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# MODIFIED NONLINEAR PREDICTIVE PI CONTROLLER FOR A CONICAL TANK LEVEL PROCESS

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## ABSTRACT
A Modified Nonlinear Predictive PI (MNPPI) control technique is a combination of two well-known control strategies of a Model Predictive Control (MPC) and Proportional Integral Derivative (PID) controller. This combination provides better control performance when compared to MPC and PID controllers. In this paper, the Modified Nonlinear Predictive PI control strategy is designed implemented to a nonlinear process and performance indices such as IAE, ISE, IATE, settling time, rise time, over shoot are compared to other controller. It's observed that, the designed controller shows lesser rise time and settling time along with ensured stability.

**Keywords:** nonlinear predictive PI, predictive PI, model predictive control, PID controller, robust stability, delay time.

## INTRODUCTION
In past decades, the PID control algorithm has been used in different areas of process control industries because of its simple structure, effectiveness of cost and easy to modify its control structure [1]. A slight modification in PID control structure is mostly acceptable by control engineer for control of nonlinear processes. However, a popular PID controller does not provide exact solution for the nonlinear processes. For this reason, the nonlinear PID controllers are designed and it could have more effective in the control of nonlinear processes. The nonlinear PID control law offers more effective in control of the process having in inherent nonlinearity.

Nonlinear PID controllers also used for control of many industrial process[2] ,like a surge vessel level control system[3], inverted pendulum control[4], six-degree of freedom (DOF) parallel manipulator[5], continuous stirred tank reactor (CSTR)[6]etc. Control of level in the conical tank system is a difficult problem and some of the researchers have implemented different control algorithms such as the conventional PI, MPC, [7, 8], predictive PI control algorithms etc., [9, 10, 11, and 12].

In this work, a modified nonlinear predictive PI controller (MNPPI) has been developed for control of conical tank level system. This control scheme (MNPPI) has been derived from a Nonlinear PI control structure. The MNPPI controller has implemented for conical tank system and simulations studies are carried out for different operating points, based on simulations result the conclusions are drawn and presented.

## THEORETICAL APPROACH OF CONTROLLER DESIGN:

### Nonlinear PI controller and modified nonlinear PI controller
General PID control law is

$$m(t) = K_c\left[e(t) + \frac{1}{\tau_I}\int e(t)dt + \tau_D\frac{de(t)}{dt}\right] \tag{1}$$

Nonlinear PI control law (error$^2$ type) is described as

$$m(t) = \tilde{K}_c(1+\beta|e(t)|)\left[e(t) + \frac{1}{\tau_I}\int e(t)dt\right] \tag{2}$$

Modified nonlinear PI control law is designed by an inclusion of a nonlinear PI controller gain ($\tilde{K}_c$) into the integral term of nonlinear PI control law. The intention of such design is to retain an invariable damping ratio. The modified nonlinear PI control law equation is represented as follows:

$$m(t) = \tilde{K}_c(1+\beta|e(t)|)\,e(t) + \frac{\tilde{K}_c}{\tau_I}\int e(t)dt \tag{3}$$

The main objective of a Modified Nonlinear PI control law (3) is to offer a simple control algorithm which can be adopted in existing industrial Nonlinear PI controllers efficiently. Modified nonlinear PI controller tuning parameters for the process in the form of $G_p(s) = \frac{k}{\tau s+1}e^{-\theta s}$ are shown in Table-1.

**Table-1.** Modified nonlinear PI control law parameters.

| Serial No. | Control structure |
|---|---|
| 1 | $k\tilde{k}_c = 0.02586 + 0.0982\,(\theta/\tau)^{-1.049}$ |
| 2 | $\tau_I/\tau = 0.997 + 0.08\,(\theta/\tau)^{2.351}$ |
| 3 | $\beta \times \Delta y_{sp} = 4.259 + 0.5362\,(\theta/\tau)^{0.667}$ |

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**DETERMINATION OF MATHEMATICAL MODEL OF THE CONICAL TANK LEVEL PROCESS**

Figure-1. shows the schematic diagram of conical tank level system. Here $F_i$ is the inlet flow rate to the tank, $F_O$ be the outlet flow rate from the tank, $F_L$ be the disturbance applied to the tank.

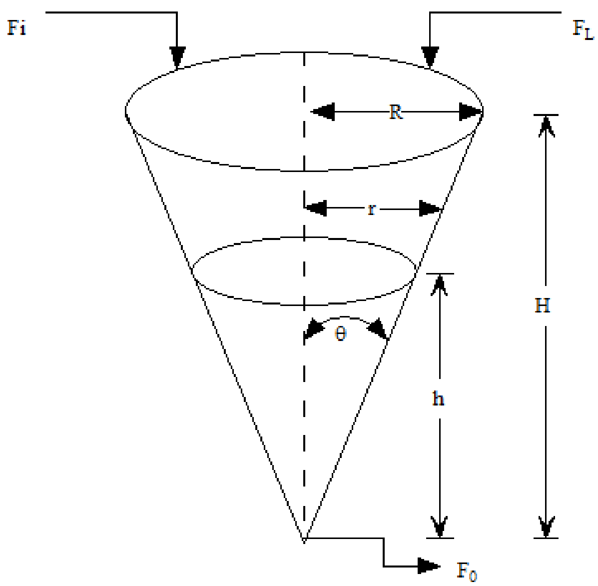


**Figure-1.** Schematic diagram of conical tank level system.

The dynamic model of the conical tank system about a nominal operating level h is given by

$$A\frac{dh}{dt} = F_i - F_o \tag{4}$$

$$F_o = b\sqrt{2gh} \tag{5}$$

$$A = \frac{\pi R^2 h^2}{H^2} \tag{6}$$

Substituting the equation (5) and (6) in (4) we get

$$\frac{dh}{dt} = \frac{F_i - b\sqrt{2gh}}{\frac{\pi R^2 h^2}{H^2}} \tag{7}$$

Where h is the height of the liquid level in the conical tank at any time, H is the total height of the conical tank, r is the radius of the conical vessel at a particular level of height h, R is the top radius of the conical tank, A is the top area of the conical tank, outlet valve ratio is b, inflow rate to the tank $F_i$ and outflow rate of the tank is $F_o$.

The model parameters are estimated using two point methods by giving specific step change at an operating point, Therefore the first order plus delay time model (8) is obtained at third operating region (50% to 75 %) of the conical tank system with respect to their height is given by

$$G_p(s) = \frac{0.925}{25.05s+1}e^{-1.09s} \tag{8}$$

**DESIGN OF MODIFIED NONLINEAR PREDICTIVE PI (MNPPI) CONTROLLER:**

The main reason for implementing the Modified Nonlinear Predictive PI (MNPPI) controller is to decrease the aggressive movement of valve, oscillatory response, cost/ complexity of control structure, maintaining simplicity and higher flexibility. Based on above objectives a MNPPI controller is presented in this work. The designed controller parameters are indicated in Table-2, and control structure of Nonlinear Predictive PI and Modified Nonlinear Predictive PI controllers shown in figures 2and 3 respectably. Controller design as follows:

Let, $G_p(s)$ is the process transfer function and

$G_c(s)$ is the controller transfer function

The closed loop transfer function $G_o(s)$ is

$$G_o(s) = \frac{G_p(s)G_c(s)}{1+G_pG_c(s)} \tag{9}$$

From equation (9)

$$G_c(s) = \frac{1}{G_p(s)}.\frac{G_o(s)}{1-G_o(s)} \tag{10}$$

Consider a process transfer function as

$$G_p(s) = \frac{K}{1+s\tau}e^{-\theta s} \tag{11}$$

Assume that the desired closed loop transfer function is

$$G_o(s) = \frac{e^{-\theta s}}{1+s\lambda\tau} \tag{12}$$

Where $\lambda$ is a tuning parameter

The controller transfer function is simplified as

$$G_c(s) = \frac{1+s\tau}{K_p(1+\lambda s\tau - e^{-\theta s})} \tag{13}$$

The Input – Output relation of the controller is

$$u(s) = \frac{1}{\lambda K}\left(1+\frac{1}{s\tau}\right)e(s) - \frac{1}{\lambda s\tau}\left(1-e^{-\theta s}\right)u(s) \tag{14}$$

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$k_c = \frac{1}{\lambda K}$ And $T_i = \tau$ are linear PI controller gain and integral time.

Controller equation (14) in time domain is

$$u(t) = k_c e(t) + \frac{k_c}{T_i}\int_0^t e(t)dt - \frac{1}{\lambda T_i}\int_0^t (1 - e^{-\theta s})u(t)dt \quad (15)$$

Thus, linear predictive PI controller is replaced by Modified Nonlinear Predictive PI controller by use of equation (3).

$$u(t) = \widetilde{k}_c\left(1 + \beta\left|e(t)\right|\right)e(t) + \frac{\widetilde{k}_c}{T_i}\int_0^t e(t)dt - \frac{1}{\lambda T_i}\int_0^t (1 - e^{-\theta s})u(t)dt \quad (16)$$

Substituting the process parameters of equation (8), the modified nonlinear predictive PI controller parameters for conical tank level system are given in Table-2.

**Table-2.** Modified nonlinear predictive PI controller parameters.

| Controller parameters | Values |
|---|---|
| $\widetilde{k}_c$ | 2.8728 |
| $K_i$ | 0.115021 |
| β | 0.043252 |
| λ | 1 |

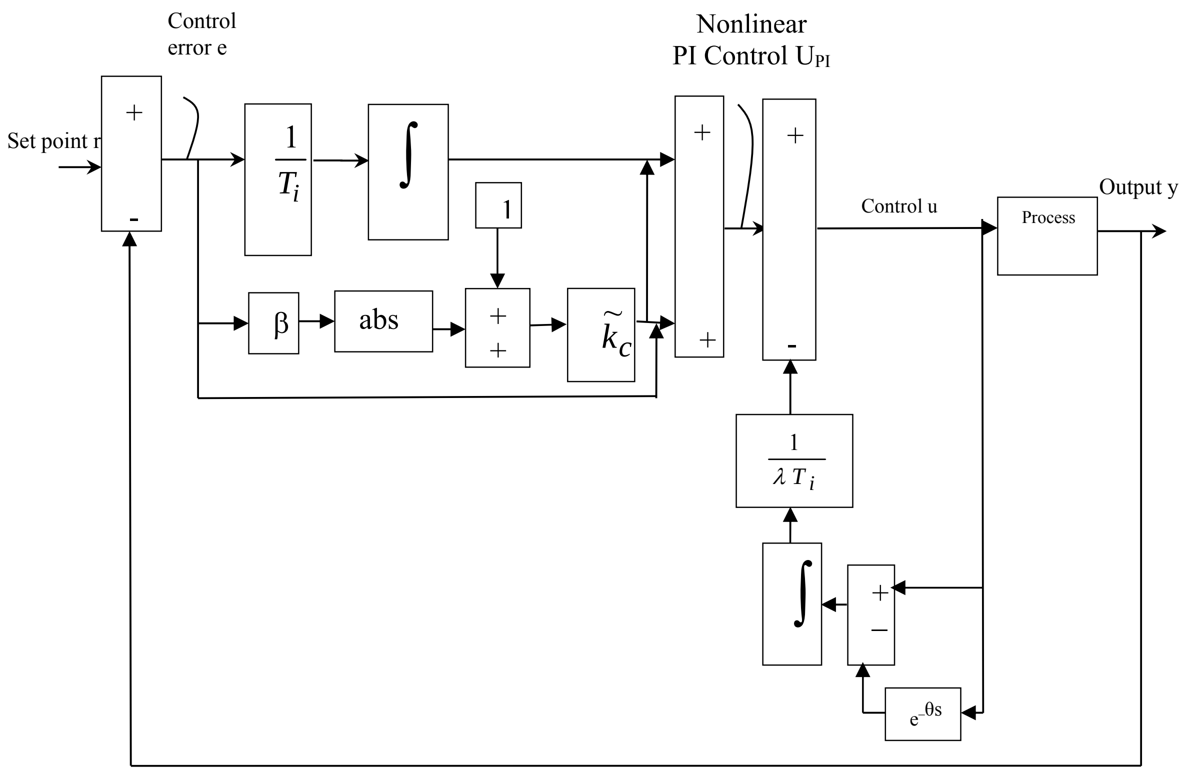


**Figure-2.** Control structure of nonlinear predictive PI.

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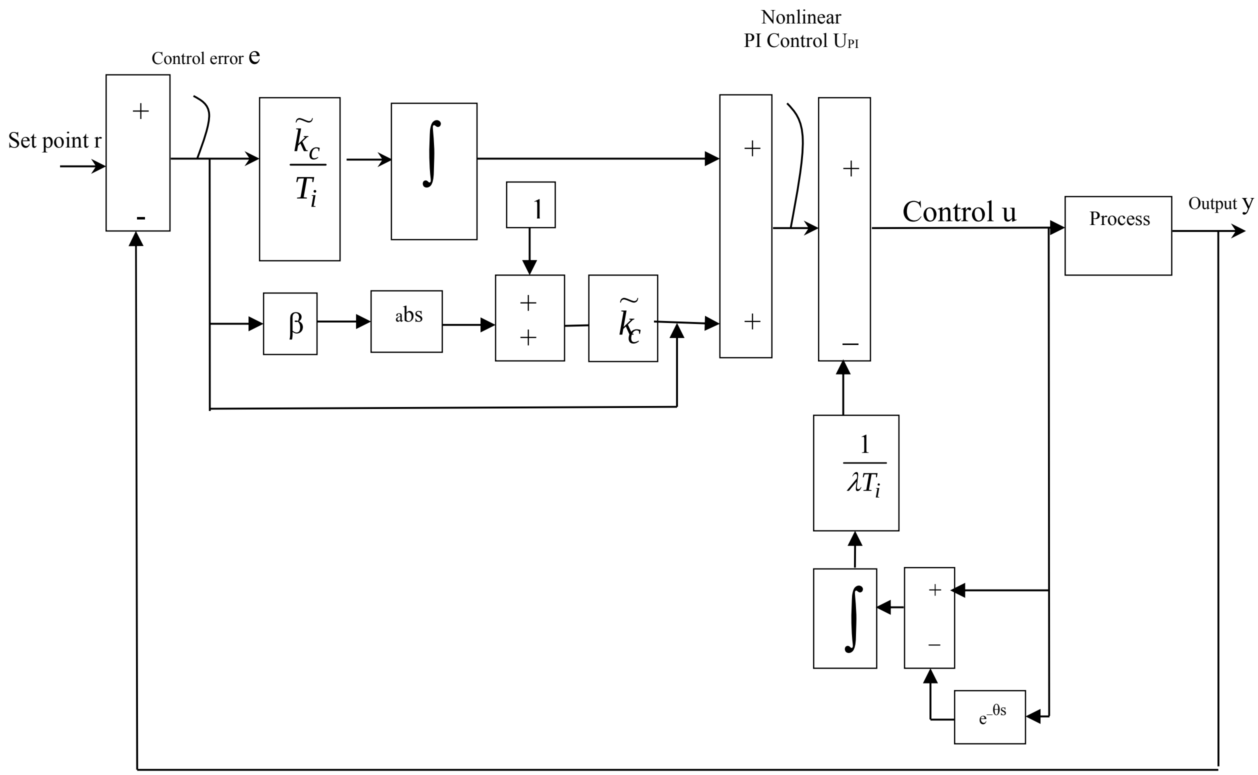


**Figure-3.** Control structure of modified nonlinear predictive PI.

## RESULT AND DISCUSSIONS

In this work, a Modified Non-linear Predictive PI controller has been designed and used for the control of level in conical tank level system. This combines the features of Model Predictive Control (MPC) and the PID control. The simulation studies were carried out on a MATLAB/Simulink environment to check the effectiveness of the designed Modified Nonlinear Predictive PI controller. The servo and regulatory responses are obtained for a various operating points which are revealed in Figures 4,5,6,7 and Figures 8,9,10,11 respectively. From the result it is observed that, the proposed Modified Nonlinear Predictive PI controller afford lesser rise time and settling time along with ensured stability. Performance index for both servo and regulatory response are summarized in Table .3 and 4 respectively. The time domain specifications of MNPPI and NPPI controllers are shown in Table-5.

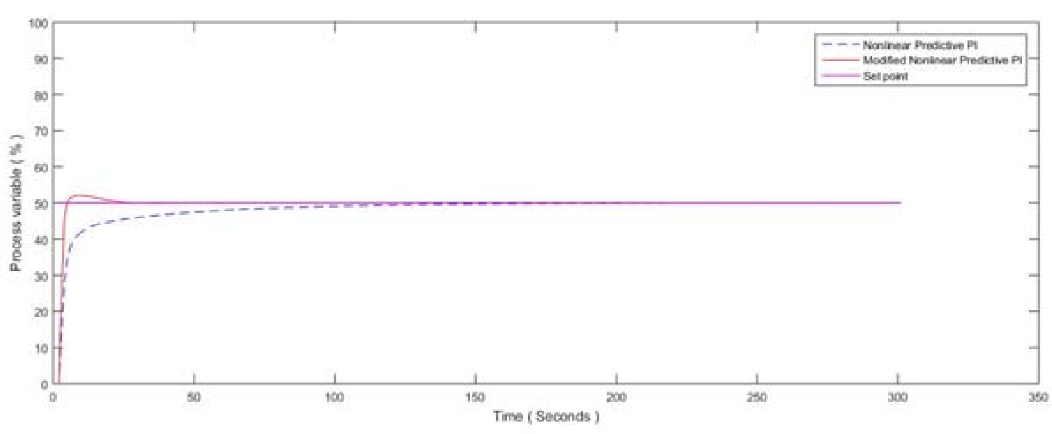


**Figure-4.** Servo response of MNPPI and NPPI controllers at 50 % of the level.

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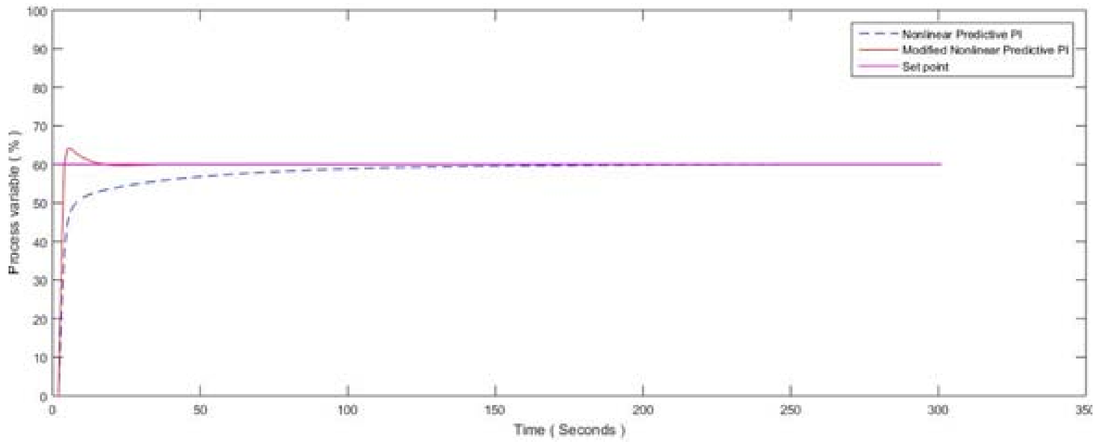


**Figure-5.** Servo response of MNPPI and NPPI controllers at 60 % of the level.

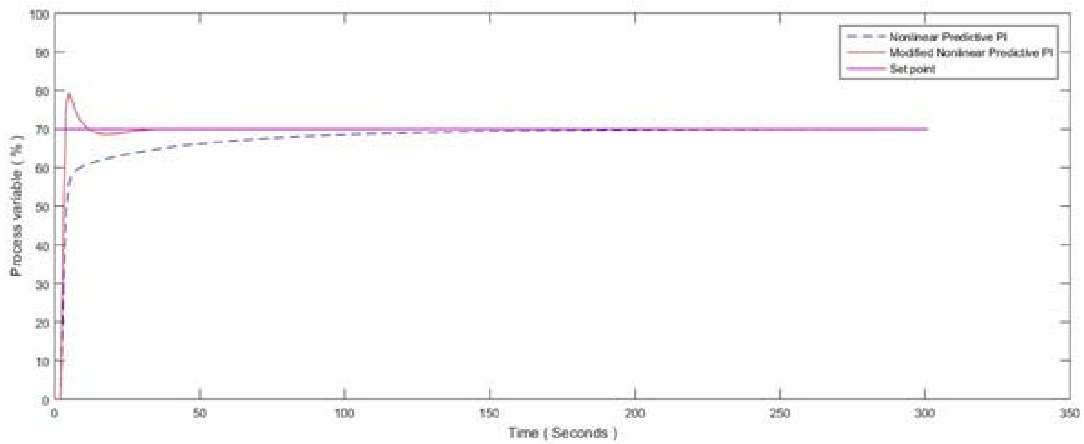


**Figure-6.** Servo response of MNPPI and NPPI controllers at 70 % of the level.

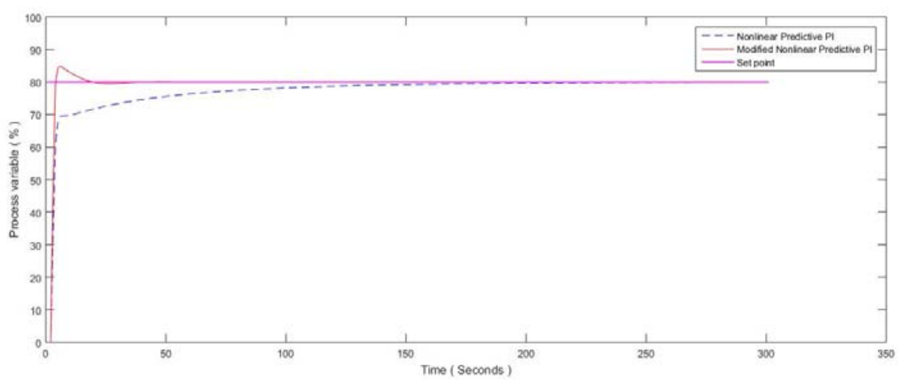


**Figure-7.** Servo response of MNPPI and NPPI controllers at 80 % of the level.

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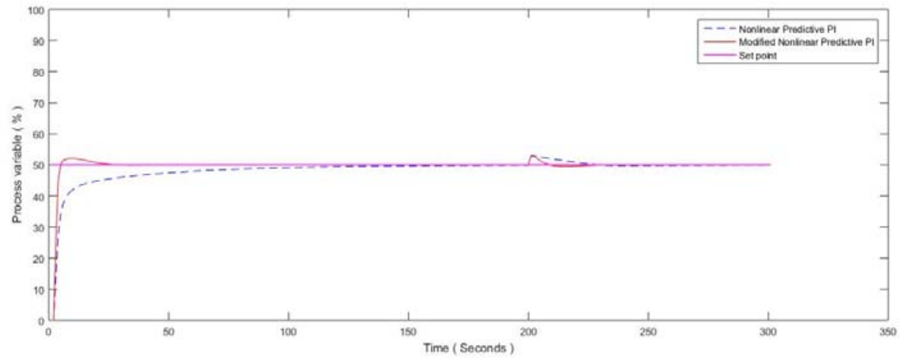


**Figure-8.** Regulatory response of MNPPI and NPPI controllers at 50 % of the level.

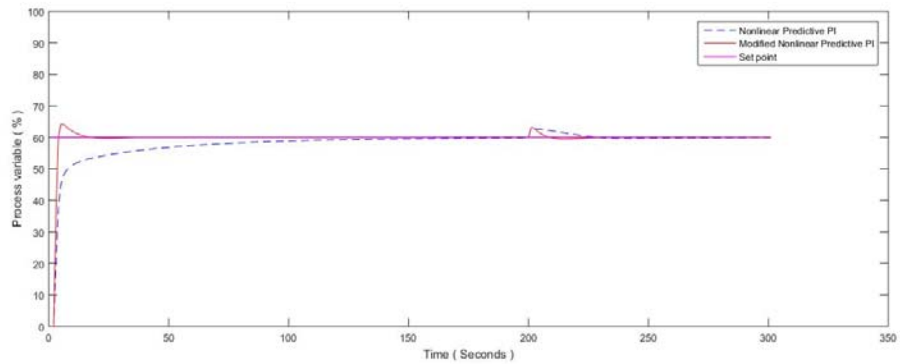


**Figure-9.** Regulatory response of MNPPI and NPPI controllers at 60 % of the level.

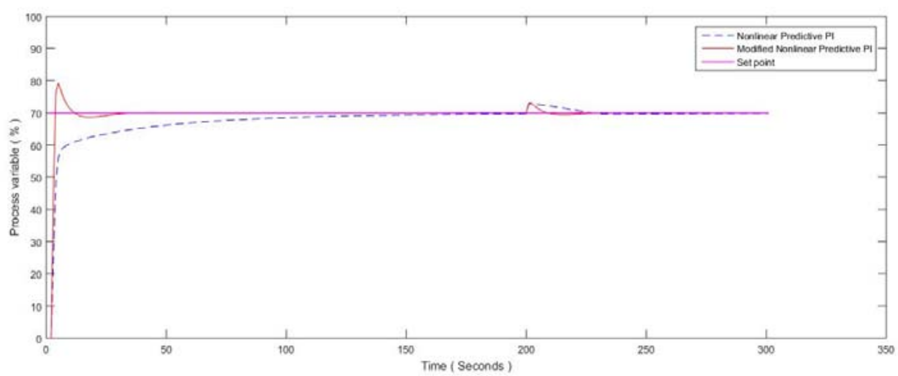


**Figure-10.** Regulatory response of MNPPI and NPPI controllers at 70 % of the level.

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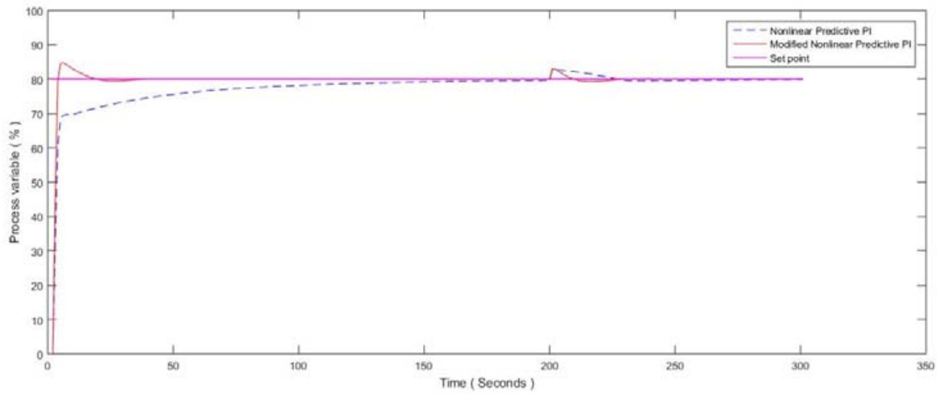


**Figure-11.** Regulatory response of MNPPI and NPPI controllers at 80 % of the level.

**Table-3.** Performance index of servo response.

| OPERATING REGIONS | METHODS | MODIFIED NONLINEAR PREDICTIVE PI | NONLINEAR PREDICTIVE PI |
|---|---|---|---|
| 50% | IAE | 130.8 | 485.2 |
| | ISE | 4283 | 7085 |
| | IATE | 473.5 | 1.655 $e^{+04}$ |
| 60% | IAE | 143.2 | 580.6 |
| | ISE | 5889 | 9506 |
| | IATE | 396.2 | 2.188 $e^{+04}$ |
| 70% | IAE | 179.1 | 674.8 |
| | ISE | 7852 | 1.217 $e^{+04}$ |
| | IATE | 688.2 | 2.766 $e^{+04}$ |
| 80% | IAE | 196.6 | 767.9 |
| | ISE | 1.05$e^{+04}$ | 1.509 $e^{+04}$ |
| | IATE | 673.1 | 3.386 $e^{+04}$ |

**Table-4.** Performance index of regulatory response.

| OPERATING REGIONS | METHODS | MODIFIED NONLINEAR PREDICTIVE PI | NONLINEAR PREDICTIVE PI |
|---|---|---|---|
| 50% | IAE | 149.8 | 530.7 |
| | ISE | 4311 | 7163 |
| | IATE | 4446 | 2.65 $e^{+04}$ |
| 60% | IAE | 162.3 | 633.2 |
| | ISE | 5917 | 9591 |
| | IATE | 4369 | 3.403 $e^{+04}$ |
| 70% | IAE | 198.1 | 729.8 |
| | ISE | 7880 | 1.226 $e^{+04}$ |
| | IATE | 4661 | 4.062 $e^{+04}$ |
| 80% | IAE | 220 | 823.5 |
| | ISE | 1.053$e^{+04}$ | 1.518 $e^{+04}$ |

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| | | | |
|---|---|---|---|
| | -IATE | 5608 | 4.722 $e^{+04}$ |

**Table-5.** Time domain specifications of MNPPI and NPPI at various operating levels.

| LEVEL IN PERCENTAGE | SPECIFICATIONS | MODIFIED NONLINEAR PREDICTIVE PI | NONLINEAR PREDICTIVE PI |
|---|---|---|---|
| 50 | Settling Time ( $t_s$ ) | 18.038 | 92.897 |
| | Rise Time ( $t_r$ ) | 1.7820 | 18.869 |
| | % Overshoot | 4.1400 | 0 |
| 60 | Settling Time ( $t_s$ ) | 11.809 | 97.368 |
| | Rise Time ( $t_r$ ) | 1.5839 | 19.023 |
| | % Overshoot | 6.9391 | 0 |
| 70 | Settling Time ( $t_s$ ) | 9.7527 | 101.31 |
| | Rise Time ( $t_r$ ) | 1.4106 | 19.241 |
| | % Overshoot | 13.13 | 0 |
| 80 | Settling Time ( $t_s$ ) | 13.35 | 103.95 |
| | Rise Time ( $t_r$ ) | 1.605 | 18.956 |
| | % Overshoot | 5.999 | 0 |

## CONCLUSIONS

A Modified Nonlinear Predictive PI controller has been adapted to conical tank liquid level process. This proposed controller provides superior control performance in terms of reduced rise time, settling time and with acceptable overshoot when compared to Nonlinear Predictive PI controller for the servo and regulatory responses at various operating conditions since the Nonlinear parameter (β) as the function of set point changes there by the Modified Nonlinear Predictive PI controller gives remarkable improvement in the response of the closed loop control system. Finally, a Modified Nonlinear Predictive PI control technique is one of the most effective control structures to prevent degradation of process dynamic output performance.

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