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END-LENGTH-OFFSET INFLUENCE ON THE BENDING RIGIDITY OF LINEAR TAPERED T-BEAMS

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ABSTRACT

In this study, a calculation system in Maxima (GNU) was implemented, based-on the numerical integration of the flexibility parameters to obtain the elements of a T-beam stiffness matrix with linear haunches including zones of infinite rigidity (rigid end off-sets) and shear/axial deformations. The length of these zones is assumed as half the width of the beam-column joint. The goal is to demonstrate that, an analysis based on the geometry between centroidal axes without considering joint stiffness, tends to exhibit greater flexural strength, overestimating the lateral deformations, which can be important when calculating the lateral displacement of frames. The flexural rigidity parameters of a T-beam with and without zones of infinite rigidity were determined through the calculation of the flexibility integrals using the three-point Gaussian quadrature rule of integration. Increases in stiffness (%) were determined when rigid end off-sets were considered.

Keywords: end length off-set, rigid zone, non-prismatic, T-beam, maxima (GNU), numerical integration, SAP2000®, flexibility integrals.

1. INTRODUCTION

In the analysis of building frames is important to model the added stiffness that is generated by the connections between beams and columns when the dimensions of these structural elements are big and affect the rigidity of the structure. In this work, the beam-column joint elements were modeled as zones of infinite rigidity by implementing the Tena-Colunga flexibility method through matrix analysis, and it was determined how the rigid zones modify the structural element stiffness coefficients, considering that shearing deformation can be generally neglected in most of the calculations.

It was shown that, for practical purposes, the zones of infinite rigidity can be modeled with enough approximation, using flexibility equations and considering shearing and bending deformations.

The added value endowed to this research, lies in the fact that the formulation developed allows to introduce the infinitely rigid zones for the analysis of variable section beams. The results of this formulation based on the numerical integration of the flexibility parameters were possible through the progress of computational science and numerical analysis; particularly, in this project, a calculation system was implemented with the powerful tool of Maxima, a computer algebra system.

In a finite element model, rigid zones are modeled with frame-End-Length-Offsets which are defined as those fractions of length of the element assumed as infinitely rigid and measured from the end of the element. The Frame-End-Length-Offsets are occasionally accompanied by stiffness factors to determine the degree of stiffness in the rigid zones, which can vary between 0 and 100%. In this article the finite element models are not shown, because the objective is to prove the variations of the stiffness factors of a T-section element with linear haunches through formulations of the numerical integration when the infinitely rigid zones are introduced in the joints.

2. METHODS

2.1 Stiffness matrix in local coordinates of the non-prismatic element with rigid end-off-sets

The stiffness matrix in local coordinates of the non-prismatic element with infinite rigid zones was defined using the flexibility method, implementing a calculation system in Maxima, the Computer Algebra System that allows to solve the numerical integration problem of the elements in the matrix. From implementing this method, the stiffness coefficients of a beam element were determined using equations (2), (3), (4), (5) and (6), taking into account the zones of infinite rigidity exemplified in Figure-1, Figure-2 and Figure-3.

Being equation (1) the flexibility matrix of the element and the zones of infinite stiffness $\alpha L'$ and $\beta L'$ equivalents to half the width of the joint,

$$[f] = \begin{Bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & f_{23} \\ 0 & f_{32} & f_{33} \end{Bmatrix} \quad (1)$$

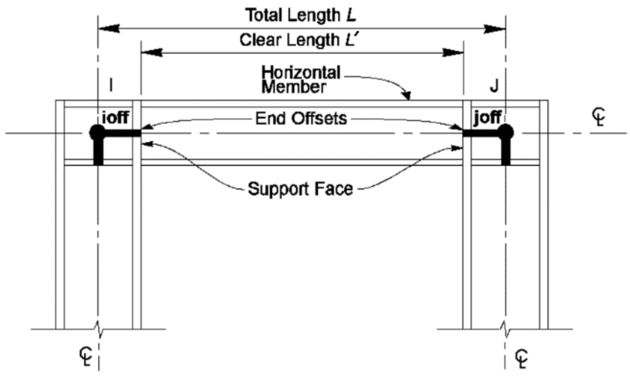

Figure-1. Frame-rigid end (length) offsets.

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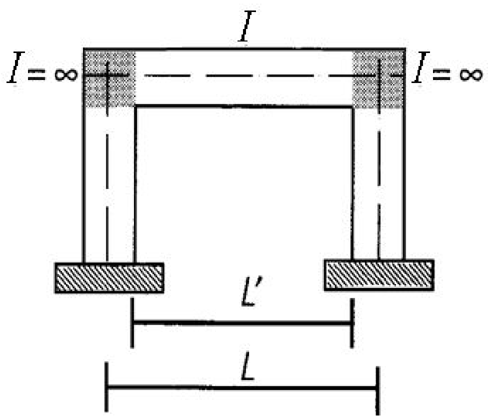

Figure-2. Rigid beam-column joint in a typical frame.

It was concluded that in the intervals $[0, \alpha L']$ y $[\alpha L' + L', L]$ where the rigid zones are located, $I_x(z) \rightarrow \infty$, y $A_{cy}(z) \rightarrow \infty$, therefore the elements of the flexibility matrix used in the calculation system can be reduced to,

$$f_{11} = \int_{\alpha L'}^{\alpha L'+L'} \frac{dz}{EA(z)} \tag{2}$$

$$f_{22} = \int_{\alpha L'}^{\alpha L'+L'} \frac{z^2 dz}{EI_x(z)} + \int_{\alpha L'}^{\alpha L'+L'} \frac{dz}{GA_{cy}} \tag{3}$$

$$f_{23} = \int_{\alpha L'}^{\alpha L'+L'} \frac{zdz}{EI_x(z)} = f_{32} \tag{4}$$

$$f_{33} = \int_{\alpha L'}^{\alpha L'+L'} \frac{dz}{EI_x(z)} \tag{5}$$

The stiffness matrix is obtained by inverting the flexibility sub matrix, so their entries are implicitly defined. The global matrix of stiffness in local coordinates of the two-node beam-column element in Figure-3 is expressed according to equation (6).

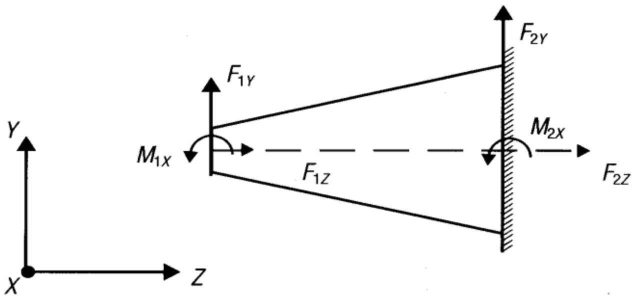

Figure-3. Tapered 2D-dimensional element. Source: [4]

$$[K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{6}$$

Stiffness submatrices in local coordinates are determined by equations (7), (8), (9) and (10).

$$[k_{11}] = \begin{Bmatrix} r_{az} & 0 & 0 \\ 0 & r_{aax} & r_{abx} \\ 0 & r_{abx} & r_{11x} \end{Bmatrix} \tag{7}$$

$$[k_{12}] = \begin{Bmatrix} -r_{az} & 0 & 0 \\ 0 & -r_{aax} & r_{bax} \\ 0 & -r_{abx} & r_{12x} \end{Bmatrix} \tag{8}$$

$$[k_{21}] = [k_{12}]^T \tag{9}$$

$$[k_{22}] = \begin{Bmatrix} r_{az} & 0 & 0 \\ 0 & r_{aax} & -r_{bax} \\ 0 & -r_{bax} & r_{22x} \end{Bmatrix} \tag{10}$$

The seven different elements of these sub matrices are defined by equations (11) to (18).

$$r_{az} = \frac{1}{f_{11}} \tag{11}$$

$$Det_x = f_{22}f_{33} - f_{23}{}^2 \tag{12}$$

$$r_{11x} = \frac{f_{22}}{Det_x} \tag{13}$$

$$r_{12x} = \frac{f_{23}L - f_{22}}{Det_X} \tag{14}$$

$$r_{22x} = \frac{f_{33}L^2 - 2f_{23}L + f_{22}}{Det_X} \tag{15}$$

$$r_{aax} = \frac{r_{11x} + r_{22x} + 2r_{12x}}{L^2} \tag{16}$$

$$r_{abx} = \frac{r_{11x} + r_{12x}}{L} \tag{17}$$

$$r_{bax} = \frac{r_{22x} + r_{12x}}{L} \tag{18}$$

Once the stiffness matrix of the variable section element in local coordinates, a calculation system is implemented (matrix or finite elements). The stiffness matrix of the element in global coordinates obtained using transformation matrices, and the connectivity between elements is defined by the assembly rule.

2.2 Mathematical model of a T-beam with straight haunches and rigid length end-off-sets

In Figure-4, a typical frame of a two-dimensional beam-column element is shown. The horizontal element is a T-beam of three straight haunches as illustrated on the Figure-5.

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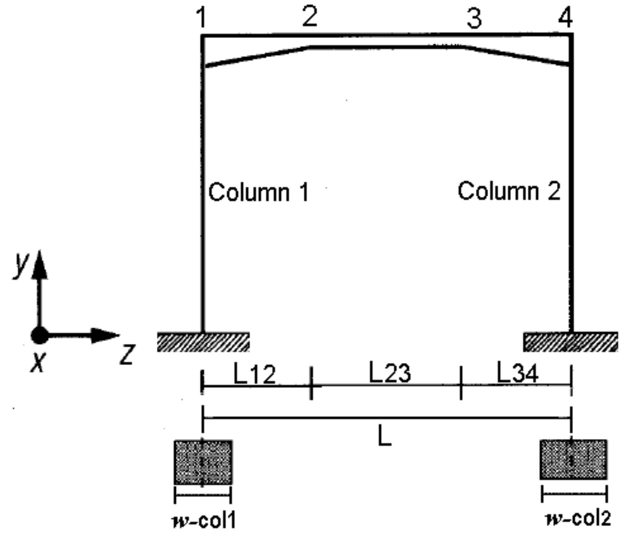

Figure-4. Typical frame with tapered beam.

In Figure-5, the height variation of a T-beam along its length is shown, from h_{w1} to h_{w4}, with the second section constant ($h_{w2}= h_{w3}$).

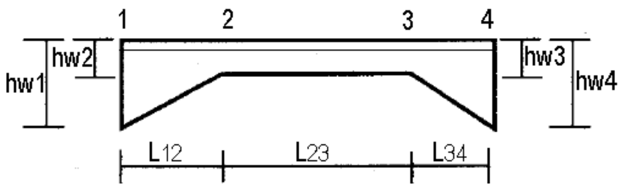

Figure-5. Tapered beam with three straight haunches.

In Figure-6 the beam model is represented by the zones of infinite rigidity in the joints. The lengths $\alpha L'$ and $\beta L'$ correspond to the rigid lengths in the joints, taken as half the column width in the joints: *0.5w-col*1 and *0.5wcol*2.

The length between the axes for the beam section is L and the free length is L'. L' is the length between the axes removing the dimensions of the rigid zones $\alpha L'$ and $\beta L'$ as presented in Figure-6 (equation 19).

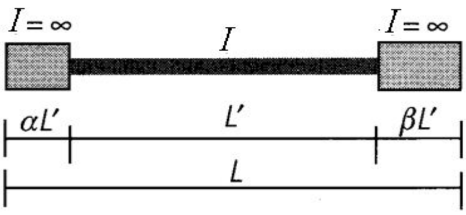

Figure-6. Beam section with infinite rigid zones.
Source: [4]

In Figure-7 constants section properties are: flange width b_f, flange thickness t_f , and web thickness b_w.. The height web h_w varies in three different sections, from h_{w1} to h_{w4}.

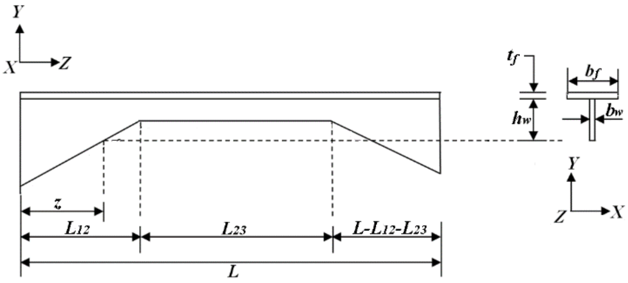

Figure-7. Tapered –T–beam cross section.

The constant dimensions of the cross section in meters are, b_f =1.10, t_f=0.05, b_w=0.3, and $h_{w1}= h_{w4}$=0.65, $h_{w2}= h_{w3}$=0.40 and the material properties are E= 1581139 T/m^2 and G=E/2.4. The lengths of the tapered sections in meters are L_{12}=2.35 and L_{34}=2.10, and the length of the beam between axes is L=7.125. The column widths in meters are *w-col*1=0.4 and *w-col*2=0.5, and the free length L'= 6.675.

$$L' = L - 0.5w\text{-}col1 - 0.5wcol2 \tag{19}$$

In the flexibility integrals, the limits of integration are carefully defined considering the free-length of beam and the rigid zone.

The linear variation of the beam web height h_w on the first and last fin plate, is represented mathematically by equations (20) and (21).

First interval, for $0 \leq z \leq L_{12}$

$$h_w(z) = \left(\frac{h_{w2}-h_{w1}}{L_{12}}\right) z + h_{w1} \tag{20}$$

Third interval, for $L_0 \leq z \leq L$, where $L_0 = L_{12} + L_{23}$

$$h_w(z) = \left(\frac{h_{w4}-h_{w3}}{L_{34}}\right)(z - L_0) + h_{w3} \tag{21}$$

Properties of area *A(z)*, the effective shear area $A_{cy}(z)$, the centroid distance *Y(z)* and the moment of inertia of the cross section beam $I_x(z)$, are introduced as function of (z) in equations (22) to (25).

$$A(z) = b_w h_w(z) + b_f t_f \tag{22}$$

$$A_{cy}(z) = b_w(h_w(z) + t_f) \tag{23}$$

$$Y(z) = \frac{\left(b_f \frac{t_f^2}{2}\right) + [b_w h_w(z)]\left[\left(\frac{h_w(z)}{2}\right) + t_f\right]}{A(z)} \tag{24}$$

$$I_x(z) = \frac{[b_f t_f{}^3 + b_w h_w(z)^3]}{3} - A(z)\left[Y(z) - t_f\right]^2 \tag{25}$$

The implementation of the calculation system in Maxima (GNU) is summarized in the following stages,

- Data entries:h_{w1}, h_{w2}, h_{w4}, b_w, b_f, t_f, L, L_{12}, L_{23}, *w-col*1 and *wcol*2, *E*, *G*.

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- Definition of variables for every section: $h_w(z)$, $A(z)$, $A_{cy}(z)$, $Y(z)$, $I(z)$.
- The integration limits of every section considering the rigid zones (end off-sets) and the calculation of the flexibility integrals for every section (approximation by the three-point Gaussian quadrature).
- Summation of the flexibility parameters for a tapered beam.
- Calculation of the elements of the stiffness matrix as a function of the flexibility factors.

3. RESULTS AND DISCUSSIONS

As shown in Table-1, the flexibility parameters calculated for a tapered beam without the infinite rigid zones are greater than those obtained when we include the zones of infinite rigidity. From these facts, we can infer that there were overestimations of deformation by lateral load applications.

Table-1. Flexibility parameters for tapered elements with and without rigid end off-sets.

f	Con rigid end off-set	Sin rigid end off-set
f11	0.002189544609591821	0.002305200739520388
f33	8.940727572553532E-06	9.188105580585336E-06
f23	0.003213509571277731	0.003311538251848125
f22	1.3949721946136	1.463148020775034

In Table-2 a comparison between the matrix elements and the flexibility parameters for tapered elements with and without rigid end off-set is presented, observing that the rigidity in the model which considers the rigid zones in the nodes increased as it was expected.

Table-2. Comparison between the elements of the matrix and the tapered element with and without rigid end off-sets.

r	Rigid end off-set	No rigid end off-set	% variation
raz	456.7159744630285	433.8017001539121	5.28
raax	4.167350329136656	3.70895979009797	12.36
rabx	1497.844561404442	1336.767640701573	12.05
r11x	650208.5861787577	590628.5173173111	10.09
r12x	417005.663821907	361818.4266825598	15.25
r22x	631361.5267032086	568611.2477594916	11.04

As shown in Table-2, the axial rigidity of the beam, considering infinite rigidity in 50% of the joint length increased by 5.28% in relation to the one not considered, whereas the other stiffness factors were about 12%, on average. Therefore, it can be concluded that the elements which do not consider rigidity in the beam-column joints tend to exhibit greater flexibility resulting in bigger deformations. This can become important when the lateral displacements of the frames are calculated.

4. CONCLUSIONS

- Numerical integration was implemented in a calculation program specially developed in the free CAS Maxima (GNU) to determine the stiffness matrix of a three tapered T-beam considering that the zones of infinite rigidity were equivalent to half the width of the joint. The results of the elements of the matrix were compared to an identical model that does not consider zones of infinite rigidity, finding important differences which led us to conclude that an analysis based on the centroidal axes of the elements without considering the stiffness factors in the nodes tend to overestimate the lateral deformations, by which it is relevant to determine the variation of the stiffness coefficients for a structural element due to its rigid zones.
- It was shown that a systematic and reasonable standard-setting calculation methodology based on numerical integration can provide reliable results without depending on robust tools of finite elements. This is a more sustainable and economic option for the analysis of tapered elements with zones of infinite rigidity in the joints.
- It was demonstrated that the calculation of the stiffness matrix of variable section elements considering the areas of rigidity in the joints, is feasible using numerical integration due to the great development that the computational field and the scientific analytical tools have had over the last decades. It was proven that with the calculation possibilities that new processors offer us today and with the advances in mathematical calculation tools, it is possible to solve high relevant problems in structural engineering without the help of expensive commercial finite element software.

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