



# THE REINFORCED CONCRETE FRAME CALCULATION WITH ALLOWANCE FOR THE ERECTION SEQUENCE, PHYSICAL NONLINEARITY AND THE CONCRETE CREEP

Olga Zavyalova<sup>1</sup> and Alexander Shein<sup>2</sup>

<sup>1</sup>Astrakhan State University of Civil Engineering, Astrakhan, Russia

<sup>2</sup>Penza State University of Architecture and Construction, Penza, Russia

E-Mail: [zavyalova\\_ob@aucu.ru](mailto:zavyalova_ob@aucu.ru)

## ABSTRACT

The calculation algorithm for the concrete frameworks of the monolithic buildings is offered. This algorithm makes allowance for the concrete creep and the modulus of instantaneous elasticity changes in accordance with [1]. The algorithm is based on the network approximation element method [2] with time effect which allows taking into account as physical nonlinearity of the concrete behavior so the concrete creep and the sequence of erection of the monolithic framework simultaneously. The examples of the realization of algorithm can be found in the program "Method of the network approximation of elements - erection and creep".

**Keywords:** loading history, reinforced concrete frame, concrete creep, tension, physical nonlinearity

## INTRODUCTION

Currently, the development of structural mechanics follows two main directions, on the one hand, the development of higher-end computational methods based on powerful software systems, on the other - the improvement of design models and original hypothesis underlying in the basis of calculation. Allowance for the material behavior of building constructions, especially concrete, allows to find out the constructional safety resource for building as well as to correct the real internal forces distribution and related strains in the newly designed buildings. The aim of the work is research of loading history effect of the reinforced concrete frames on the strain-stress distribution of its elements. The sequence of a building as well as maturing process and concrete creep as time functions, and physical nonlinearity of the concrete behavior are taking into account at the calculations.

## METHODS USED IN THE CALCULATION

In all cases the calculation was made in accordance with strained scheme by the method of the network approximation of elements [2, 5-7]. This method allows entering any integral and differential relationships into the equations to characterize material properties. The method also takes into account geometrical nonlinearity in the elements of building straining. The program allows defining the de-sired number of intermediate sections (8, 16, 32, 64 and so on) of bar elements. In the example each element is divided into 8 sections. The system of nonlinear equations includes: boundary-condition equations; nodes equilibrium equations; compatibility equations for strain in the nodes; equilibrium equations for external and internal forces in the given cross-sections which taking into account the axial and lateral elements strains and representing differential equations of the first and second orders. The finite-difference approximation is used to reduce the differential equation system to the algebraic form. The system of nonlinear equations is solved by

Newton method with the use of Jacobi nonsingular matrix and step-by-step iteration. The defined norm for the successive iterations difference is used as the convergence criterion of calculations. To compare results the calculation of the same frame was made by finite element method without taking into account the above-listed factors.

## THE STATEMENT OF THE PROBLEM AND BASIC DATA

The article describes the calculation of the plane six-storied single-bay monolithic frame:

- allowing for the sequence of erection;
- allowing for the changes of concrete modulus of instantaneous elasticity in Harutyunyan's exponential relationship [1];
- allowing for the concrete creep, according to [1] for compressed and bending elements.
- allowing for the concrete physical nonlinearity.

The frame for calculation contains the next characteristic properties:

The frame bay - 6m, the number of bays - 1, story height - 6m, the column section - 40x40cm, symmetrical reinforcement:  $A_s=28,26\text{cm}^2$ ,  $h_s=17\text{cm}$ ; the girder sections - 30x60cm, symmetrical reinforcement,  $A_s=42,39\text{cm}^2$ ,  $h_s=27\text{cm}$ . Concrete Class is B-20, the standard concrete modulus of elasticity is  $E_0=27\text{GPa}$ , the reinforcement modulus of elasticity is  $E_s=200\text{GPa}$ , the uniformly distributed load through the girders is  $q=30\text{kN/m}$ .

## ALLOWANCE FOR THE SEQUENCE OF ERECTION

In the internal forces definition in the transverse frames of civil and industrial buildings the designer works with frame design diagram with project number of stories. The girder loading of the frame by constant and temporary loads is carried out for all stories simultaneously. To



define the real resources of the constructional building security with allowance for actual distribution of the internal forces with its deformations during their erections it is necessary to take into account the loading history with set up increasing design diagram and change of physical properties of materials in time. The accounting for the sequence of precast reinforced-concrete frames is studied in [3]. The increasing monolithic frame calculation is being built in a relatively short time is done in this work.

The periodicity of frame floor erection at 9 day intervals is accepted for the calculation. It actually corresponds to the terms of the building erection of the monolithic multistoried hotel at Astrakhan. The zero reference point is a day. At this period the first floor columns are erected, the second floor columns are erected on the 9th day, the third floor columns are erected on the 18th day, etc. The girders are erected on the 5th, 14th, 23rd days, etc. When the formwork is removed from the first floor of monolithic slabs and girder (at the moment of actual beginning of its operation under load) there are two overlying the girders with slabs are already erected. The time of the next girder loading is taken 18 days, i.e. for the girder and column of the first floor it is the 23rd day from the zero reference point, for girder and column of the second floor it is the 32nd day, etc. at intervals in 9 days. The girder of the last sixth floor is erected on the 50th day and begins to work on the 68th day.

**ALLOWANCE FOR MATURING AND CONCRETE CREEP**

The volume of monolith construction in recent years much you have grown, both in Russia and abroad. The possibility to reduce the time of buildings construction by increasing the speed of the concrete work (sometimes speed up to 3 ... 5 days per floor) is no small part. There is a valid question: what influence of the concrete loading in the early stages is on VAT, when its structure is not yet fully formed, and the strength and stiffness are far from standard values? In other words, it's important to know how are working stresses far from the limiting means in the currentage of concrete.

The software provides the possibility to take into account the change of the concrete modulus of instantaneous elasticity through time and the concrete maturing simultaneously with concrete creep effect.

The changing of the concrete modulus of elasticity in the process of maturing is adopted by an exponential function [1]:

$$E_b(t) = E_0 \cdot (1 - e^{-\alpha t}) \tag{1}$$

Allowance for the concrete creep is made with the help of the coefficients  $B_z(t_1, t)$ , derived from relationships for compression and bending [1]. It should be noted that in the method [1], the similar coefficients are used to correct the intensity of stresses in the concrete and reinforcement by multiplication of elastic instantaneous stresses, calculated in a conventional manner, by these coefficients. In this work, based on the formula

$$B_z \sigma_b = B_z (E_b \cdot \varepsilon) = (B_z E_b) \cdot \varepsilon, \tag{2}$$

allowance for maturing and concrete creep can be produced at each stage of loading, by multiplication of the concrete modulus of elasticity at the appropriate age  $E_b(t)$  by the coefficient  $B_z(t_1, t)$ :

$$B_z(t_1, t) = 1 - \frac{\gamma \mu E_s n_0 \varphi(t_1)}{1 + \mu n_0 m(t_1)} \times \int_{t_1}^t e^{-\int_{t_1}^{\tau} \left( \gamma + \frac{\gamma \mu E_s n_0 \varphi(u)}{1 + n_0 \mu m(u)} + \frac{\mu n_0 m'(u)}{1 + \mu m(u)} \right) du} d\tau \tag{3}$$

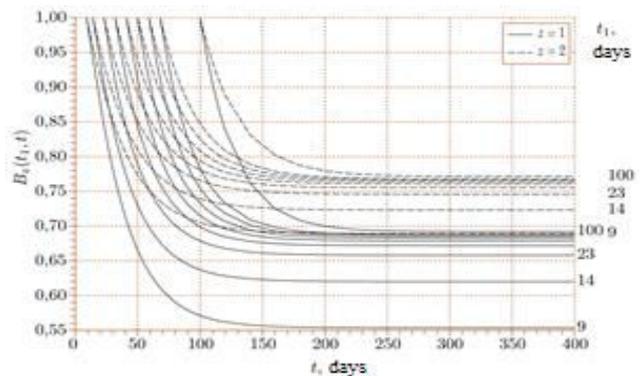
At these formulae:

$t_1$  - the age of concrete at the moment of loading,  $t$  - the age of concrete at the moment of observation;  $\mu$  - the coefficient of reinforcement of cross section;  $\varphi(t) = C_0 + A_1/t$  - the function taking into account the concrete creep measure;  $C_0$  - the limiting mean of the concrete creep measure for material;  $A_1, \gamma, \alpha$  - constant parameters of the concrete creep measure;  $m(t) = E_s/E_b(t)$  - the ratio of the reinforcement modulus of elasticity to concrete modulus of elasticity (at the age of concrete  $t$ -days);  $n_0$  - the coefficient is taken into account under bending ( $z=1$ ) and undertension and compression ( $z=2$ ):

$$n_0 = \begin{cases} 1 + h_s^2 \cdot A_b / I_b, & \text{under } z = 1; \\ 1, & \text{under } z = 2. \end{cases}$$

Let take the constant values (for heavy-weight concrete of the natural maturing):  $C_0 = 0, 09 \cdot 10^{-7} \text{ kPa}^{-1}$ ;  $\gamma = 0, 026$ ;  $A_1 = 4, 83 \cdot 10^{-7} \text{ day/kPa}$ ;  $\alpha = 0, 03 \text{ day}^{-1}$ .

The coefficient  $B_1$  takes into account the bending,  $B_2$  - tension and compression. The graphic charts for  $B_1$  и  $B_2$ , depending on the age of concrete under loading ( $t_1 = 9, 14, 23 \dots 100$  days.), are shown at (Figure-1):



**Figure-1.** The coefficients of creep (full line graph -  $B_1$ , dashed line -  $B_2$ ).



The graphic charts for concrete modulus of elasticity depending on the age of concrete at the moment of loading allowing for the coefficients of creep are shown at Figure-2.

The estimated stiffness of the frame elements is determined by the formulae:

$$EI(t_1; t) = E_{b1}(t_1; t) \cdot I_b + E_s I_s; \quad (4)$$

$$EA(t_1; t) = E_{b2}(t_1; t) \cdot A_b + E_s A_s; \quad (5)$$

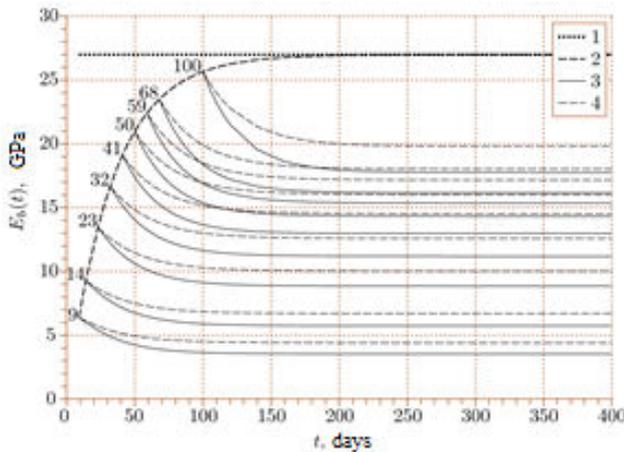


Figure-2. The concrete modulus of elasticity  $E_b$ :

- 1 -standard value;
- 2 -allowing for the maturing:  
 $E_b(t) = E_0 \cdot (1 - e^{-\alpha t})$ ;
- 3 -under bending allowing for the concrete creep  
 $E_{b1}(t_1; t) = E_{b0}(t_1) \cdot B_1(t_1; t)$ ;
- 4 -under tension and compression allowing for the concrete creep  
 $E_{b2}(t_1; t) = E_{b0}(t_1) \cdot B_2(t_1; t)$ .

**ALLOWANCE FOR THE PHYSICAL NONLINEARITY OF CONCRETE BEHAVIOR**

For diagram approximation of concrete behavior  $\sigma - \epsilon$  let use formula [4]:

$$\sigma_b = E_b \epsilon - A_3 \epsilon^3, \quad (6)$$

which was modernized to take into account time effect

$$\sigma_b(t) = E_b(t) \epsilon - A_3(t) \epsilon^3 \quad (7)$$

$$A_3(t) = \frac{4}{27} \frac{E_b^3(t)}{R_b^2(t)}. \quad (8)$$

In this formula  $E_b(t)$  - the current initial-value modulus of elasticity of work material, calculated using the formula (1);  $R_b(t)$  - the ultimate concrete strength for current age, calculated using the known logarithmic dependence  $R_b(t) = 0,7R_b \lg t$ .

Simultaneous allowance for the physical nonlinearity of concrete behavior and concrete creep leads to the concrete modulus of elasticity is the two-variable function - the age of concrete under loading ( $t_1$ ) and current time  $t$ , i.e.  $E_b = E_b(t_1; t)$ . Besides, the meaning  $A_3$  used at the equation of moments is different from  $A_3$  used at the equation of axial forces. The coefficient  $B_1$  is for the bending stiffness,  $B_2$  is for stiffness in tension and compression.

The bending stiffness of cross section with allowance for reinforcement deformations which come about from the bending moment and the axial force is found as:

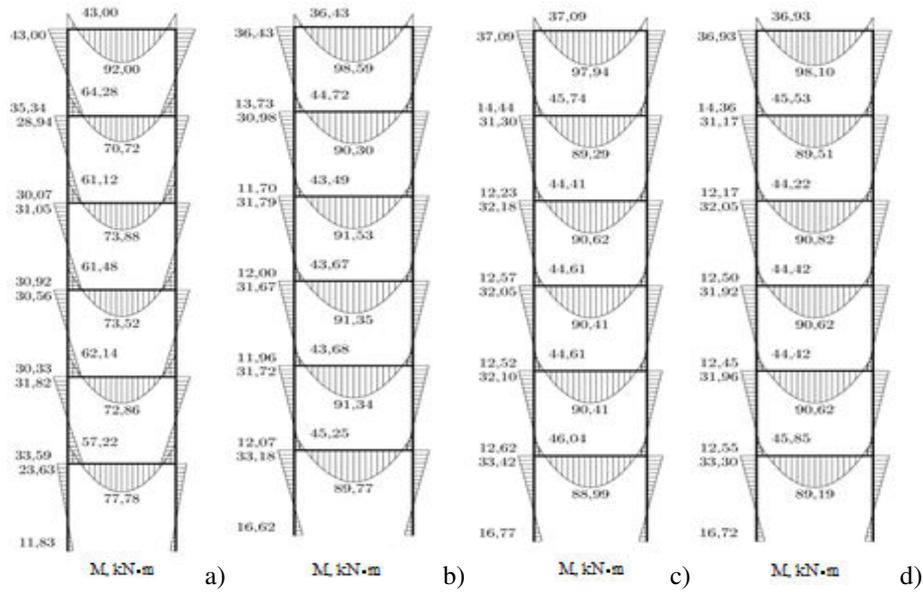
$$EI(t_1; t) = E_{b1}(t_1; t) \cdot I_b - A_3(t_1) \times \times (v'')^2 \frac{bh^5}{80} + E_s I_s + \sum E_s A_{sj} y_{sj} \frac{u'}{v''} \quad \dots\dots\dots (9)$$

The stiffness in tension and compression with allowance for reinforcement deformations which come about from the bending moment and the axial force is found as:

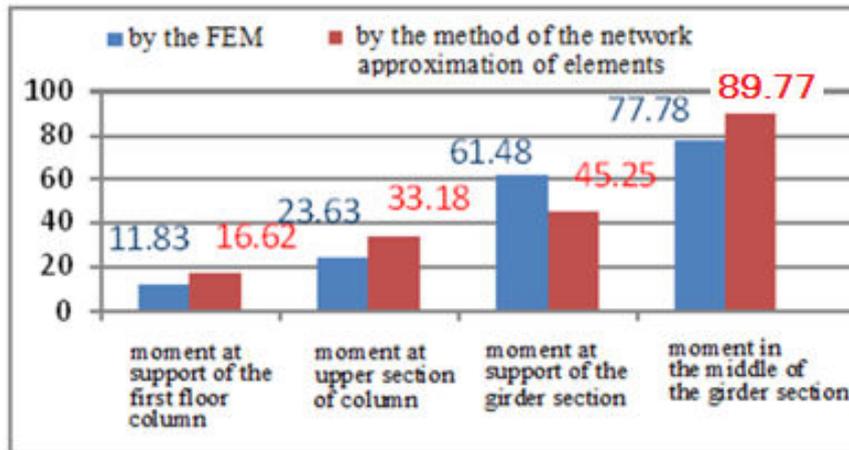
$$EA(t_1; t) = E_{b2}(t_1; t) \cdot A_b - A_b A_3(t_1) \times \times (u')^2 + E_s A_s + \sum E_s A_{sj} y_{sj} \frac{v''}{u'} \quad (10)$$

**THE ANALYSIS OF ACCOUNTING RESULTS OF MONOLITHIC FRAME**

The pre-testing accounting was performed without allowance for the loading sequence by the finite element method (FEM) (the program SCAD) and by the method of the network approximation of elements.



**Figure-3.** The bending moment curve: a - calculated by the FEM; b, c, d - calculated by the method of the network approximation of elements with allowance for the sequence of erection: b - with constant function  $E_b=E_0$ ; c - with allowance for maturing and concrete creep; d - with allowance for physical nonlinearity, maturing and concrete creep

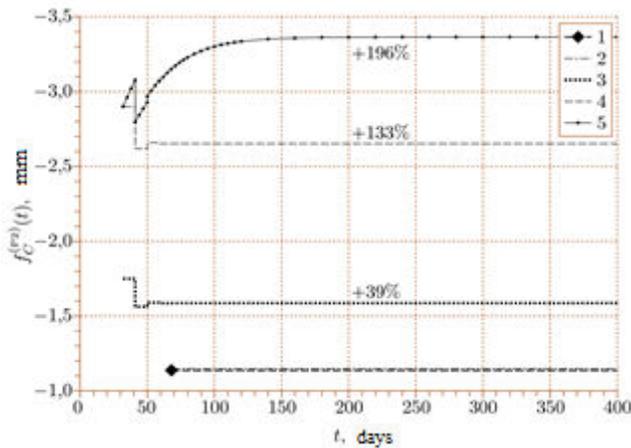


**Figure-4.** The bending moment value (kN·m) from loadings at the stage of erection ( $q_1 = 30$  kN/m): blue columns - value got by the FEM (traditional calculation), red columns - by the method of the network approximation of elements with allowance for the sequence of erection and loading.

The accounting results are the same. It allows concluding about accuracy of calculations made by the proposed program. From the Figures 3 and 4 we can see that the most change of the bending moment value in comparison with the standard calculation takes place at the first floor column - the moment at support increases by 41, 7%, the moment at upper section of column increases by 40, 4%, at the girder sections the moment at support

decreases by 20...28%, simultaneous increase of moments in the middle of the girder sections is 16...24%.

Allowance for maturing and concrete creep doesn't give obvious increase of the bending moment value, but leads to the sizeable stress redistribution between concrete and reinforcement. Besides there is a considerable increase of displacements (Figure-5).



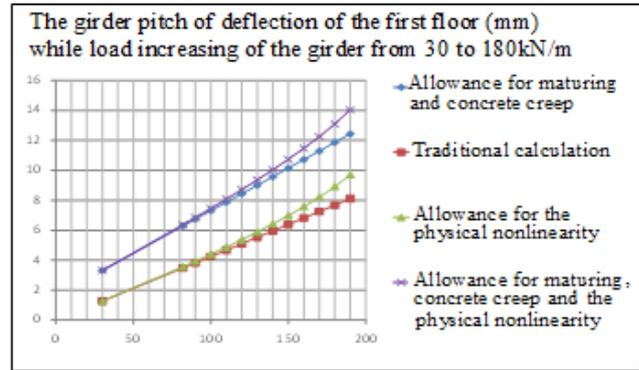
**Figure-5.** The girder pitch of deflection of the second floor (from loading at the stage of erection):

- 1 - by the FEM;
- 2 - by the method of the network approximation of elements;
- 3 - with allowance for the sequence of erection;
- 4 - with allowance for maturing of concrete;
- 5 - with allowance for concrete creep

At Figure-5 the first and second graphs are coincided. They were got for six-storied frame simultaneously loaded at all floors by uniform loading  $q_1=30$  kN/m. The 3-5 graphs take into account the sequence of loading. It explains the discontinuous jumps of deflection in the opening days. For example, the second floor girder was put into operation on the 32nd day, when the formwork was removed from it; its deflection acquired a certain value. On the 41st day, the third-floor girder was engaged. The second floor girder received a small upward bend, i.e. its deflection decreased. The next girder increased the load of the second floor girder a little more.

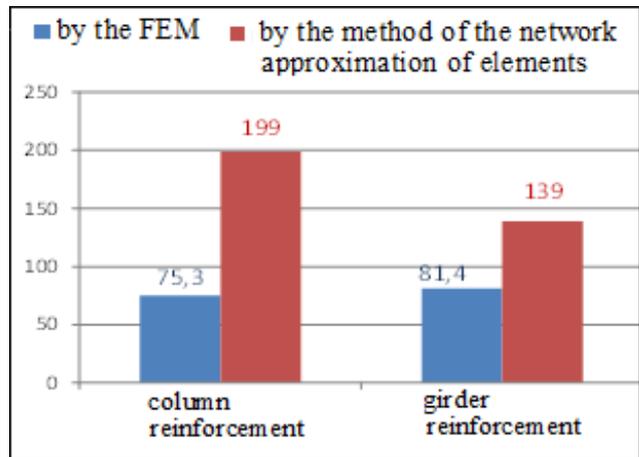
The comparative diagram of deflections for the traditional calculation with allowance for all the factors considered in the work of the frame has the form (Figure-6).

Let make the frame calculation of a high-rise factory, taking into account the corresponding characteristics of the elements and load intensity on the floor structure. The maximum load on the flooring joist is 120 kN/m, for roof collar beam - 60 kN/m. The section of columns is 600x400 mm,  $A_s = 42.39$  cm<sup>2</sup>, the section of girders is 600x300 mm,  $A_s = 42.39$  cm<sup>2</sup>, the reinforcement of columns and girders sections are defined as symmetrical. Concrete B-20.



**Figure-6.** The girder pitch of deflection of the first floor.

Stress analysis of the most loading frame elements made with allowance for loading history shows the high stress increase in the reinforcement of supporting column sections and girder middle sections (Figure-7).

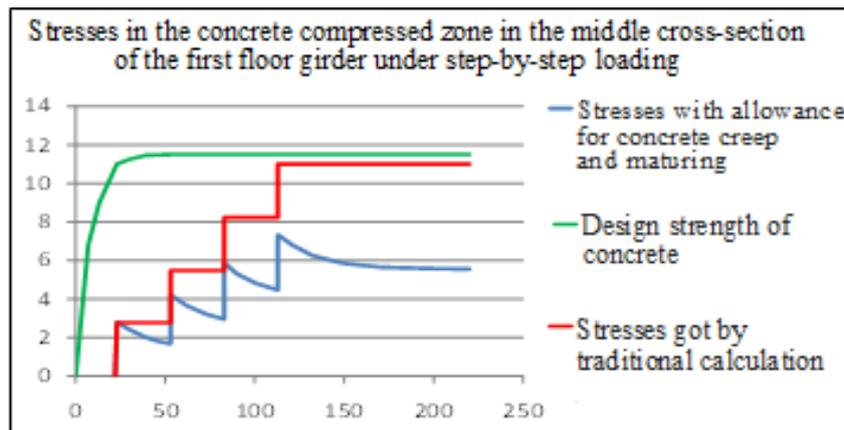


**Figure-7.** Stress in reinforcement of the supporting column section and the middle girder section, MPa.

The loading of the first floor girder is made in four stages with an interval of one month

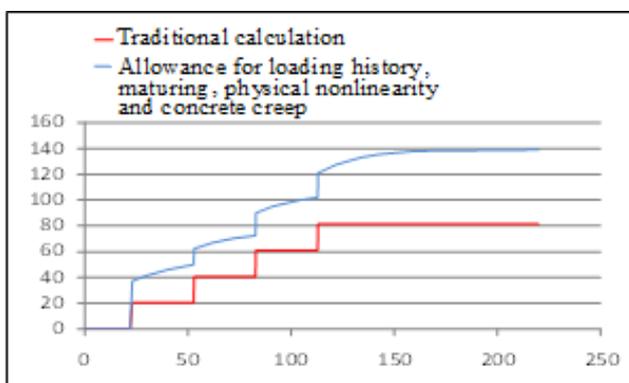
- at the age of 23 days the load is  $q_1=30$  kN/m;
- at the age of 53 days the added load is  $q_2=30$  kN/m;
- at the age of 83 days the added load is  $q_3=30$  kN/m;
- at the age of 113 days the added load is  $q_4=30$  kN/m.

At each stage, the elastic instantaneous stresses in the concrete compressed zone were calculated with allowance for the actual concrete modulus of elasticity. Then to take into account for concrete maturing and creep the Harutyunyan's relationships are used. Figure-8 shows the stress changes in the concrete compressed zone in the middle cross-section of the first floor girder.

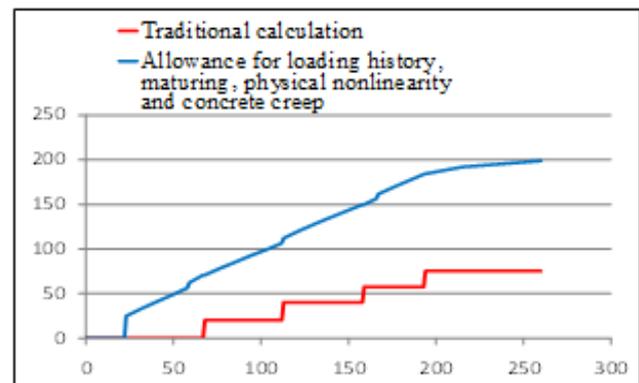


**Figure-8.** Stress diagram in the concrete compressed zone in the middle cross-section of the first floor girder under step-by-step loading (MPa).

It can be seen from Figure-8 that the stresses got by the traditional calculation at the fourth stage of loading approach to the design strength of concrete under compression. Calculation with allowance for the maturing and concrete creep shows that the actual stresses in the concrete compressed zone at all stages of loading are much lower. Rapidly leaked creep and stress concrete relaxation is appeared within a period of 30 days from the moment of load application of the next stage. The step-by-step load application in small portions during construction, let avoid "peaks" on the diagram  $\sigma(t)$ . After 200 days in the analyzed frame the normal concrete stresses are 50% that of its design strength. Obviously, the acceleration of the construction time of reinforced concrete frames with allowance for the actual work of the concrete, does not cause fears from the concrete stresses, but in addition it is well to bear in mind the significant increase of unaccounted stresses in the reinforcement of the frame elements (Figures 9, 10).



**Figure-9.** Stresses (MPa) in the reinforcement of the middle cross-section of the first floor girder under step-by-step loading with allowance for the actual concrete behavior



**Figure-10.** Stresses (MPa) in the compressed column reinforcement of the first floor (upper section) while step-by-step loading with allowance for the actual work of the concrete.

In this example the exceedance of the actual stresses in the girder reinforcement was by 1.7 times, in the column reinforcement - by 2.6 times. It should be noted that by using formula (1) to take into account the real concrete modulus of elasticity, an additional increase of the elastic instantaneous stresses in the reinforcement was about 20% in comparison with the traditional calculated stresses, i.e. with the standard value  $E_b$ . To reveal the real stress distribution in the material of structures in accordance with the work schedule, and perhaps to adjust this plan, the suggested calculation technique realized in the method of the network approximation of elements will help.

## CONCLUSIONS

- Allowance for loading history of monolithic concrete constructions, erected in a relatively short time, shows the decrease of constructional reliability resource of such buildings.
- The suggested calculation technique by use of the method of the network approximation of elements let take into account the loading history and reveal the value of additional stresses in the most important



structural elements. For example, the increase of the stresses in the girder reinforcements from the loads applied at the stage of erection was by 2.5 times and in general in girders - by 1, 7 times.

- c) The increase in stresses in the column reinforcement of the 1st floor in comparison with the traditional calculation was by 2.6 times.
- d) The active increase of stresses in the reinforcement caused by the concrete creep loaded in the early periods is manifested within 150-180 days from the moment of loading and stabilizes at the age of concrete 220-270 days, which should be especially taken into account when determining the stress-strain state of multistoried monolithic structures.
- e) The increase of the girder pitch of deflection of the monolithic frame with allowance for the concrete creep reaches 70-95% compared with the traditional calculation.
- f) Allowance for erection sequence of monolithic frame gives up to 40% of the divergence of internal forces from the loads taken into account at erection stage, in the most loaded elements, in comparison with the determined traditional calculation. Of particular concern are the columns of the 1st floor and the middle cross-sections of the girders. In the multi-span frames, the support cross-sections of the girders of the outermost spans in the support points on the middle columns are also overloaded.
- g) The proposed technique allows to more accurately assess the stress state in the elements of monolithic frameworks, erected in a short time, and to reveal how close the stresses in concrete and reinforcement to the ultimate strength at each stage of the load application.

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