



# RELEVANCE OF CONSTANT INTENSITY DISTRIBUTION TO CONTOURING REFLECTING PLATES WITH INTENSITY INTEGRATION TECHNIQUE

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## ABSTRACT

Intensity Integration Technique (IIT) is an image correlation technique to contour mirror-like reflective surfaces. In the case of one-dimensional surfaces, curvature at any point on it can be expressed in terms of the ratio of reflected intensities from it from two (load, unload) states of the surface. As a consequence, it is shown, that if and when one of the reflected intensity distributions is uniform (constant), the reflected intensity value of the other at every point represents curvature to some scale. For now, as constant intensity distributions are not feasible, a viable equivalent is proposed and the hypothesis verified experimentally.

**Keywords:** intensity integration technique, reflecting plates, constant intensity, reconstruction, line summation.

## 1. INTRODUCTION

Digital correlation techniques attempt to obtain deformation fields of loaded specimens by comparing their reflected intensity distributions before and after deformation. Literature abounds with a large body of information in this field. Among them the most well-known is DIC (Digital Image Correlation). (See for example, Chu *et al* [1], Sutton [2]). In this, a random speckle pattern is sprayed on matte like non-reflective surfaces [3] to find the deformation field by tracking the change in position of patterns in subsets. The deformation field is established for transparent specimens also by projecting speckle patterns on and through them [4], [5]. On the other hand, Intensity Integration Technique (IIT) [6], tries to track the deformation fields associated with mirror-like reflecting surfaces. It relies on the premise that the quantity of light reflected off the specimen is the same before and after loading and so the integrated (cumulative) intensity could serve as a correlation parameter.

Subramanian and Jagannath [6] propose this technique to contour large reflective loaded cantilever surface. They use diverging laser beam to illuminate a Plexiglas specimen. The reflected intensity images from the specimen falling on a screen are captured before and after loading. Treating the cantilever as one-dimensional, the image intensities summed up across the specimen widths are arranged sequentially along the length of the cantilever to form a one-dimensional digital array. The length of this array for the loaded reflected image varies slightly from the unloaded, depending upon whether it is loaded forward or backward. The shift in the location of any point on the one-dimensional array for the unloaded image is identified with a corresponding point for the loaded image when the cumulative intensity sums for these two locations are the same.

The plot of cumulative intensity values usually is non-linear as the intensity distribution from the light source is arbitrary and *never* a constant. Further, if the specimen is illuminated by a diverging beam of light, the reflected beam carries an apparent curvature distribution

which gets added to the curvature distribution due to load. Instead, if the illumination is from a collimated source, apparent curvature is either zero or negligible. Following ref. [6], curvature ( $d^2w/dy^2$ ) of a loaded specimen surface is expressed as (with screen to specimen distance as  $D$ )

$$(I_1/I_2) = 1 + 2D (d^2w/d^2y) \quad (1)$$

$I_1$  and  $I_2$  are the reflected intensity values from the same point on the specimen captured by its loaded and unloaded images.

Herein, first, an *arbitrary* unloaded specimen intensity distribution is assumed for a tip loaded cantilever and a simulation is carried out to form the deformed integrated intensity distribution by theoretical slope-distance estimation

As a second case, the un-deformed cumulative intensity distribution is assumed to be a straight line. This is possible only if the values for illuminating intensity are all the same constant over the specimen surface. (In reality, this is infeasible and somewhat futuristic).

Here too, the deformed cumulative integrated intensity curve is developed using the theoretical slope-distance estimates. By differentiating this curve the intensity distribution for the deformed state is obtained. The significance of constant reflected intensity distribution is investigated and a method to imitate such a distribution with projected equal-spaced lines is mooted. The procedure is termed Line Summation in this study.

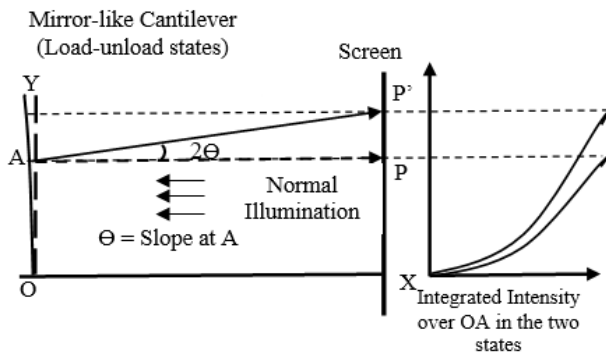


Figure-1. IIT schematic.

### 1.1 Revisiting intensity integration technique briefly

Intensity Integration Technique (IIT) states that the total quantity of light reflected off any portion of a mirror-like surface is the same for any of its nearby states. IIT is a double exposure method needing intensity distributions of images of the surface before and after loading.

In Figure-1, a cantilever beam reflects a collimated beam of light incident on it to a matte screen kept at some large distance  $D$  (distance  $OX$ ) from it. The point  $O$  represents the root of the cantilever.

When the specimen is bent, say, backward, the incident ray at any point  $A$ , shifts its reflection on the screen from  $P$  to  $P'$  due to the rotation of the surface normal at  $A$ . For any point  $A$  on the beam the integrated (cumulative) intensity over a distance  $XP$  on the screen for state 1 (unloaded) must equal the integrated intensity over a distance  $XP'$  for state 2 (loaded). Expressed mathematically,

$$\int_0^P I_1 dy = \int_0^{P'} I_2 dy \quad (2)$$

Here distance  $(XP' - XP) = PP' = a$  is referred to as slope-distance at  $A$  for the cantilever.

## 2. SIMULATION OF INTENSITY INTEGRATION TECHNIQUE

By integrating Euler-Bernoulli [7] beam expression, for a tip-loaded uniform cantilever of length  $L$ , in experiments one can express the theoretical slope along its length in terms of applied tip-deflection. The tip slope  $\theta_{TIP}$  is expressed in terms of tip deflection  $W_{TIP}$  as  $(3W_{TIP}/2L)$ .

Defining  $\xi (=y/L)$ , as the non-dimensional  $y$  distance, the slope equation becomes,

$$\theta = (2\xi - \xi^2) * \theta_{TIP} = (a/2D) \quad (3)$$

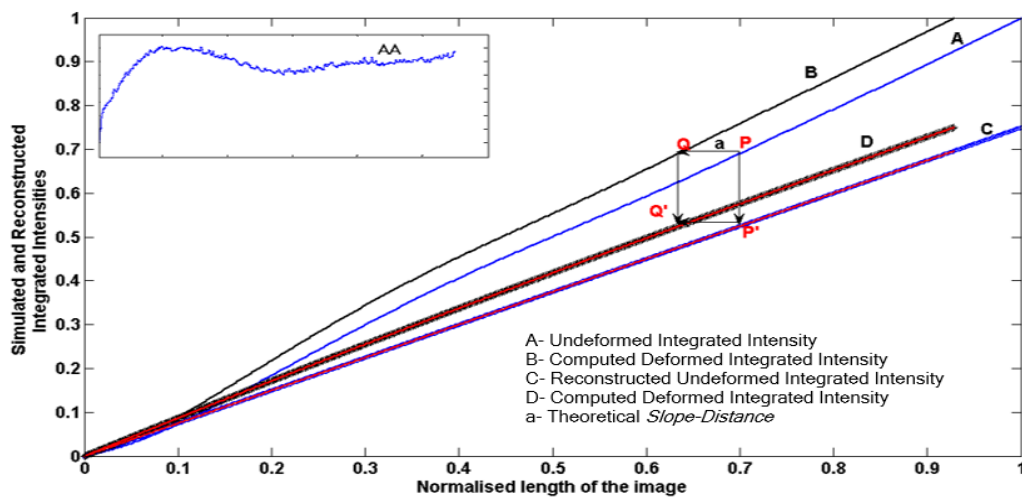
Therefore slope-distance  $a$  takes the form

$$a = (3D/L) * (2\xi - \xi^2) * W_{TIP} \quad (4)$$

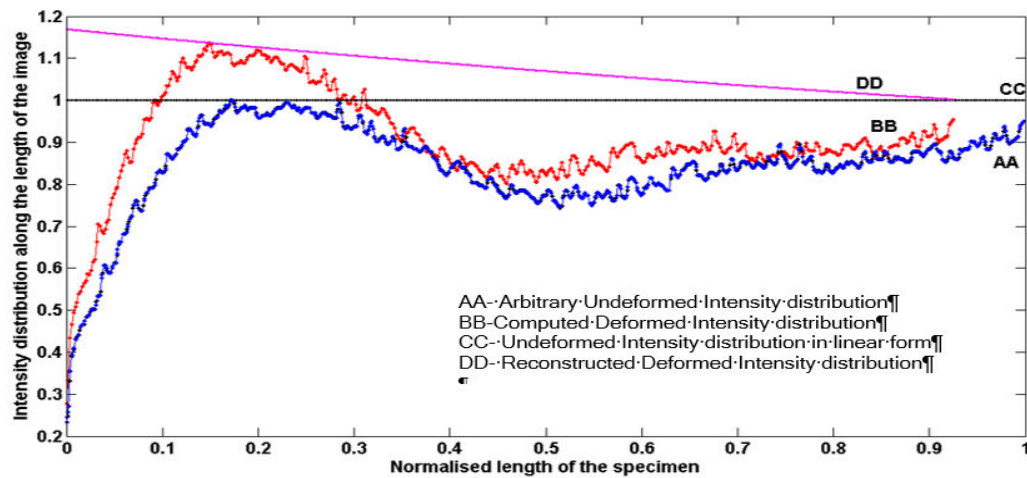
## 3. RECONSTRUCTING INTENSITY DISTRIBUTION FOR DEFORMED SURFACE

In the following, a simulation of IIT is attempted with arbitrary un-deformed intensity data (see inset in Figure-2). The cumulative sum of intensities of the curve  $AA$  forms the integral intensity curve  $A$  in Figure-2. Instead, if the illuminating intensity distribution were uniform (constant) the reflected intensity distribution off an unloaded flat surface also will be likewise; integrated intensity then would be linear. In the method of reconstruction proposed here, the integral of intensities for the un-deformed specimen are assumed to form a straight line of some desired slope and the deformed integrated intensity curve is simulated by knowing the theoretical slope-distances between the two at every point. In Fig.2, for any point  $P$  on curve  $A$ ,  $Q$  is the corresponding point on curve  $B$  satisfying IIT. In the curves  $C$  and  $D$  the corresponding points are identified such that  $PQ = P'Q' = a$ , the slope-distance. All points on the curve  $D$  satisfy this requirement. For an assumed constant intensity illumination, the curve  $C$  is thus the linear form of curve  $A$  representing the unloaded state and curve  $D$  is the modified form of curve  $B$ , the integral intensity curve for the loaded state. By differentiating these integral intensity curves ( $A$  to  $D$ ) in Figure-2, one gets the corresponding intensity distributions ( $AA$  to  $DD$ ) for loaded and unloaded surfaces in Figure-3.

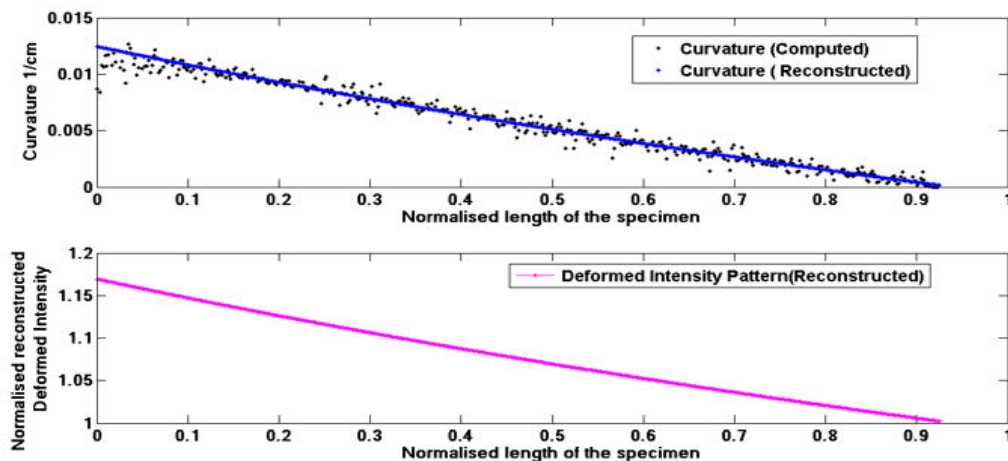
The curve  $BB$  in Figure-3 indicates the 'deformed' intensity distribution corresponding to the computed 'deformed' integrated intensity curve  $B$  in Figure-2. This shows that the reflected intensity distribution also is irregular. Whereas, significantly, the linear form for cumulative intensity curve  $C$  and computed deformed intensity pattern  $D$  in Figure-2 respectively yield constant un-deformed intensity distribution  $CC$  and uniformly varying deformed intensity distribution  $DD$  in Figure-3. This is very desirable in practice, as there is no scatter and is possible only if the illuminating intensity distribution is a constant.



**Figure-2.** Simulated and reconstructed deformed integral intensity curves corresponding to arbitrary undeformed intensity distribution as in the inset.



**Figure-3.** Simulated and reconstructed intensity distribution along the length of the image (Undeformed, Deformed).



**Figure-4.** (a) Reconstructed curvatures (b) Deformed intensity pattern.



#### 4. MEASUREMENT OF SLOPE AND CURVATURE OF A CANTILEVER BEAM

In IIT, curvature is related through the local values of intensity in the loaded image ( $I_2$ ) to the local values of intensity in the unloaded image ( $I_1$ ) at the corresponding locations of the cumulative integral intensity curves in the two states. The curvature values obtained in ref [6] are widely scattered as they depend on the ratio of two irregularly varying intensity values at pixel locations.

If the un-deformed intensity distribution is constant, the curvature values are expressed in terms of deformed intensity values only and so the scatter reduces. In Fig.4a the simulated curvature values are scattered as the assumed un-deformed reflected intensity distribution is not uniform. But the curvature values from reconstructed data form a continuous line with almost no scatter when the un-deformed intensity distribution is a constant.

In Figure-4b, reconstructed deformed intensity distribution pattern represents curvature to some scale. It depends on the slope of the straight line C. It is obvious that if the intensity distribution for the loaded image resembles curvature, the integration of curvature leads to slope and hence the integrated intensity pattern of the loaded image in turn should resemble slope. That is, referring to Figure-2, if points on the curve D are plotted with straight line C as the axis of reference, it turns out to be the slope curve to some scale. It is easy to see that the ordinates of points on D from C are now a constant times the slope distance  $a$ , the constant being the cotangent of the angle the line C makes with X-axis.

#### 5. LINE-SUMMATION

The projected interpretation to IIT is applicable only if the intensity distribution over an object surface corresponding to any one of its states is a constant and not arbitrary. It is obvious that this condition is futuristic and not met in practice with any known manner of illumination. Therefore to verify the hypothesis an alternative is proposed here along the lines of well-known 'reflecting grid method' [8]-[10] which can be thought to emulate (discrete) constant intensity increment with every grid line location over the surface. This satisfies mathematically the conditions for the proposed interpretations to hold. For this, a set of uniformly spaced lines, say, guidelines or gridlines are projected onto the reflecting specimen surface and the images off the object surface are captured. The cumulative line count (line-summation, which is interpreted as the equivalent of integrated intensity in IIT); in the unloaded state for the object is always a straight line. This is taken to imitate in a discrete sense constant intensity distribution for the unloaded state of the surface studied. The shift of these lines gives slope and the Eulerian strain of the gridlines gives curvature [9].

For an application of the line-summation, clipped middle portions of images (Figure-5a,b) of a centrally loaded clamped circular plate [11] are studied. Lines are counted by applying image-processing in MATLAB (by means of denoising, thresholding, skeletonizing,

complimenting and labelling as required). The number of lines (line-sum/count) is plotted against pixel position for the two states in Figure-5(a) and 5(b)

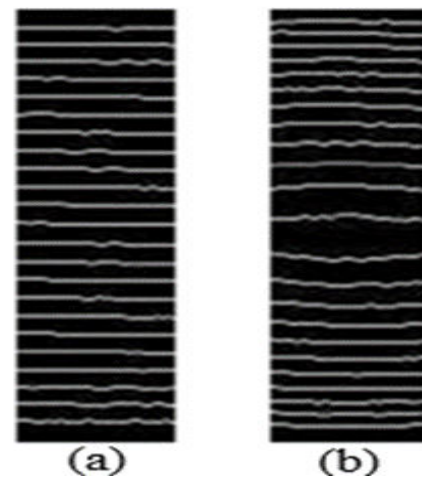


Figure-5.Guidelines in centrally loaded clamped circular plate: unloaded and loaded.

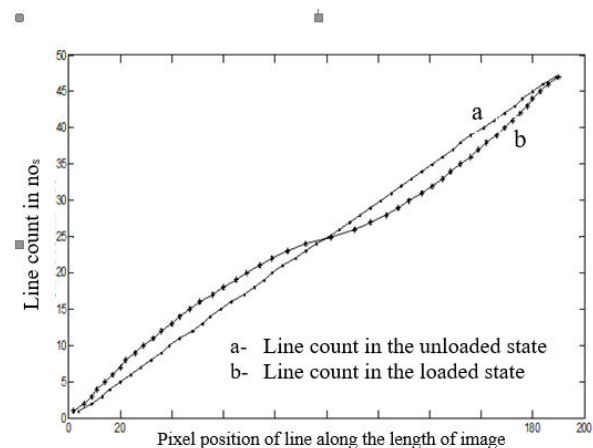


Figure-6.Line count vs its position along the length of the image.

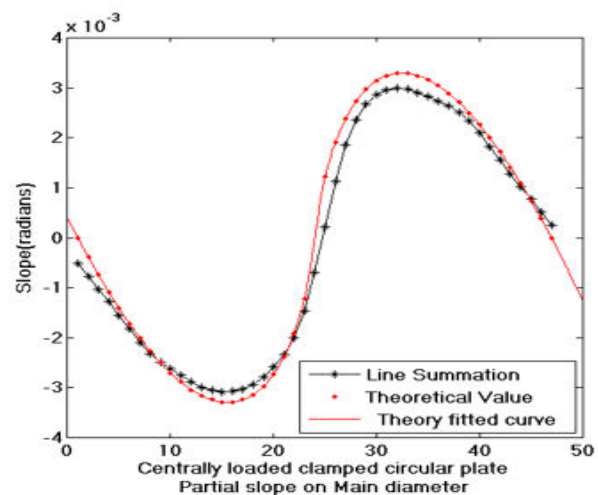
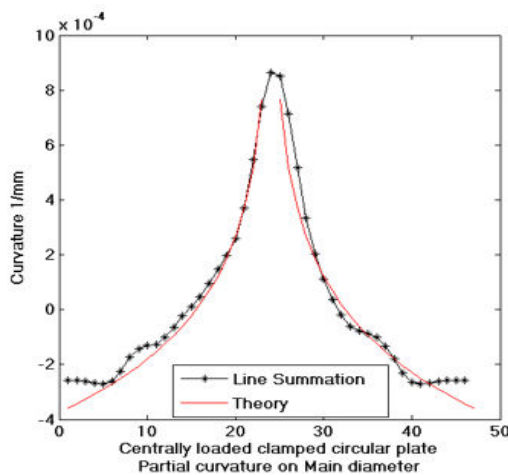


Figure-7.Slope on vertical diameter of centrally loaded clamped plate using Line Summation.





**Figure-8.**Curvature on vertical diameter of centrally loaded clamped plate using Line Summation.

In Figure-6, the line-count graph for unloaded image is practically linear and for the loaded image it is the anti-symmetric curve b with reference to line a. The shape of the curve b, by virtue of the previous result, yields slope. By measuring slope-distances between the curve b and the straight line a, and dividing by twice the distance D between the object and screen, slope ( $dw/dy$ ) on the vertical diameter of the circular plate is obtained. Differentiating the slope curve one gets the principal curvature ( $d^2w/dy^2$ ) along this diameter. A comparison of the experimental results with theory shows the mean error is of the order of 3% (Figure-7 and Figure-8).

## 6. CONCLUSIONS

The Intensity Integration Technique (IIT) for slope and curvature is rationalized here from a novel stand point hypothesizing a requirement that constant intensity illumination for any one state of the object is imposed. Well-known reflecting grid method [9] is called in as analogous to IIT in a discrete sense, when a state of constant intensity illumination condition is seen to simplify analysis but not feasible to generate in practice. With the help of experimental images its applicability is demonstrated. The accuracy of results from using line-summation is dependent on the number of lines projected.

Thus, in this analysis two distinct results are presented based on IIT. The first one, establishes that the cumulative intensity curve can represent slope distribution to some scale if in a theoretical sense one of the reflected intensity distributions (values) from a mirror-like surface (unload state) is constant. For now, as this is not feasible in practice, the second one identifies a viable substitute based on the equivalence between IIT and reflecting grid method and verifies it experimentally.

Iterative Intensity Integration Technique (IIIT) [12], upgraded version of IIT, which is used for contouring two-dimensional reflective surfaces without projecting grid lines on the specimen, could be benefitted with this proposal of constant intensity distribution in undeformed state.

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