}^{m_8 y} + K_{45} \text{e}^{m_9 y} + K_{46} \text{e}^{m_{10} y} \] (50)

\[ C_{21}(y) = C_{21} \text{e}^{m_1 y} + C_{22} \text{e}^{m_2 y} + C_{23} \text{e}^{m_3 y} + C_{24} \text{e}^{m_4 y} + C_{25} \text{e}^{m_5 y} + C_{26} \text{e}^{m_6 y} + C_{27} \text{e}^{m_7 y} + C_{28} \text{e}^{m_8 y} + C_{29} \text{e}^{m_9 y} + C_{30} \text{e}^{m_{10} y} \] (51)

**RESULTS AND DISCUSSIONS**

The Numerical evaluations of the Analytical results reported in the previous section was performed and the set of results is reported graphically in Figures 1 to 7 for the unsteady free convective two immiscible fluid flow through a horizontal channel on the upper porous channel and non-porous lower channel bounded by two infinite horizontal parallel plates under the influence of magnetic field and sorpt effect by assigning different numerical values such as \( Gr=5 \), \( Gc=5 \), \( Pr=1 \), \( Sc=0.78 \), \( F=3 \), \( \beta=1 \), \( \alpha_1=1 \), \( \alpha_2=1 \), \( \alpha_3=1 \), \( \alpha_4=1 \), \( \alpha_5=1 \), \( \alpha_6=1 \), \( \beta_1=1 \), \( \beta_2=1 \), \( \beta_3=1 \), \( \beta_4=1 \), \( \beta_5=1 \), \( \beta_6=1 \), \( \beta_7=1 \), \( \beta_8=1 \), \( \beta_9=1 \), \( \beta_{10}=1 \), \( \gamma_1=1 \), \( \gamma_2=1 \), \( \gamma_3=1 \), \( \gamma_4=1 \), \( \gamma_5=1 \), \( \gamma_6=1 \), \( \gamma_7=1 \), \( \gamma_8=1 \), \( \gamma_9=1 \), \( \gamma_{10}=1 \), \( \phi_1=1 \), \( \phi_2=1 \), \( \phi_3=1 \), \( \phi_4=1 \), \( \phi_5=1 \), \( \phi_6=1 \), \( \phi_7=1 \), \( \phi_8=1 \), \( \phi_9=1 \), \( \phi_{10}=1 \), \( \eta_1=1 \), \( \eta_2=1 \), \( \eta_3=1 \), \( \eta_4=1 \), \( \eta_5=1 \), \( \eta_6=1 \), \( \eta_7=1 \), \( \eta_8=1 \), \( \eta_9=1 \), \( \eta_{10}=1 \), \( \omega_1=1 \), \( \omega_2=1 \), \( \omega_3=1 \), \( \omega_4=1 \), \( \omega_5=1 \), \( \omega_6=1 \), \( \omega_7=1 \), \( \omega_8=1 \), \( \omega_9=1 \), \( \omega_{10}=1 \), \( \xi_1=1 \), \( \xi_2=1 \), \( \xi_3=1 \), \( \xi_4=1 \), \( \xi_5=1 \), \( \xi_6=1 \), \( \xi_7=1 \), \( \xi_8=1 \), \( \xi_9=1 \), \( \xi_{10}=1 \), \( \zeta_1=1 \), \( \zeta_2=1 \), \( \zeta_3=1 \), \( \zeta_4=1 \), \( \zeta_5=1 \), \( \zeta_6=1 \), \( \zeta_7=1 \), \( \zeta_8=1 \), \( \zeta_9=1 \), \( \zeta_{10}=1 \), \( \mu_1=1 \), \( \mu_2=1 \), \( \mu_3=1 \), \( \mu_4=1 \), \( \mu_5=1 \), \( \mu_6=1 \), \( \mu_7=1 \), \( \mu_8=1 \), \( \mu_9=1 \), \( \mu_{10}=1 \), \( \nu_1=1 \), \( \nu_2=1 \), \( \nu_3=1 \), \( \nu_4=1 \), \( \nu_5=1 \), \( \nu_6=1 \), \( \nu_7=1 \), \( \nu_8=1 \), \( \nu_9=1 \), \( \nu_{10}=1 \), \( \lambda_1=1 \), \( \lambda_2=1 \), \( \lambda_3=1 \), \( \lambda_4=1 \), \( \lambda_5=1 \), \( \lambda_6=1 \), \( \lambda_7=1 \), \( \lambda_8=1 \), \( \lambda_9=1 \), \( \lambda_{10}=1 \), \( \sigma_1=1 \), \( \sigma_2=1 \), \( \sigma_3=1 \), \( \sigma_4=1 \), \( \sigma_5=1 \), \( \sigma_6=1 \), \( \sigma_7=1 \), \( \sigma_8=1 \), \( \sigma_9=1 \), \( \sigma_{10}=1 \), \( \tau_1=1 \), \( \tau_2=1 \), \( \tau_3=1 \), \( \tau_4=1 \), \( \tau_5=1 \), \( \tau_6=1 \), \( \tau_7=1 \), \( \tau_8=1 \), \( \tau_9=1 \), \( \tau_{10}=1 \), using MATLAB.

Further the values of \( \epsilon = 0.0007 \) and the frequency parameter \( \omega = 30 \) are fixed for all the graphs.

The influence of heat absorption parameter \( H1 \) and Sorpt effect \( sr \) are displayed through the velocity profiles in Figures 1 to 5 respectively.

From these figures it is seen that an increase in either of the Heat absorption parameter or the sorpt effect leads to a delay in the velocity field while it enhances with an increase in the value of the sorpt number.

Figure-1 and Figure-2 displays the effect of the Grashof number \( Gr \) and \( Gc \) for Heat and Mass transfer respectively on the velocity field. It is clearly seen that an increases in the Region I and decreases in the lower nonporous region II channel for various values .The behavior of the fluid velocity \( u \) perpendicular to the channel length on \( Gr \) is observed. It is seen that when \( Gr \) increases, \( u \) diminishes towards the downward direction of the channel. Also it is clear that the Grashof number under Heat transfer increases the velocity of the fluid more than for Mass transfer.

Figure-3 describes the effect of Permeability parameter on the velocity \( u_1 \) in region I and suppress the velocity \( u_2 \) in region-II. the velocity is low for a less than unity Permeability Parameter further increase above unity reports causes an increase in the velocity.

Figure-4 exhibit velocity profile for various values of sorpt number. It is observed that the velocity decreases with larger velocity boundary layer in Region II as compared to region I to the end of the boundary layer. This observation concludes with the fact that increase in the thickness of a fluid reduces the velocity field of that fluid.

In Figure-5, the momentum diffusivity gradually dominates the thermal diffusivity, the velocity of the flow decreases with slight alteration in the porous region while the variation in the velocity is not significant even if the Prandtl number increases for region II.

Figure-6 shows the variation of temperature profile for different values of the Prandtl number. As the value of \( m \) increases, the temperature of the fluid increases in the both regions, one can easily see that the temperature of the fluid in the region I is lesser than the temperature of the fluid in the region II.

Figure-7 represents the effects of Soret number on the concentration profile. As the Soret number increases, the concentration profile of the flow is having a slight change in the Region I and in the upper part of the clear Region II one can see the difference of various parameters. It also shows us that the increase in the value of the concentration of the fluid increases in the boundary layer region but no effect is observed from onwards in the Figures.
Figure-2. Unsteady free convective two immiscible fluid flow through a horizontal channel with $Gr=3$.

Figure-3. Unsteady free convective two immiscible fluid flow through a horizontal channel with $K=1$.

Figure-4. Unsteady free convective two immiscible fluid flow through a horizontal channel with $Sr=.95$.

Figure-5. Unsteady free convective two immiscible fluid flow through a horizontal channel with $Pr=5$.
Figures-6. Unsteady free convective two immiscible fluid flow through a horizontal channel with Pr=1.

Figure-7. Unsteady free convective two immiscible fluid flow through a horizontal channel with Sr=1.

CONCLUSIONS
In this paper, the effect of soret is mainly studied by using various parameters under unsteady mixed convective flow of an immiscible fluid through a Horizontal channel in a porous and non-porous channels. The fluid is electrically conducting through a porous medium in the presence of uniform magnetic field. Soret Effect is added by its mathematical form. The governing equations o is solved analytically. The analytical results are derived for the flow field, heat transfer, mass transfer, by using the perturbation technique. The features of the flow characteristics are analyzed by plotting graphs and discussed in detail. The velocity profiles increases the value of Grashof number, Prandtl Number, Permeability parameter but they are decreasing based on the values of heat source parameter, radiation parameter Also an increase in Soret number increases the velocity profiles, concentration and temperature profile. The effect of porous decreases the flow in both regions.

REFERENCES


