



# A NEW METHOD OF LOCATION ESTIMATION FOR FINGERPRINTING LOCALIZATION TECHNIQUE OF INDOOR POSITIONING SYSTEM

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### ABSTRACT

The conventional way of finding the closest pair of points is the use of brute force method that simply computes the distances of all pair of points in the plane and finds the points with the minimum distance. An improved version of this method was the use of divide and conquer algorithm. However, the expedition for improving the computational cost of this problem continues to grow because of potential applications in location estimation and sequence matching. This paper attempted to develop and analyze a new method called closest coordinate scheme to determine the estimated position for indoor positioning system. The enhanced fingerprint localization technique was linked with the closest coordinate scheme to test its value in terms of accuracy and efficiency. Results showed that the closest coordinate scheme is efficient and accurate. Future endeavor may focus on the time and space complexities of closest coordinate scheme and find out similar applications.

Keywords: brute force, dynamic warping, received signal strength, trilateration.

### INTRODUCTION

One of the earliest problems of computational geometry is to find out the closest pair of a point (CPP).It has several applications in the field of image processing, graph partitioning, speech recognition, pattern matching, and pattern identification and intrusion detection. Most of the solutions are focused on the running time as the basis of efficiency of the developed algorithms. The study of [30], showed that CPP has been explored well and several studies resulted an efficient expected running time compared with other CPP algorithms. The following information is some CPP applications in different fields. The closest pair and post office problem for stochastic points of [25] used Euclidean space to solve the closest pair of points and applied it to post office problem using stochastic points. Another application of CPP is in the field of location problem of alternate fuel store of [26] wherein an efficient heuristic algorithm was developed to locate the alternative-fuel stations. The improvement of trimmed iterative closest point algorithm was proposed by [28] and showed high accuracy compared with several other algorithm of similar functions in variety of situations. It is then followed by the scan matching of [27] used iterative closest point [ICP] algorithm which seeks to minimize the misalignment between two point cloud data sets. The following information is useful for the formulation of the new method of CPP.

#### Overview of brute force method

The brute force method is one of the most popular way to solve the problem wherein the computation of distances of all pairs of points and the determination of minimum distance are done to find out the closest pair. The information below are examples on how to determine the closest pair of points using brute force.



Figure-1. Coordinates on the xy-plane.

For the purpose of illustration the 10 sample points are used. The points are A(1,2), B(3,1), C(10,7), D(5,5), E(6,1), F(7,7), G(7,10), H(2,9), I(1,6) and J(8,3). The brute force method of determining the closest pairs of points is to compute the distances of all pair of points [23]. Tables below presented the all the pair of points and the computed values of its distances.

# \_\_\_\_\_

Α	В	С	D	Е	F	G	Н	Ι	J
AB									
AC	BC								
AD	BD	CD							
AE	BE	CE	DE						
AF	BF	CF	DF	EF					
AG	BG	CG	DG	EG	FG				
AH	BH	СН	DH	EH	FH	GH			
AI	BI	CI	DI	EI	FI	GI	HI		
AJ	BJ	CJ	DJ	EJ	FJ	GJ	HJ	IJ	

www.arpnjournals.com **Table-1.** All pair of points.

Table-2. Computed distances of the pairs of points.

Α	В	С	D	Е	F	G	Н	Ι	J
(1,2)	(3,1)	(10,7)	(5,5)	(6,1)	(7,7)	(7,10)	(2,9)	(1,6)	(8,3)
2.24									
10.3	9.22								
5	4.47	5.39							
5.1	3	7.21	4.12						
7.81	7.21	3	2.83	6.08					
10	9.85	4.24	5.39	9.06	3				
7.07	8.06	8.25	5	8.94	5.39	5.1			
4	5.39	9.06	4.12	7.07	6.08	7.21	3.16		
7.07	5.39	4.47	3.61	2.83	8.94	7.07	8.49	7.62	

To determine the total number of comparisons is to use the arithmetic series by multiplying the number of terms and the average of first and last terms.

$$Sum_{as} = total number of terms(\frac{(first term+last term)}{2})$$

The example above has 9 number of terms since point J or the last point was not included and the first and last terms are 1 and 9 respectively. Using the formula above, the total number of comparisons is equal to 45. It was simplified in the following equation:

$$KC = \frac{(nd-1)nd}{2}$$

Where KC is the total comparisons and nd is the total compared distances. This formula is also related with the computational cost wherein the number of processes are counted as the number of comparisons.

# Overview on divide and conquer algorithm

The divide and conquer algorithm by Shamos and Hoey [23] and [31] was an enhancement of brute force method. This method reduced the number of comparisons. **Overview on RSS fingerprinting localization technique** The Received Signal Strength (RSS)-based fingerprinting positioning system technique that requires site survey to determine the RSS signal of the environment. These localization technique may use Wireless Fidelity (WiFi) and Radio Frequency Identification (RFID) technology as emphasized by [17]. The figure below is the RSS-based fingerprinting technique according to [13].



Figure-2. The RSS-based fingerprinting technique.



Figure-2 shows the RSS fingerprinting localization technique. It is composed of offline and online phases. The offline phase is responsible for collection of RSS signals. These signals are stored to fingerprint database while the online phase is responsible for capturing of the RSS from mobile sensor and determining the estimated position. Table below shows the fingerprinting localization technique advantage and drawback by [17]. This method is laborious in the collection and calibration stage that made the researchers to come up with an enhancement.

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### Overview on dynamic time warping

Dynamic Time Warping (DTW) is a kind of dynamic programming used in sequence and pattern matching. The DTW was used to increase the performance in terms of efficiency of indoor location estimation. DTW study revealed that 20% increase in accuracy was obtained by [12].

# Applications of indoor positioning system

The study of [11] reduced the RSSI fluctuation that contributes negative effect to accuracy of position estimation. It provides precise and less error compared with other similar methods and the unsupervised indoor localization project of [12] was the improvement of [11].It was then followed by the use of ZigBee of [8] for tracking the landmarks. The inertial navigation system (INS) and Kalman filter improved the accuracy of operation. Another breakthrough in the field of position estimation was stated in [15] with the aid of dead reckoning and motion sensors to collect and store RSS to radio-map database of the environment. Some artificial intelligence system was used for location estimation maximum likelihood. K-nearest neighbors, weighted K-nearest neighbor, support vector machines and artificial neural networks. The method are the source of estimated location of indoor positioning systemused by [4], [6] and [14]. These artificial intelligence algorithms require training stage which contribute to the complexity of the system. The filtering techniques on the sources for additional sensor data, Gaussian Mixture Model, data Dempster Shafer fusion theory, fusion technique, Kalman filter or particle swarm filters by [1], [2], [3], [4], [7], [9], [10], [16], and [18]were also explored which introduced filtering of RSS signal to improve the RSS of the radio-map database on the collection side. All of the above mentioned literature are already improved the accuracy but the laborious collection of radio map and complexities of the system were still present.

In this study, a new way of improving the position estimation of fingerprinting technique in terms of accuracy and efficiency was introduced. The new method of CPP was designed for position estimation.

# **RESEARCH METHOD**

Three phases were considered in order to improve the fingerprinting localization technique.

### Phase 1 - The basis of location estimation

This phase is the offline phase which is responsible for the setting up of radio map of fingerprint technique of indoor positioning system. A class room with 16 seats was used as a model.



Figure-3. Floor plan of classroom of 16 seats.

### The coordinates of model class room

The coordinates-map is the modified version of radio map. The classroom environment has the assigned coordinates as shown by the figure below.



Figure-4. Assigned coordinates of classroom

# The structure of coordinates-map

Table-3 shows the structure of coordinates-map and stored on the computer.

Table-3. Coordinates-map.

<b>P</b> ( <b>x</b> , <b>y</b> )	Location	<b>P</b> ( <b>x</b> , <b>y</b> )	Location
1,1	Near Door	6,6	Near table5
1,2	Near Door	6,2	Near table5
1,3	Back of chair1	6,3	Table5
1,4	Back of chair1	6,4	Table5
1,5	Back of chair2	6,5	Table5

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1,6	Between chair2 & 3	6,6	Between Table5 &6
1,7	Back of chair3	6,7	Table6
1,8	Back of chair 3	6,8	table6
1,9	back of chair 4	6,9	Table6
1,10	back of chair 4	6,10	Near table6
2,1	Near Door	7,1	Near chair13
2,2	Near Door	7,2	Near chair13
2,3	Table1	7,3	Chair13
2,4	Table1	7,4	Chair13
2,5	Table1	7,5	Chair14
2,6	Table1	7,6	Between chair14&15
2,7	Between table1 & 2	7,7	Chair15
2,8	Table 2	7,8	Between Chair15&16
2,9	Table2	7,9	Chair16
2,10	Near Table2	7,10	Near chair16
3,1	Near table3	8,1	Near Table7
3,2	Near table3	8,2	Near table7
3,3	Chair5	8,3	Table7
3,4	Chair5	8,4	Table7
3,5	Chair6	8,5	Table7
3,6	Between Chair6 &7	8,6	Between Table7&8
3,7	Chair7	8,7	Table8
3,8	Between chair7 &8	8,8	Table8
3,9	Chair8	8,9	Table8
3,10	Near chair8	8,10	Near table8
4,1	Near table3	9,1	Near table9
4,2	Near table3	9,2	Near table9
4,3	Table3	9,3	Near table9
4,4	Table3	9,4	Near table9
4,5	Table3	9,5	Table9
4,6	Between table3 &4	9,6	Table9
4,7	Table4	9,7	Table9
4,8	Table4	9,8	Near table9
4,9	Table4	9,9	Near table9
4,10	Near table4	9,10	Near table9
5,1	Near chair9	10,1	Front Wall
5,2	Near Chair9	10,2	Front Wall
5,3	Chair9	10,3	Front Wall
5,4	Chair9	10,4	Front Wall

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5,5	Chair10	10,5	Front Wall
5,6	Between Chair 9&10	10,6	Front Wall
5,7	Chair11	10,7	Front Wall
5,8	Between Chair11&12	10,8	Front Wall
5,9	Chair12	10,9	Front Wall
5,10	Near Chair12	10,10	Front Wall

# Phase 2 - The information about RSS collection

This phase is the online phase that collects data. An android phone and computer were used as mobile sensor and server respectively. A Wifi RSS sampling application of [6] was used and installed to mobile sensor to retrieve the RSS and MAC address from each of the wireless access point.

# **RSS** data collection system diagram

Three wireless access points were responsible to send RSS signals to mobile sensor. An application installed to an android phone served as the mobile sensor that collects RSS from wireless access points. The computer served as the server that is responsible for the determination and display of estimated location.



Figure-5. System diagram of data collection.

# Determination of the mobile coordinates

The Free-space path loss (FSPL) is used to compute for the distance between the mobile sensor and the wireless access point determination. The formula is: FSPL (dB) = 20Log (r) + 20 Log (f) + 92.45Log(r) = FSPL(dB) - 20 Log(f) - 92.45

Where FSPL is the free space path loss in terms of dB, f is the frequency in terms of GHz. and r is the distance in terms of kilometer.



Figure-6. Trilateration setup of mobile sensor.

The distance (r1, r2, r3) between the mobile sensor and the wireless access points are computed using trilateration method as follows:

$$r1^{2} = (x1 - x)^{2} + (y1 - y)^{2}$$
$$r2^{2} = (x2 - x)^{2} + (y2 - y)^{2}$$
$$r3^{2} = (x3 - x)^{2} + (y3 - y)^{2}$$

Rearranging the equations:

$$H^{2} = x1^{2} + y1^{2} - r1^{2}$$
$$I^{2} = x2^{2} + y2^{2} - r2^{2}$$

$$J^2 = x3^2 + y3^2 - r3^2$$

$$x = \frac{H(y3 - y2) + I(y1 - y3) + J(y2 - y1)}{2[x1(y3 - y2) + x2(y1 - y3) + x3(y2 - y1)]}$$

$$y = \frac{H(x3 - x2) + I(x1 - x3) + J(x2 - x1)}{2[y1(x3 - x2) + y2(x1 - x3) + y3(x2 - x1)]}$$

Where:

x - x-coordinate of mobile sensor y - y-coordinate of mobile sensor

### Phase 3 - Position estimation

This phase include the development of a new approach called Closest Coordinate Scheme (CCS) to determine the estimated position of fingerprinting localization technique. The use of the concept of dynamic warping and Euclidean distance were employed to increase the efficiency of position estimation of indoor positioning system. An n x m matrix was used. The size of the matrix depends on the coordinates-map assignments. To illustrate the CCS method, the computed mobile sensor coordinates are x and y were assumed. To determine the estimated location, the distance between the mobile sensor coordinates-map are necessary. Figure below presents the sample grid table.



Figure-7. Matrix of distance.

### Procedure on how to determine the estimated position

- a) Start from the lower left corner of the matrix following the heuristic step below.
- b) Follow the heuristic step of Figure-8.



Figure-8. Heuristic step.

- c) Determine the minimum between Start, Dxnyn1, Dxnyn2 and Dxnyn3 cells.
- d) Move to the next level but be sure that the Start cell must be the minimum value of the heuristic step
- e) Repeat procedure 2, 3 and 4 and proceed to procedure 6 if there is no change in the movement of heuristic step.
- f) Determine the final minimum value between the contents of the Start, Dxnyn1, Dxnyn2 and Dxnyn3 cells and determine the corresponding coordinates of the minimum value.

### **RESULTS AND ANALYSIS**

The following discussions present the results of the experiments that determines the possible enhancements.

# The modified fingerprint localization technique for indoor positioning system



Figure-9. Modified fingerprint technique.

Fingerprinting localization technique of [13] starts with the collection of RSS signals up to the determination of the estimated position. The modified version of fingerprinting localization technique also starts with the collection of RSS signals up to the determination of the estimated position. The method of the collection of RSS reading of mobile sensor from Wireless Access Points (AP1, AP2, AP3) is the same with [6]. The modification are focused on two things; First is the creation of coordinates-map instead of radio map as mentioned by [13]. This enhancement lessen the effect of external noise interference and environmental change during the collection of RSS as mentioned by [14], [15] and [17]. The site survey are also eliminated which lessen the laborious collection of radio map and complexities of the system. Second is the development of new method on how to determine the position of the mobile sensor called CCS.

### The closest coordinate scheme

For the purpose of simulation, it is assumed that the computed mobile sensor coordinates are x = 4.4 and y = 6.3. The computed value of distances are the distance between the mobile sensor coordinates and coordinates map coordinates arranged in the grid below.



Figure-10. Matrix of distance.

The steps of section 2.3.3 has the following outputs:





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Figure 17. Seventh heuristics steps.

The coordinates P (4.4, 6.3) are mobile sensor coordinates. The result shows that the estimated position of the coordinates P (4.4, 6.3) is located between table 3&4 since the closest coordinates of mobile sensor coordinates is P (4, 6) of coordinates map and the number of comparisons is 36. Please refer to table 2 for the coordinates map and its location.

### The test of the number of comparisons

The study used the number of comparisons on how to determine the minimum distance as the basis for the estimated position. The number of comparisons are the basis of the efficiency of the execution because the comparisons are the major process of the position estimation. The following presents the result of test and comparisons between the closest CCS, brute force and divide and conquer CPP algorithm.

The total number of comparison of Closest Coordinate Scheme is determined by:

$$Ka = HS(\frac{(nd-1)nd}{2})$$

Where Ka is the number of comparison, HSis the number of heuristic steps and nd is the total number of distances.

The total number of comparison of Brute Force Method and Divide and Conquer algorithm is given by:

$$KC = \frac{(nd-1)nd}{2}$$

Where k is the number of comparisons and n is the total number of distance to be compared.

### Sample computations of the number of comparisons of three CPP algorithms Let:

# Point to be compared with the coordinates map is P(1.1, 2)

Number of points to be compared with P (1.1, 2) is 100 points

Number of distances between P (1.1, 2) and the points of coordinates map is 100

### A. Closest coordinate scheme

Y											
10	7.1	6.4	5.7	5.2	4.9	4.7	4.8	5.0	5.4	6.0	
9	6.5	5.7	5.0	4.4	3.9	3.7	3.8	4.1	4.6	5.2	
8	5.9	5.1	4.3	3.5	3.0	2.7	2.8	3.2	3.8	4.6	
7	5.6	4.6	3.7	2.9	2.1	1.7	1.8	2.4	3.2	4.1	
6	5.3	4.4	3.4	2.4	15	08	1.0	1.8	2.8	3.8	
5	5.3	4.3	3.3	243	1.5	• 0.4	• 0.8	1.7	2.7	3.7	
4	5.5	4.5	36	2.6	• 1.8	1.3	1.5	2.1	3.0	3.9	
3	5.8	49	4.0	• 3.3	2.6	2.3	2.4	2.9	3.5	4.4	
2	62	5.4	4.7	4.0	3.5	3.3	3.4	3.7	4.3	5.0	
1	6.8	• 6.1	5.4	4.9	4.5	4.3	4.4	4.6	5.1	5.7	
	1	2	2	4	r.	c	7	Ö	0	10 V	~

Figure-18. Summary of heuristic steps for CCS.

A= 6 steps; based on the schematic steps of figure 19

$$Ku = 6\left[\frac{(4-1)4}{2}\right]$$
  
= 36 comparisons

### **B.** Brute force method

$$k = \frac{(100 - 1)100}{2}$$
  
= 4950 comparisons

### C. Divide and conquer algorithms with 2 divisions

$$k = 2[\frac{(50-1)50}{2}]$$

Coordinates of the MS P(x,y)	Number of schema-tic steps	Closest coordinate scheme	Brute force method	Divide and conquer algorithm
P (1.1, 2)	2	12	4950	2450
P (1.5,1)	2	12	4950	2450
P (2.4, 3)	3	18	4950	2450
P (2.4, 6)	6	36	4950	2450
P (10,10)	9	54	4950	2450
P(9,9)	9	54	4950	2450
P(7,7)	6	36	4950	2450

Table-4. Result of the test of comparisons of different CPP algorithm.

P(6.3,5.3)	6	36	4950	2450
P(1,1)	1	6	4950	2450
P(2.2,2)	2	12	4950	2450

Table-4 shows that the CCS has the least number of comparisons compared with the brute force and divide and conquer algorithm. It shows that CCS is the most efficient among the three presented CPP algorithm in terms of number of comparisons.

### The test of accuracy

Table-5. Result of the test of accuracy.

Coordinates of the Mobile Sensor P(x,y)	Location base on Coordinates Map	Remarks
P (2, 10)	Near table1	Accurate
P (1.5,1)	Near Door	Accurate
P (2.4, 3)	Chair5	Accurate
P (2.4, 6)	Between Chair6&7	Accurate
P (10,10)	Front Wall	Accurate
P(9,9)	Near Table9	Accurate
P(7,7)	Chair15	Accurate
P(6.5,5)	Chair14	Accurate
P(1,1)	Near Door	Accurate
P(2.2,2)	Near Door	Accurate

Based on the results stated in table 5, all of the simulation tests achieved high accuracy on location estimation.

# CONCLUSIONS

The researchers presented an enhanced method of fingerprinting localization technique of indoor positioning system. This lessens the laborious collection of RSS signals. A new method called closest coordinate scheme on the position estimation was also introduced as a method of position estimation. The CCS algorithm was efficient in terms of number of comparisons than brute force and divide and conquer algorithms. Result shows that the series of test achieved high accuracy on the position estimation using the enhanced technique. Future endeavor may focus on the development of a system for the CCS to find out the time complexity and space of the implementation of the algorithm to find out its value to the field of similarity matching.

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