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DAMAGE DETECTION USING LAPLACIAN OPERATOR ON INTERPOLATED MODE SHAPE CURVATURE

Fawazi N.[1,2], Hasrizam, C. M.[1] and Hafiz Amran[1]
[1]Intelligent Dynamic System Ikohza, Department of Mechanical Precision Engineering, Malaysian Japan International Institute of Technology, University Teknologi Malaysia Kuala Lumpur, Jalan Sultan Yahya Petra, Kuala Lumpur, Malaysia
[2]Tokyo City Universities, Tokyo, Japan
E-Mail: fawazi.kl@utm.my

ABSTRACT

In this paper, a mode shape curvature based damage detection algorithm for estimating damage location in a beam is presented. The computational modal analysis is used to obtain the natural frequencies and the mode shapes for both undamaged and damaged structure. The Laplacian difference equation is used to estimate the difference of the calculated mode shapes. Considering the limited number of possible measured points during the modal analysis, Akima's interpolation is used. This is to estimate the mode shape displacements at the unmeasured positions which are a challenging problem for crack detection using mode shape data. The Gaussian wavelet for one-dimensional problem is used in this study which leads to a clear visualization of the position of the damaged structure.

Keywords: damage detection, mode shape, Akima's interpolation, continuous wavelet transform.

INTRODUCTION

In recent years, significant efforts have been made in the area of vibration-based damage detection methods. These developed methods are based on the dynamic characteristics, i.e., natural frequencies, mode shapes and modal damping, which are directly related to the stiffness of the structure [1,8].

In general, mode shape-based and curvature mode shape-based methods can be used for damage localization. In order to precisely locate and size the damage, the mode shape-based methods have to rely on certain computational algorithms or signal processing techniques. Thus, research on systematic damage detection algorithms have become a trend and researchers are focusing on the improvement of the existing damage detection algorithm.

Previous literature [1-5] show that mode shapes and corresponding mode shape curvatures are highly damaged sensitive and can be used not only for damage detection and but also for evaluation. These investigations also show that damage location and size can be assessed by employing the mode shape curvature data from the undamaged and damaged structures. These works calculate the difference in the mode shape data, between the healthy and the damaged state of a structure in order to localize the damage position.

In present, many research studies have been focusing on utilizing the changes in the mode shapes. The idea of using mode shapes as a crack identification tool is based on the presence of a crack which causes changes in the modal characteristics. The changes in mode shapes are much more sensitive in comparison to the changes in natural frequencies. This is because the mode shapes contain local information, which makes them more sensitive to local damages and enables them to be used directly in multiple damage detection. Not only that, the mode shapes is less sensitive to environmental effects, such as temperature, than natural frequencies [7].

However, the measurement of the mode shapes requires a series of sensors. For a long or huge structure, the experimental measurement can be a very tedious work. Thus, the measurement may fail to detect any crack in a structure.

In this study, as an alternative solution to solve the need of a high number of sensors or measured points, the Akima's interpolation is introduced in the computation. Not only extending the previous work [4], this work attempts to produce more efficient visualization using wavelet transformation in order to locate the existing damage. This work compares the healthy structure as a benchmark to a damaged structure by utilizing the mode shape information. The different mode shape information can be a useful guideline in order to develop an algorithm for crack detection.

DAMAGE DETECTION COMPUTATION

Mode shape curvature

Generally, mode shapes information can be used for damage localization. It is a fact that, any damage in most cases will affect the mode shapes curvature. This mode shape curvature not only can be used to identify the existence of the crack but also to localize the crack. Some previous research works only refer to the cracked mode shape curvature for crack detection, and some calculate the difference of the mode shape curvature between the undamaged and damaged structure to clearly visualize the crack position.

In this study, both mode shape curvature of the healthy structure and damaged structure are referred to. From the mode shape information of the two healthy and damaged structure, a difference function can be calculated.

Laplacian operator and laplacian curvature

The data of the mode shape curvature are discrete in space. Thus, the change in slope can be estimated using a finite difference approximation. This slope characteristic can be used to clearly visualize the damage location in a structure. In this paper, this curvature which is calculated using Laplacian Operator is referred to as Laplacian Curvature. For one-dimensional Laplacian, $\mathcal{L}_i$ of the discrete mode shape, y_i is given by [4]

$$\mathcal{L}_i = (y_{i+1} + y_{i-1}) - 2y_i \quad (1)$$

Akima interpolation

As stated n the previous section, it is somehow difficult to identify the location of the crack within the structure that has a relatively small damage. The location of the damage is hard to be localized either in the mode shape curvature or the difference function between the healthy and damaged structure [7]. Thus, in this study, the Akima's interpolation is introduced in the crack detection computation. This work starts by referring to the existing example in Ref. [4] which provide experimental results as a comparison to the present numerical computation.

The Akima interpolation [2,3] is a continuously differentiable sub-spline interpolation which was developed by Hiroshi Akima. This interpolated computation is developed based on piecewise polynomials which result in a smooth curve that passes through the given points. The slope of the curve is calculated at each given point, and the interval between the given points is determined by the coordinates of the slopes of the referred points

Only data from the next neighbor points are referred to in order to determine the coefficients of the interpolation polynomial. Having the advantage of no need to solve large equation systems, this interpolation method can be considered as computationally very efficient. As proven in [2], this developed computation method not only results in a smooth curve fitting but also consider the relatively small number of points as the basis of the method.

Taking the advantage of this interpolation method, the obtained mode shape data at each respective natural frequencies are interpolated in order to achieve a relatively smooth curve fitting. In the real application, the location of the crack is not visible which can cause a mis-unmeasured mode shape data at the specific position where the crack is. The suitable interpolation method is needed especially in detecting the small crack.

In this study, by taking examples in the previous work [4], the interpolated data is used to detect the crack position by using the difference function of the mode shapes between the damaged and undamaged beam. A further comparison is also done to see the difference of the Laplacian curvature which can be obtained using equation (1).

Difference function (mode shape and Laplacian Curvatre)

This study compares the difference function of the mode shapes curvatures $\delta_{\text{mode shape}}$ between the healthy structure (undamaged structure) and the damaged structure. This simple expression can be expressed as follows;

$$\delta_{\text{mod}eshape} = \varphi_{damaged} - \varphi_{undamaged} \quad (2)$$

where φ is the mode shape data. Similarly, the difference function of the Laplacian Curvature $\delta_{Laplacian}$ between the healthy structure (undamaged structure) and damaged structure is expressed as follows;

$$\delta_{\text{Laplacian}} = \mathcal{L}_{\text{damaged}} - \mathcal{L}_{\text{undamaged}} \quad (3)$$

Continuous wavelet transform (CWT)

In order to explore how far this method can be extended in crack detection, the Continuous Wavelet Transform is applied. This work is different in comparison to Ref. [5] where the work only employs the difference of the mode shapes data and presents the work using CWT. In this work, the difference of the Laplacian curvatures is referred to.

In this study, the Gaussian wavelet of order 4 is used in order to clearly visualize the location of the damage.

NUMERICAL SIMULATION

Undamaged and damaged beam

In this study, a computational simulation was conducted by referring to the experimental work done by Ref. [4]. The same model was referred to where a free-free steel beam, 0.6m x 0.25m x 4mm thickness was used. By having a uniform mesh of 20 x 7, two different beams were modeled as a healthy structure (undamaged beam) and a damaged beam. The damage was defined by cutting a through-thickness slot (the thickness of a saw blade) in the middle across approximately half the width, and 0.2 m from one end with 30% damage.

Since this work attempts to further extend the experimental work of Ref. [4] using computational simulation, only 30% of damage was considered. The rest will be considered in the future study. This work is only to investigate the capability of Akima's interpolation in order to detect the crack.

Similar to Ref. [4], the first two bending natural frequencies are referred to. The comparison between the experimental modal analysis and the present work shows good agreement as shown in Table-1. The obtained mode shape data between the undamaged beam and the damaged beam using the FEA are then used to calculate the difference of the mode shapes between the undamaged and damaged beam.

ARPN Journal of Engineering and Applied Sciences

Table-1. Effected damage on the natural frequencies between Ref. [4] and FEA (present).

Mode	**Natural frequency** [Hz], **Experiment** (Ratcliffe, 1997) **[4]**		**Natural frequency** [Hz], **FEA** (Present)	
	Undamaged	**Damaged**	**Undamaged**	**Damaged**
1	60.82	54.82	59.357	54.624
2	167.65	151.88	164.89	154.7

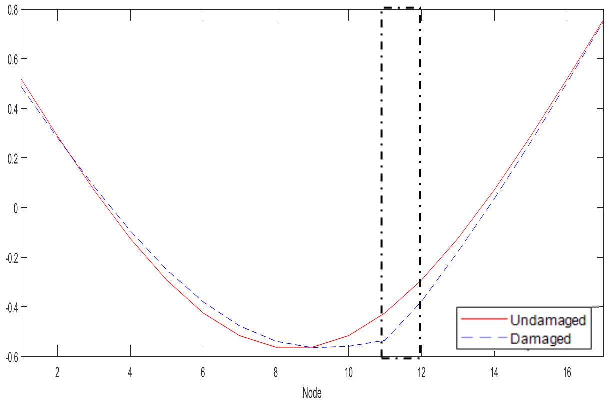

Figure-1. Mode shape 1 between undamaged and damaged beam.

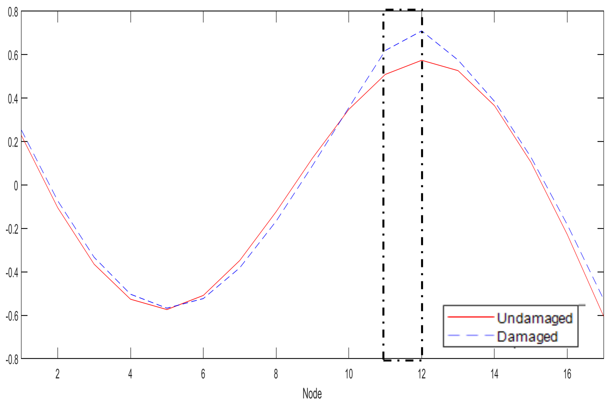

Figure-2. Mode shape 2 between undamaged and damaged beam.

Figure-1 and Figure-2 show the first and second mode shape curvatures between the undamaged and damaged beam. It can be seen there is a slight difference between the healthy and the damaged beam. Based on these two Figs., the position of the damage lies somewhere within node 10 to 13. Even though the difference is relatively small, this range shows a clear difference for crack detection.

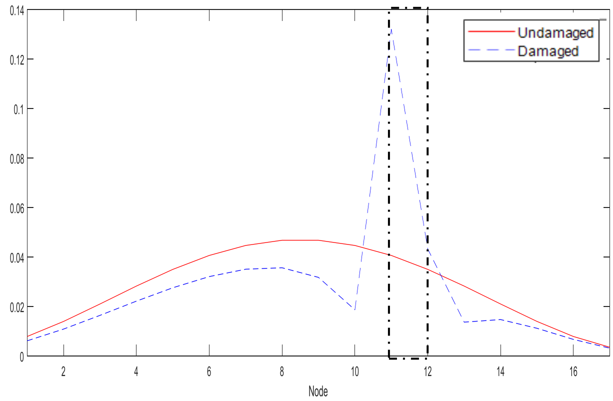

Figure-3. Laplacian for mode shape 1 between undamaged and damaged beam.

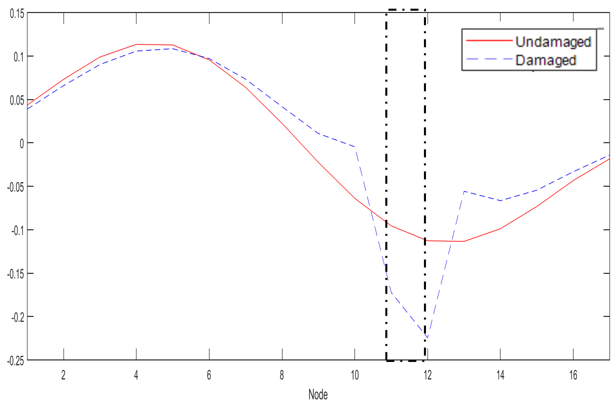

Figure-4. Laplacian for mode shape 2 between undamaged and damaged beam.

On the other hand, the Laplacian Curvatures between undamaged and damaged beam are shown in Figures 3 and 4. In comparison to the mode shape curvature, the difference can be identified more clearly. The damage can be identified within the range of node 10 to node 13. This clearer visualization is very important for damage detection especially when a structure has relatively small damage which results in a relatively small difference in mode shape curvature.

As stated in the previous section, the number of measured nodal points may affect the accuracy of the mode shape curvature. This accuracy can be increased by increasing the number of the measured nodal points. However, increasing the measurement point can be a tedious work especially during real experimental works. Thus, Akima's interpolation is used in this study as an

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attempt to obtain more measured point data which may increase the accuracy.

Figure-5 and Figure-6 show the interpolated function of the first and second mode shape curvatures. It is clear the interpolated data is different and not as smooth as a parabolic line in the previous figures. On the other hand, the location of the damage can also be identified within the node 10 and 12.

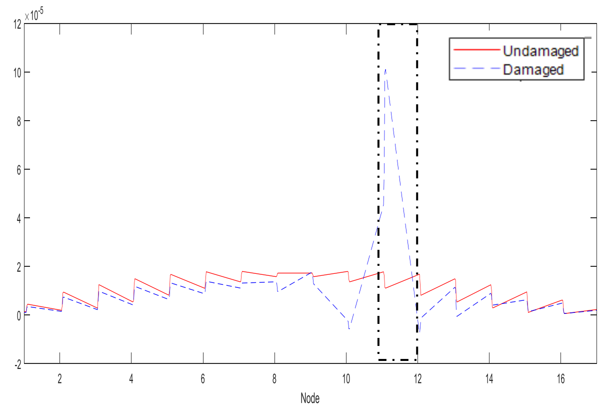

Figure-5. Laplacian for mode shape 1 between undamaged and damaged beam (interpolated using Akima interpolation).

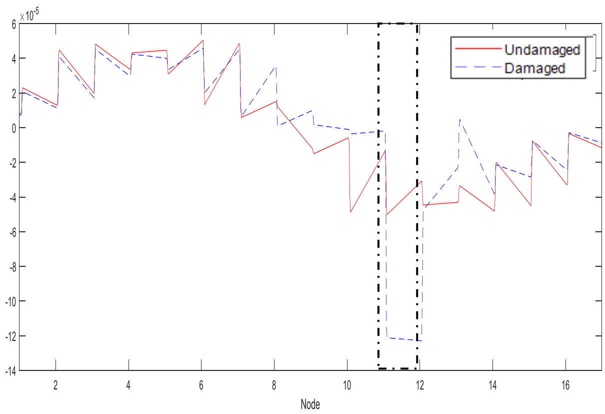

Figure-6. Laplacian for mode shape 2 between undamaged and damaged beam (Interpolated Using Akima interpolation)

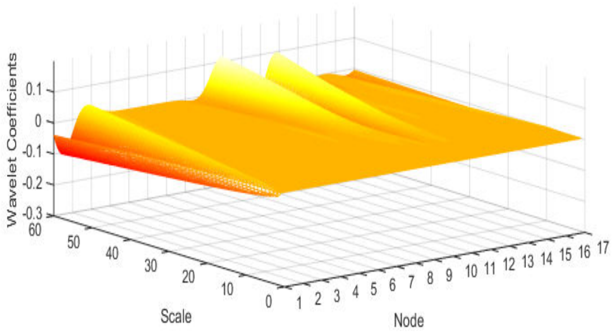

Figure-7. Wavelet transform modulus for difference function of mode shape 1 between undamaged and damaged beam (3d).

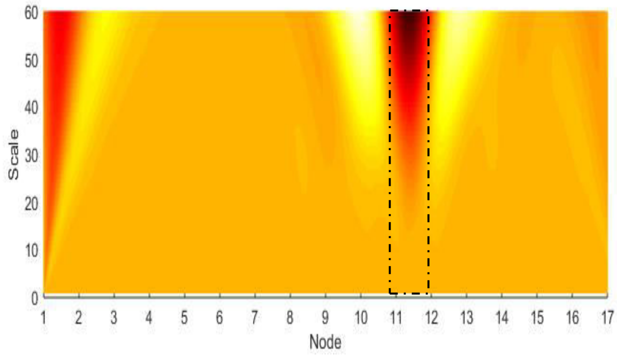

Figure-8. Wavelet transform modulus for difference function of mode shape 1 between undamaged and damaged beam(2d).

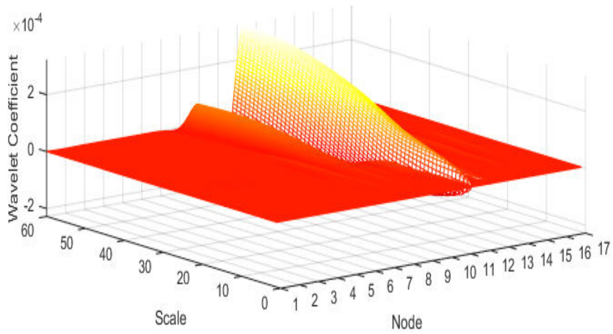

Figure-9. Wavelet transform modulus for difference function of laplacian for mode shape 1 between undamaged and damaged beam (3d)

The difference function of the mode curvatures and the Laplacian curvatures are then visualized using continuous wavelet transformation. The wavelet transformation modulus for mode shape 1 and mode shape 2 are shown in Figure 7,8,11 and 12. The other figures are the wavelet transformation for Laplacian Curvature for mode shape 1 and 2 respectively.

In comparison to the mode shape curvature based CWT, it is easier to detect the location of the crack by referring to the Laplacian Curvature based CWT. The location of the damage is also clearly detected between the node 11 and node 12.

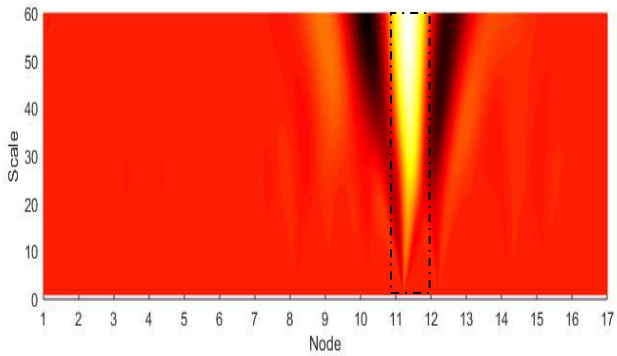

Figure-10. Wavelet transform modulus for difference function of laplacian for mode shape 1 between undamaged and damaged beam (2d).

ARPN Journal of Engineering and Applied Sciences

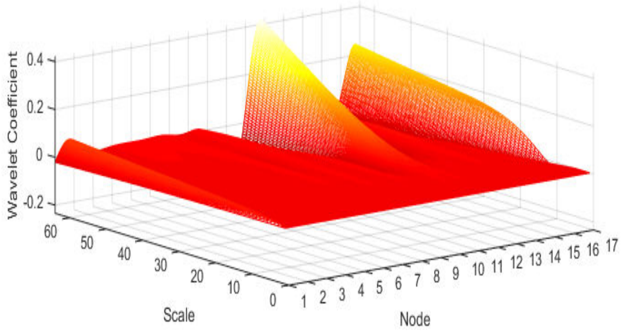

Figure-11. Wavelet transform modulus for difference function of mode shape 2 between undamaged and damaged beam (3d).

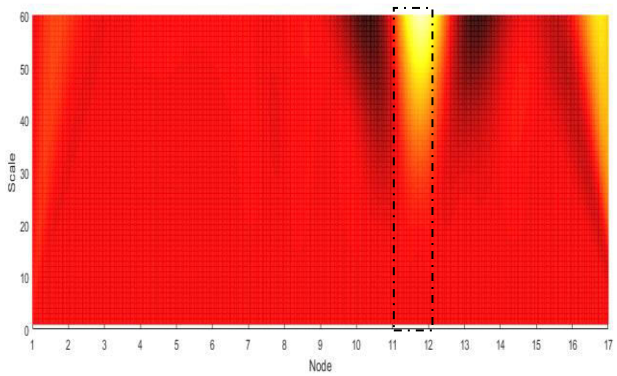

Figure-12. Wavelet transform modulus for difference function of mode shape 2between undamaged and damaged beam (2d).

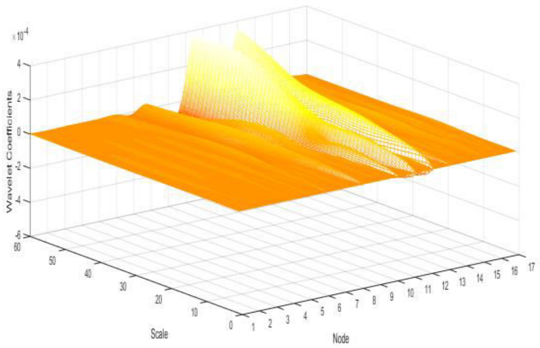

Figure-13. Wavelet 0

(3d).

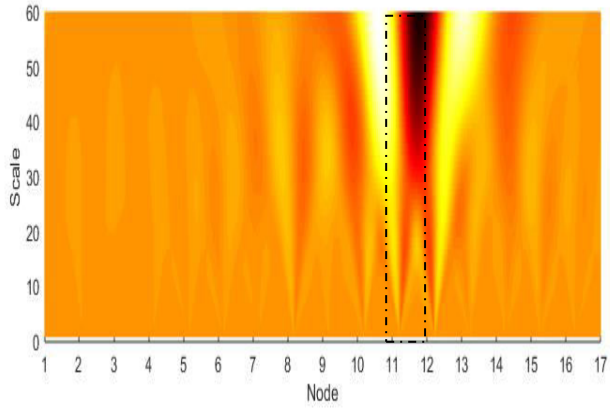

Figure-14. Wavelet transform modulus for difference function of laplacian for mode shape 2 between undamaged and damaged beam (2d).

CONCLUSIONS

The finite difference Laplacian Curvature can be used for damage detection in comparison to the mode shape curvature. The location of damage can be clearly identified.

Akima's interpolation can be a good tool to interpolate mode shape data in order to increase the accuracy measured mode shape curvature. This can also be an alternative to the tedious measurement work which man measured points are needed.

By using Continuous Wavelet Transformation, clearer visualization of the damage position can be obtained. Not only that, the location of the damage is more accurate in comparison to the Laplacian Curvature with no interpolated computation.

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