CONTRIBUTION OF GEOSTATISTICAL ANALYSIS FOR THE ASSESSMENT OF RMR AND GEOMECHANICAL PARAMETERS

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ABSTRACT
Geotechnical and engineering geology practitioners are always looking out for tools which can help understand and reduce the large uncertainty and variations in rock masses after complex geological processes. Relying on traditional interpolation techniques for geotechnical variables may lead to large uncertainty and major stability risk in the mining phase. The present paper proposes a direct and indirect methodology based on geostatistical estimation and simulation techniques to determine the expected Rock Mass Rating (RMR) and its underlying parameters, each geostatistical model identifies potential risk-prone areas in which failures could be experienced, superposing the different resulting maps allowed us to define low-risk conservative RMR model. A total of 115 underground rock blocks samples from five mining openings were examined for the rock mass quality using the RMR, Q and RMi characterization systems. Cross-validation and jack-knifing techniques showed that the proposed indirect estimation and simulation methods outperformed the more frequently used direct approach and shows a more accurate map with a low error coefficient which makes them adequate for RMR modeling. The resulting map of the indirect approach allowed taking into account the nonlinear nature, directional behavior of the RMR and its constitutive parameters which can be used to assist engineers in proposing suitable excavation techniques and an appropriate support system. The developed model help to assess different geomechanical parameters that can use to develop numerical models that explicitly consider the rock mass heterogeneity.

Keywords: geomechanics, rock mass rating, geostatistical analysis.

List of symbols or abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>OK</td>
<td>Ordinary kriging</td>
</tr>
<tr>
<td>SK</td>
<td>Simple kriging</td>
</tr>
<tr>
<td>OK_i</td>
<td>Indirect ordinary kriging</td>
</tr>
<tr>
<td>OK_d</td>
<td>Direct ordinary kriging</td>
</tr>
<tr>
<td>OCK</td>
<td>Ordinary co-kriging</td>
</tr>
<tr>
<td>OCK_i</td>
<td>Ordinary co-kriging (Q_* as a covariate)</td>
</tr>
<tr>
<td>OCK_d</td>
<td>Ordinary co-kriging (RM_* as a covariate)</td>
</tr>
<tr>
<td>CGS</td>
<td>Conditional Gaussian simulation</td>
</tr>
<tr>
<td>GCS_i</td>
<td>Indirect Gaussian conditional simulation</td>
</tr>
<tr>
<td>GCS_d</td>
<td>Direct Gaussian conditional simulation</td>
</tr>
<tr>
<td>IDW</td>
<td>Inverse distance weighting</td>
</tr>
<tr>
<td>v</td>
<td>Spherical variogram model</td>
</tr>
<tr>
<td>Exp</td>
<td>Exponential variogram model</td>
</tr>
<tr>
<td>Gaus</td>
<td>Gaussian variogram model</td>
</tr>
<tr>
<td>Ls</td>
<td>Varioigram Lag size</td>
</tr>
<tr>
<td>C_0</td>
<td>Nugget semivariance</td>
</tr>
<tr>
<td>C_00</td>
<td>Nugget semivariance of the Primary variable</td>
</tr>
<tr>
<td>C_11</td>
<td>Cross-variogram Nugget semivariance</td>
</tr>
<tr>
<td>St_00</td>
<td>The sill of the primary variable varioigram</td>
</tr>
<tr>
<td>St_01</td>
<td>The sill of the covariate variogram</td>
</tr>
<tr>
<td>St_11</td>
<td>Cross-variogram sill</td>
</tr>
<tr>
<td>RMR</td>
<td>Rock mass rating</td>
</tr>
<tr>
<td>RMi</td>
<td>Rock mass index</td>
</tr>
<tr>
<td>Q</td>
<td>Rock mass quality</td>
</tr>
<tr>
<td>UCS</td>
<td>Uniaxial compressive strength</td>
</tr>
<tr>
<td>RQD</td>
<td>Rock quality designation</td>
</tr>
<tr>
<td>Jn</td>
<td>Joint set number</td>
</tr>
<tr>
<td>Jr</td>
<td>Joint roughness number</td>
</tr>
<tr>
<td>Ja</td>
<td>Joint alteration number</td>
</tr>
<tr>
<td>Jw</td>
<td>Joint water reduction factor</td>
</tr>
<tr>
<td>Je</td>
<td>Joint surface condition</td>
</tr>
<tr>
<td>RSS</td>
<td>Residual sum of squares</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean squared error</td>
</tr>
<tr>
<td>MAPE</td>
<td>Mean absolute percent error</td>
</tr>
<tr>
<td>NMAE</td>
<td>Normalized mean-absolute error</td>
</tr>
<tr>
<td>S_k</td>
<td>Skewness</td>
</tr>
<tr>
<td>K_r</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>GSI</td>
<td>Geological strength index</td>
</tr>
<tr>
<td>E_m</td>
<td>Rock mass deformation modulus</td>
</tr>
<tr>
<td>(\sigma_{cm})</td>
<td>Rock mass compressive strength</td>
</tr>
<tr>
<td>(\varphi_{cm})</td>
<td>Internal friction angle of rock mass</td>
</tr>
<tr>
<td>(\gamma_{cm})</td>
<td>The cohesion of rock mass</td>
</tr>
<tr>
<td>(r^2)</td>
<td>Coefficient of determination</td>
</tr>
<tr>
<td>Q_*</td>
<td>Q-system transformed covariate variable</td>
</tr>
<tr>
<td>RM_*</td>
<td>RMi transformed covariate variable</td>
</tr>
<tr>
<td>p</td>
<td>IDW exponent value</td>
</tr>
<tr>
<td>(\gamma^*(h))</td>
<td>Experimental Semivariogram</td>
</tr>
<tr>
<td>Min</td>
<td>Minimum value</td>
</tr>
<tr>
<td>Max</td>
<td>Maximum value</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Geotechnical engineering practitioners are always looking out for tools which can improve the design and help understand and reduce the large uncertainty and variations in rock masses. In the literature, only a few applications can be found aiming the estimation of RMR (Ferrari et al., 2014) [1] (Marisa Pinheiro et al., 2016) [2] (Hesameddin Eivazy, 2017) [3]. These authors have concluded that rock mass parameters are reasonably predictable in unsampled locations using geostatistical interpolation methods. These analyses have been applied to specific geotechnical problems on limited area and localized sites.

Rock masses are characterized by uncertainty and heterogeneity after complex geological processes. Relying on traditional interpolation methods may lead to large uncertainty and major stability risk in the mining phase. The tackling of the described problems can be made using a sophisticated analysis taking into account the spatial relationship of a modeled the rock mass rating which is a very significant criterion to evaluate the stability of an underground mining area.

Rock mass rating was developed by Bieniawski [4], it is an index of rock mass competency based on the rating of five parameters:

- \( A_1 \) Intact rock strength
- \( A_2 \) Rock quality designation (RQD)
- \( A_3 \) Joint spacing (Js)
- \( A_4 \) Joint surface condition (Jc)
- \( A_5 \) Groundwater condition (Jw)

\[
RMR_{basic} = \sum_{i=1}^{5} A_i
\]  

This paper focuses on applying and comparing different geostatistical techniques to predict basic RMR values, directly and indirectly, performs quantifying uncertainty prediction taking into account spatial variability. The analysis is made in an attempt to use these interpolations for assessment of rock mass geomechanical parameters.

The interpolation results obtained by direct and indirect approach are statistically compared via cross-validation and jack-knifing techniques in order to measure the benefits of using geostatistics.

2. GEOLOGICAL SETTING AND CASE STUDY

Imiter silver mine is located on the northern side of the Precambrian JbelSaghro inlier (eastern Anti-Atlas, Morocco) (Figure-1), north of the West African Craton. Two major lithostructural units are recognized: the lower complex made of Middle Neoproterozoic detrital sediments intruded by ca. 570–580 Ma diorite and granodiorite plutons, the upper complex composed by lava flows and ash-flow tuffs associated with cogenetic granites. The Imiter deposit, localized along N070–090° E-trending regional faults system is assumed to be a Late Neoproterozoic epithermal deposit, hosted by both lower and upper complexes. The ore deposition is genetically associated with the felsic volcanic event, dated at 550 Ma, and assumed to result from a regional extensional tectonic regime.

The Imiter fault system, 10-km long, is localized at the contact between lower and upper complexes (Figure-1b). It consists in the association between N090°E and N060–070°E faults that define a succession of apparent left-lateral pull-apart texture, at map scale (Figure-1). At Imiter II, the main B3 corps (sampling area) is oriented N065°E with a south dipping and intercepted by four sets of discontinuities (Figure-3).

Figure-1. a) Schematic map of the Moroccan Anti-Atlas and localization of the Imiter silver mine and other mineral bearing indices of the JbelSaghro. (b) Simplified geological map of the Imiter silver mine.
The study area is at a depth of 500m is characterized by:

- The common rock type are meta-siltstones
- There are prominent foliations over all the rock mass in Est-West direction.
- There are multiple discontinuity sets, many of which intersect each other.

The underground geotechnical surveys method provides a great opportunity for engineers and geologists to observe and sample the rock mass on the excavation faces. A total of 115 rock blocks samples from five mining openings were examined for the rock mass quality using the RMR (Figure-4), Rock Quality system (Q) and Rock Mass Index (RMi) characterization systems, the outcrop mapping was carried out on freshly exposed parallel faces in the horizontal south to north direction to simplify variables with directional behavior.

3. EXPLORATORY STATISTICAL ANALYSIS

To provide an overview of the measured selected variables data, a basic statistical study was made. Classical descriptors were determined, such as mean, maximum, minimum, standard deviation and skewness ($S_k$) of data distribution.

The descriptive statistics of the rock mass rating data suggested that the RMR may vary from 27 to 74 with a mean value of 46.9% and standard deviation of 11.72%, the geomechanical quality of the rock mass varies from poor to good. The summary of the statistics for rating parameters is shown in (Table-1).

The coefficients of variation of RMR parameters ranged from 7% to 43%, It was noted the presence of a strong spatial variability of RMi and Q ratings with a variation coefficient of 103% and 154% respectively.

Once the data set was checked, the next step was generating the histogram to study the symmetry and pattern of the frequency distribution and to determine how much percent of the samples are far from the central value. The histogram of RMR, RMi and Q values are displayed in (Figure-5).

The uniaxial compressive strength (UCS), Q, and RMi data were positively skewed, the rest of other parameters...
Table-1. Descriptive statistics of rock mass collected data.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>UCSD</th>
<th>Min</th>
<th>Standard deviation</th>
<th>Median</th>
<th>Average</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>4.91</td>
<td>3.63</td>
<td>0.29</td>
<td>3.76</td>
<td>3.87</td>
<td>0.08</td>
<td>1.40</td>
<td>1.49</td>
</tr>
<tr>
<td>RQD</td>
<td>20.00</td>
<td>5.00</td>
<td>4.36</td>
<td>8.00</td>
<td>9.72</td>
<td>19.00</td>
<td>0.41</td>
<td>-1.16</td>
</tr>
<tr>
<td>Js</td>
<td>10.00</td>
<td>5.00</td>
<td>1.70</td>
<td>8.00</td>
<td>7.32</td>
<td>2.88</td>
<td>-0.25</td>
<td>-1.07</td>
</tr>
<tr>
<td>Jc</td>
<td>28.00</td>
<td>2.00</td>
<td>5.55</td>
<td>12.00</td>
<td>13.04</td>
<td>30.77</td>
<td>0.33</td>
<td>-0.45</td>
</tr>
<tr>
<td>Jw</td>
<td>15.00</td>
<td>9.00</td>
<td>2.48</td>
<td>15.00</td>
<td>12.94</td>
<td>6.16</td>
<td>-0.38</td>
<td>-1.87</td>
</tr>
<tr>
<td>RMR</td>
<td>74.91</td>
<td>27.63</td>
<td>11.72</td>
<td>47.78</td>
<td>46.90</td>
<td>137.4</td>
<td>0.08</td>
<td>-0.92</td>
</tr>
</tbody>
</table>

Figure-5. Frequency distribution for modelling variables.

were normally distributed (skewness of between -1 and 1), there are some maxima, but the entire distribution can be treated as having a single peak.

4. TREND ANALYSIS

Many geostatistics techniques assume of spatial stationarity, the validity of this property (i.e. The absence of regular trends in space) needs to be verified by projecting the sample locations on an x, y plane. The RMR value of each sample is given in the z dimension (Figure-6).

Figure-6. RMR trend analysis projections.

The Trend Analysis of RMR data does not present any systematic trend or change in space, because the values cannot be interpolated by a monotone ascending or descending function in the studied domain, this leads to assume a “trend” free case in this study where stationarily condition can be applied and kriging can be allowed.

5. MODEL EVALUATION

The evaluation of RMR variograms and maps accuracy is based on the following parameters:

- The root-mean-square error (RMSE):
  \[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (A_t - F_t)^2} \]  
- The Residual sum of squares (RSS):
  \[ RSS = \sum_{i=1}^{n} (A_t - F_t)^2 \]  
- The normalized mean-absolute error (NMAE):
  \[ NMAE = 100\% \sum_{i=1}^{n} \frac{A_t - F_t}{A_t} \]  
- The mean absolute percentage error (MAPE):
  \[ MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{A_t - F_t}{A_t} \right| \]

Where, \( A_t \) Is the actual value, \( F_t \) Is the predicted value of the output variable and \( n \) is the number of samples.
6. SPATIAL ANALYSIS

6.1. Geostatistical estimation

McDonnell & Burrough (1998) [5] have shown that for applications in geosciences, Kriging is the best linear unbiased estimator (Journel and Huijbregts, 1978) [6] (Isaaks and Srivastava, 1989) [7]. The construction of the kriging estimator is done by successively imposing these features (linearity, unbiasedness, optimality). Variations of the estimate are achieved by imposing a known or unknown mean and allowing local variations of it (Goovaerts, 1997) [8].

6.2 Geostatistical simulation

Geostatistical simulation is a stochastic, nonlinear modeling method that obtains multiple plausible realizations of spatial variability based on the same input data according to the following criteria (Dowd, 1993) [9]:

- At all sampled locations they honor the real values,
- They have the same spatial dispersion (i.e. same variogram, as the true values),
- They have the same distribution as the true values

6.3 Variography analysis

Spherical, exponential and Gaussian isotropic theoretical functions were fitted to the sample variograms depending on the shape using a weighted least squares method (Robertson, 1987) [10] procedure and cross-validation technique. The parameters of the model: nugget semivariance, range, and sill or total semivariance were determined.

To define different classes of spatial dependence for RMR, the ratio between the nugget semivariance and the sill was used (Cambardella et al, 1994) [11]. If the ratio was ≤25%, the variable was considered to be strongly spatially dependent, or strongly distributed in patches, the RMR was considered strongly spatially dependent.

7. DIRECT APPROACH

In the direct approach, the RMR basic values were used directly as inputs to inverse Distance Weighting, geostatistical estimation, and simulation in order to create 2D models of RMR in the mining area. This approach neglects the nonlinear nature of RMR underlying variables. However, it is considered as a simple, quick and practical solution.

7.1 Inverse distance weighting

Inverse Distance Weighting (IDW) interpolation is a deterministic, nonlinear interpolation technique that uses a weighted average of the attribute values from nearby sample points to estimate the magnitude of that attribute at non-sampled locations, assuming that each measured point has a local influence that diminishes with distance. It weights the points closer to the prediction location greater than those farther away. IDW is calculated as:

\[ Z'(u) = \sum_{i=1}^{n} \lambda_i Z(u_i) \]  

where:
- \( Z(u_i) \) are the conditioning data
- \( n \) is the number of sample points
- \( \lambda_i \) are the assigned weights

The weights are determined as:

\[ \lambda_i = \frac{1}{d_i^p} \]  

\[ \sum_{i=1}^{n} \lambda_i = 1 \]  

\( d_i \) are the Euclidian distances between estimation location and sample points

\( p \) is the power or distance exponent value.

The optimal power value is determined by minimizing the root mean square error and the mean absolute percentage error calculated from RMR cross-validation. Figure-7 shows that IDW with power 10 is an optimal distance exponent value that produces the minimum RMSE and MAPE. As \( p \) increases only the immediate surrounding data points will influence the RMR prediction.

Figure-8. IDW interpolation map for RMR data.
Table-2. Results of OK variography analysis.

<table>
<thead>
<tr>
<th>Models</th>
<th>$L_s$</th>
<th>$C_0$</th>
<th>Range</th>
<th>Sill</th>
<th>$C_0$/Sill %</th>
<th>RSS</th>
<th>$R^2$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sph</td>
<td>1.41</td>
<td>0</td>
<td>11.35</td>
<td>139.15</td>
<td>0</td>
<td>794</td>
<td>0.958</td>
<td>3.912</td>
</tr>
<tr>
<td>Exp</td>
<td>2.76</td>
<td>0</td>
<td>22.12</td>
<td>170.53</td>
<td>0</td>
<td>310</td>
<td>0.995</td>
<td>4.38</td>
</tr>
<tr>
<td>Gaus</td>
<td>1.43</td>
<td>11.42</td>
<td>8</td>
<td>125.49</td>
<td>9</td>
<td>1742</td>
<td>0.964</td>
<td>4.505</td>
</tr>
</tbody>
</table>

7.2 Ordinary kriging

Ordinary kriging (OK) is the most widely used kriging method. It serves to estimate a value at a point of a region for which a variogram is known, using data in the neighborhood of the estimation location. The spatial distribution of the rock mass rating data was analyzed using geostatistics. Spatial patterns are usually described using the experimental Semivariogram $\gamma^*(h)$, which measures the average dissimilarity between RMR data separated by a vector $h$.

$$\gamma^*(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [z(x_i) - z(x_i + h)]^2$$  \hspace{1cm} (9)

where $n$ is the number of pairs of sample points separated by the distance $h$.

Semivariogram sample is calculated from the data sample using the following equation:

$$Z^*(x) = \sum_{i=1}^{n} \lambda_i z(x_i)$$  \hspace{1cm} (10)

$Z^*(x)$ : prediction location

$$\sum_{i=1}^{n} \lambda_i = 1$$  \hspace{1cm} (11)

$\lambda_i$ : unknown weight of measured value of pairs of point

$Z(x_i)$ : measured value of pairs of point

$n$ : number of measured values

The most important step in the use of geostatistical methods is to obtain the variogram with high correlation.

This step has a significant impact on the behavior and results of the model. Spherical, exponential, Gaussian models were fitted to the RMR empirical semivariograms assuming isotropic condition.

Figure-9 shows the empirical variogram, which is a plot of the values of as a function of $h$, gives information on the spatial dependency of the RMR variable. The Spherical model with the smallest RMSE=3.912 (Table-2) was selected to describe the RMR spatial dependency (Figure-10).
Theoretical

Table 3. Results of the RMR-Q* cross-variography analysis.

<table>
<thead>
<tr>
<th>Variogram parameters</th>
<th>Goodness-of-fit</th>
<th>Cross-validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
<td>L_s</td>
<td>C_00</td>
</tr>
<tr>
<td>Sph</td>
<td>1.57</td>
<td>0.078</td>
</tr>
<tr>
<td>Exp</td>
<td>2.5</td>
<td>0</td>
</tr>
<tr>
<td>Gaus</td>
<td>1.49</td>
<td>10.93</td>
</tr>
</tbody>
</table>

Table 4. Results of the RMR-RMi* cross-variography analysis.

<table>
<thead>
<tr>
<th>Variogram parameters</th>
<th>Goodness-of-fit</th>
<th>Cross-validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
<td>L_s</td>
<td>C_00</td>
</tr>
<tr>
<td>Sph</td>
<td>1.49</td>
<td>0</td>
</tr>
<tr>
<td>Exp</td>
<td>2.34</td>
<td>0</td>
</tr>
<tr>
<td>Gaus</td>
<td>1</td>
<td>3.54</td>
</tr>
</tbody>
</table>

7.3 Ordinary cokriging

The ordinary cokriging (OCK) procedure is an extension of kriging when a multivariate variogram model and multivariate data are available. The spatial dependence between the RMR and Q-system or RMI was estimated by means of a cross-variogram given by the following equation (Deutsch and Journel, 1992) [12] (Yates and Warrick, 1987) [13]:

\[
γ_{AB}^*(h) = \frac{1}{2N(h)} \sum_{i=1}^{n} \sum_{j=1}^{m} \{z_A(x_i) - z_A(x_j)\} \{z_B(x_i) - z_B(x_j)\}
\]

(12)

\[
γ_{AB}^*(h) : \text{is the estimated cross-variogram value at distance } h
\]

\[
N(h) \text{ is the number of data pairs of observations of the first variable } z_A \text{ and the second variable } z_B \text{ at locations } x_i \text{ and } x_j \text{ when the distance between } x_i \text{ and } x_j \text{ fits in a distance class } h.
\]

The cokriging estimate is a linear combination of both the RMR and the secondary variables Q or RMI given by:

\[
Z^*(x) = \sum_{i=1}^{n} w_i z_A(x_i) + \sum_{j=1}^{m} \sum_{i=1}^{n} v_{ij} u(x_{ij})
\]

(13)

Subject to one of the following sets of linear constraints:

\[
\sum_{i=1}^{n} w_i + \sum_{j=1}^{m} \sum_{i=1}^{n} v_{ij} = 1
\]

(14)

Where, \(w_i\) are the kriging weights associated with the \(n\)-nearest neighbors, \(v_{ij}\) are the cokriging weights associated with the \(m\) auxiliary variables, \(u_{ij}\) That are spatially correlated to the variable of interest.

In the present study, we consider the following transformation for covariate variables:

\[
Q^* = \ln(Q) + 3.5
\]

(15)

\[
RMI^* = \ln(RMI) + 3.5
\]

(16)

Finding cokriging theoretical models that fit best the experimental Semivariogram (Figures 11, 12) and cross-variograms (Figure-13) with less error coefficient is, however, a difficult exercise. The selection was made on the basis of cross-validation parameter (RMSE) to assess the precision of the interpolation method (Tables 3, 4).

When models are fitted to the experimental semi- and cross-variograms, the Cauchy-Schwartz equation must be checked to guarantee a correct Cokriging estimation variance in all circumstances (Deutsch and Journel 1992) (Isaaks and Srivastava 1989)

\[
|γ_{RMR-Q}| \leq \sqrt{γ_{RMR}γ_Q}
\]

(17)

\(γ_{RMR-Q}\) is the cross-variogram value

\(γ_{RMR}\) is Semivariogram value of RMR

\(γ_Q\) is Semivariogram value of Q-system

Figure-11. OCK Experimental Semivariogram and fitted model for RMR data.
After evaluating different models, it was demonstrated that the exponential model best suited for both variables RMR and Q with the lowest RMSE= 3,443 (Table-3) and therefore, it was selected as a best-fitted model for cokriging map (Figure-14).

### 7.4 Sequential Gaussian simulation

In this paper, geostatistical simulation is used in order to give an advantage to spatial correlation of RMR data; Sequential Gaussian simulation is the most commonly used method to develop conditional simulations.

This method requires a Gaussian transformation of the original RMR data set, to accomplish this, the RMR sample data were normalized using the two steps Approach for Transforming Continuous Variables to Normal was conducted (Templeton, 2011) [14], histograms, box-plot Q-Q plot was graphical ways to judge whether RMR transformed data are normally distributed (Figures 15, 16).

<table>
<thead>
<tr>
<th>Models</th>
<th>$L_s$</th>
<th>$C_0$</th>
<th>Range</th>
<th>Sill</th>
<th>$C_0$/Sill %</th>
<th>RSS</th>
<th>$R^2$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sph</td>
<td>1.35</td>
<td>0</td>
<td>10.86</td>
<td>138.49</td>
<td>0</td>
<td>323</td>
<td>0.982</td>
<td>4.374</td>
</tr>
<tr>
<td>Exp</td>
<td>2.4</td>
<td>0</td>
<td>19.27</td>
<td>160.53</td>
<td>0</td>
<td>537</td>
<td>0.975</td>
<td>4.495</td>
</tr>
<tr>
<td>Gaus</td>
<td>1</td>
<td>16.3</td>
<td>8.22</td>
<td>140.8</td>
<td>11.5</td>
<td>807</td>
<td>0.959</td>
<td>4.418</td>
</tr>
</tbody>
</table>

Figure-15. Histogram of RMR original data.
Figure-16. Q-Q and box plots of RMR original data.

The RMR data in Figure-15 shows a left-skewed histogram has a peak to the right of center, more gradually tapering to the left side. In Figure-16, the left and right ends of pattern are below and above the line y=x respectively, this departure from linearity described by a long tails at both ends of the data distribution.

Figure-17. Histogram of RMR transformed data.

In order to produce a conditional simulation, a new variogram must be developed with the Gaussian distribution then fitted (Figure-19) and cross-validated (Table-5). Variogram development, and simulations are performed on Gaussian transformed data and results must be back-transformed.

Figure-18. Q-Q and box plots of transformed RMR

Table-6. Normality parameters of RMR data.

<table>
<thead>
<tr>
<th>RMR</th>
<th>Kurtosis</th>
<th>Skewness</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original data</td>
<td>-0.916</td>
<td>0.077</td>
<td>0.007</td>
</tr>
<tr>
<td>Transformed data</td>
<td>-0.302</td>
<td>-0.001</td>
<td>0.998</td>
</tr>
</tbody>
</table>

The results of conditional simulation are expressed by a number of equally probable maps, each realization obtained by using the conditional simulation method is equally valid, in this research, the number of RMR realizations is set to one thousand, so that the post-processing outputs (mean of the realizations and conditional probabilities) could be calculated with a reasonable approximation (Figure-20).

A major limitation to the previous kriging methods that minimize the variance, inherently underestimates the variability and smoothen out local details of RMR spatial variation, this can be a problem when trying to map sharp spatial poor-quality zones, therefore, conditional simulation is a robust way to create RMR exceedance probability maps (Figure-21).
8. INDIRECT APPROACH

RMR is a sum of ratings assigned non-linearly to several geotechnical parameters; the direct use of geostatistical techniques to estimate RMR does not account for the non-linearity property and may carry some estimation errors. However, each of the underlying components is an additive variable and therefore, can be directly averaged and modeled separately.

The indirect approach to spatial prediction, in this case, would be the following:

- Gaussian transformation of each RMR parameter $R_i$ for conditional simulation, (Table-7)
- Variography analysis for OK (Table-8) and Simple kriging (SK) (Table-9)

Table-7. Normality coefficients of RMR parameters.

<table>
<thead>
<tr>
<th></th>
<th>RQD</th>
<th>UCS</th>
<th>Js</th>
<th>Jc</th>
<th>Jw</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMR</td>
<td>$S_k$</td>
<td>$K_t$</td>
<td>$S_k$</td>
<td>$K_t$</td>
<td>$S_k$</td>
</tr>
<tr>
<td>Original data</td>
<td>0.408</td>
<td>-1.163</td>
<td>1.399</td>
<td>1.495</td>
<td>-0.253</td>
</tr>
<tr>
<td>Transformed data</td>
<td>0.265</td>
<td>-1.069</td>
<td>0.028</td>
<td>-0.311</td>
<td>0.255</td>
</tr>
</tbody>
</table>

Figure-21. Exceedance probability map of RMR>40%
- Estimate and simulate each of RMR parameters which are linear variables (Figure-22).
- Assign a rating to the estimated or simulated value of each parameter.
- Back transformation of each simulated parameter $R_i$.
- Obtain the final estimated (Figure-23) and simulated (Figure-24) rating RMR, as the sum of the ratings obtained.

Table-8. Results of the indirect ordinary kriging variography analysis for estimation.

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>Variogram parameters</th>
<th>Mod</th>
<th>$L_s$</th>
<th>$C_0$</th>
<th>Range</th>
<th>Sill</th>
<th>$C_0$/Sill</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD</td>
<td>Spherical</td>
<td></td>
<td>1.26</td>
<td>1.6</td>
<td>10.02</td>
<td>15.89</td>
<td>10%</td>
</tr>
<tr>
<td>UCS</td>
<td>Spherical</td>
<td></td>
<td>1.05</td>
<td>0.005</td>
<td>10.16</td>
<td>0.086</td>
<td>6%</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Spherical</td>
<td></td>
<td>2.98</td>
<td>0.001</td>
<td>8.42</td>
<td>2.48</td>
<td>0%</td>
</tr>
<tr>
<td>$J_c$</td>
<td>Spherical</td>
<td></td>
<td>1.89</td>
<td>3.17</td>
<td>9.46</td>
<td>32.52</td>
<td>10%</td>
</tr>
<tr>
<td>$W_i$</td>
<td>Spherical</td>
<td></td>
<td>1.93</td>
<td>0.78</td>
<td>6.38</td>
<td>5.85</td>
<td>13%</td>
</tr>
</tbody>
</table>

Table-9. Results of indirect simple kriging variography analysis for simulation.

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>Variogram parameters</th>
<th>Mod</th>
<th>$L_s$</th>
<th>$C_0$</th>
<th>Range</th>
<th>Sill</th>
<th>$C_0$/Sill</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD</td>
<td>Sph</td>
<td></td>
<td>1.59</td>
<td>0.61</td>
<td>12.74</td>
<td>12.24</td>
<td>5%</td>
</tr>
<tr>
<td>UCS</td>
<td>Sph</td>
<td></td>
<td>1.21</td>
<td>0.002</td>
<td>9.72</td>
<td>0.089</td>
<td>2%</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Sph</td>
<td></td>
<td>1.06</td>
<td>0.208</td>
<td>8.52</td>
<td>1.93</td>
<td>11%</td>
</tr>
<tr>
<td>$J_c$</td>
<td>Exp</td>
<td></td>
<td>1.64</td>
<td>0.51</td>
<td>13.17</td>
<td>33.77</td>
<td>2%</td>
</tr>
<tr>
<td>$W_i$</td>
<td>Sph</td>
<td></td>
<td>1.23</td>
<td>0.81</td>
<td>9.89</td>
<td>2.23</td>
<td>36%</td>
</tr>
</tbody>
</table>

9. VALIDATION

9.1 Cross-validation

Several authors recommend the cross-validation technique for evaluating the accuracy of an interpolation technique (Webster and Oliver, 2001) [15] (Kravchenko and Bullock, 1999) [16]. Cross-validation is a leave-one-out technique that uses all of RMR data to estimate the autocorrelation models by removing data from 115
locations, taken all of the available data from other locations and then estimating the value of the removed locations data using those remaining data (Olea, 1999) [17].

In cross-validation analysis, a graph can be constructed between the estimated and actual values for each sample location in the domain. The cross-correlation plots showed that ordinary cokriging with the transformed Q covariate outperforms other direct predictions (Figure-27).

For all points, cross-validation compares the measured and predicted values. In this research, RMSE, MAPE, and NMAE are used to evaluate predicted value with IDW, okd, and OCK and helps us for determination the best RMR model.

The overall performance of RMR co-kriging with the transformed Q covariate was outstanding with exponential semi-variogram which gave the lowest error coefficients in all the cases (Figure-29).

9.2 Jack-knifing validation

With the aim of comparing results and selecting the most suitable interpolation technique out of above mentioned seven different methods, the jack-knifing process has been performed, using new samples. An independent data set of 13 RMR surveys have been carried out in the research area to be compared with the estimated data point (Figure-30).
The RMSE, MAPE, and NMAE were employed as criteria to evaluate the accuracy and effectiveness of RMR prediction maps via jack-knifing data points.

Comparing the predicted values of all previous methods with the new sampling results showed that the indirect approach gives more accurate results (Figure-31).

10. ENGINEERING POST-PREDICTION APPLICATIONS

Each geostatistical model identifies some potential risk-prone areas, with the aim of reducing the conceptual model uncertainty and risks in geotechnical design and underground mining operations, each grid point in the study area is given a minimum RMR value based on the previous interpolation maps considering the possible worst scenario (Figure-32):

$$RMR = \text{Min} \{ \text{IDW}, \text{OK}, \text{OK}_{d}, \text{OCK}_{1}, \text{OCK}_{2}, \text{GCS}_{d}, \text{GCS}_{i} \}$$  \hspace{1cm} (18)$$

The RMR resulting map can be a useful tool to derive rock mass geomechanical parameters such as uniaxial compressive strength ($\sigma_{c,m}$) and equivalent young modulus ($E_{m}$) that can be used for further numerical simulation and engineering work as if it were an equivalent continuous medium.

When numerical models are used as a tool of mining stability analysis, rock mass strength is defined in terms of a strength envelope that may be the linear case, like Mohr-Coulomb or non-linear like that suggested by Hoek.

10.1 Generalized Hoek-Brown parameters

In most cases, it is not practically possible to carry out triaxial tests on rock masses at a large scale which is necessary to obtain direct values of Generalized Hoek-Brown parameters. The Hoek–Brown criterion relates the strength envelope to the rock mass classification through the Geological strength index (GSI) or RMR indexes and allows strength assessment based on the previous interpolated maps. The Generalized Hoek-Brown criterion parameters (Hoek, Carranza-Torres, Corkum, 2002) [18], are given by the following equations:

$$m_b = m_i \cdot \exp \left( \frac{GSI - 100}{24 - 14D} \right)$$  \hspace{1cm} (19)$$

For $RMR_{76} < 18$ (Hoek, Kaiser, and Bawden, 1995) [19]:

$$GSI = RMR_{76}$$  \hspace{1cm} (20)$$

$S$ and $a$ are constants for the rock mass given by the following relationships:

$$s = \text{Exp} \left( \frac{GSI - 100}{9 - 3D} \right)$$  \hspace{1cm} (21)$$

$$a = \frac{1}{2} + \frac{1}{6} \left( e^{\frac{-GSI}{15}} - e^{\frac{-20}{3}} \right)$$  \hspace{1cm} (22)$$
The deformation modulus is a required input parameter for different types of numerical analyses; therefore it is necessary to obtain realistic values of rock mass young modulus using the previously calculated parameters.

The generalized equation (Hoek, Diederichs, 2006)\[20\]:

\[
E_m (MPa) = E_i \left(0.02 + \frac{1 - \frac{D}{2}}{1 + e^{\left(\frac{60 + 15D - GSI}{11}\right)}}\right)
\]

(23)

Using the version 2002 of the Hoek-Brown equation, the ratio of the strength of the rock mass and the intact rock is:

\[
\sigma_{cm} = s^a \sigma_c
\]

(24)

10.2 Mohr-coulomb parameters

In case of using the Mohr-Coulomb failure criterion, it is necessary to estimate the cohesion ($\varphi_m$) and the friction angle($\psi_m$) parameters of the rock masses which are related to the RMR value according to Bieniawski [21]

Aydan and Kawamoto [22] proposed a linear relationship between the internal friction angle and the RMR value:

\[
\varphi_m = 20 + 0.5RMR
\]

(25)
In this case, the cohesion can be calculated from the friction angle and the rock mass strength by the following equation:

\[ c_m = \frac{\sigma_{cm} \left( 1 - \sin \phi_m \right)}{2 \cos \phi_m} \]  

(26)

**Figure-38.** 2D map of rock mass friction angle.

**Figure-39.** 2D map of rock mass cohesion.

**11. CONCLUSIONS**

The accurate prediction of geotechnical variables and the risk level is very important for any underground mining project, in this paper, we presented an alternative of traditional approaches, methodologies based on geostatistical estimation and simulation techniques were carried out to determine the expected RMR and its underlying parameters and provides a risk failure analysis with a measure of the uncertainty at any target location.

The proposed indirect estimation and simulation methods outperformed the more frequently used direct approach and shows a more accurate map with low error coefficients which makes them adequate for RMR modeling. However, it should be noted that is high computational and more pre-processing time needed.

This research showed that each geostatistical model identifies potential risk-prone areas in which failures could be experienced, superposing the different resulting maps allowed us to define low-risk conservative RMR model.

The resulting map of the indirect approach allowed taking into account the nonlinear nature, directional behavior of the RMR constitutive parameters which can be used to assist engineers in proposing suitable excavation techniques and an appropriate support system. The developed model help to assess different geomechanical parameters that can use to develop numerical models that explicitly consider the rock mass heterogeneity.

**REFERENCES**


