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RHEOLOGICAL MODEL OF CREEP AND RELAXATION IN ASPHALTIC MIXTURES USING THE TRANSFORMED CARSON LAPLACE, MAXIMA AND MAPLE

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ABSTRACT

In this article, the results of a creep and relaxation studies in asphalt mixtures are presented. Initially, a description of the conceptual model of the material and the details of the viscoelastic characterization in asphalt mixtures using the Creep test are shown. Then, the creep phenomenon is represented using a generalized Kelvin model with calibrated parameters [9]; also the relaxation model is developed using the direct and inverse transformation of Carson Laplace. For the modeling process of the two phenomena, a symbolic calculation code was programmed using MAXIMA-CAS and Maple software. Finally, we conclude that the generalized Kelvin rheological model fits the Creep laboratory data, and, the calculated relaxation function simulates the phenomenon correctly. This function was also compared with the approximate analytical model of Schapery (1965) and the result data were satisfactory.

Keywords: carson-Laplace transform, creep, relaxation, asphalt mix, maxima.

1. INTRODUCTION

The simplest and most fundamental test for the "viscoelastic characterization" of an asphalt mixture is the recovery test developed at a constant temperature of 40 ° C. This is a constant compression uniaxial load test (100 kPa) applied for an approximate time of 3600 seconds; then the load is suspended and the relaxation phase begins for a time of 1800 seconds. The result of the test is a graph of axial deformation in time represented in two phases. In the first phase, the function obtained is fluency and in the second phase the function is relaxation.

Figure-1 shows the typical assembly of the test. The displacement sensors "LVDT" are responsible to measure the elongation.

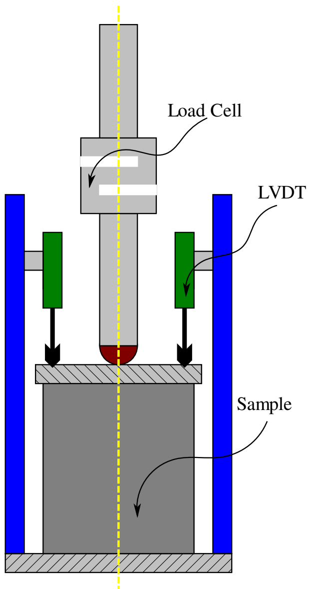

Figure-1. Assembly for creep test.

Figure-2 shows a typical curve of the recovery test with its two steps: Creep and Relaxation.

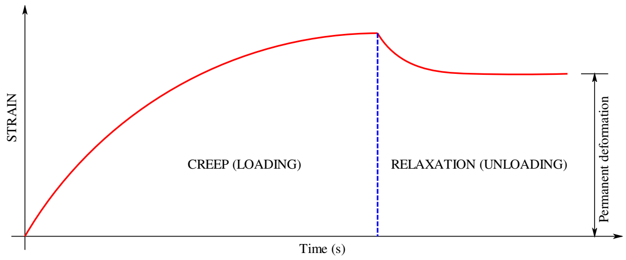

Figure-2. Curve obtained from the recovery test (English school).

2. METHODS

Below are presented, the formal Creep function of Kelvin generalized; the Carson-Laplace transform process for the determination of the relaxation function; algorithms to calculate the Creep function, the relaxation function and the characteristic times using MAXIMA and MAPLE, and experimental data .

2.1 Formal creep function of the generalized Kelvin model

The generalized Kelvin model can be mechanically represented as shown in Figure-3,

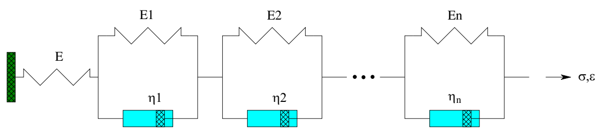

Figure-3. Mechanical model of Kelvin.

The model consists of the serial association of a spring element and n elements of Kelvin. The deformation is then a sum that is described using Equation (1).

$$f(t) = \left[\frac{1}{E} + \sum_{i=1}^{n} \frac{1}{E_i}\left(1 - e^{\left(\frac{E_i}{\eta_i}t\right)}\right)\right] y(t), \tag{1}$$

The discrete spectrum of characteristic time in creep is described using Equation (2),

$$[\tau_f]_i = \frac{\eta_i}{E_i}, \tag{2}$$

Evaluating at t=0 and infinity we can obtained Equations (3) and (4),

$$\mathrm{f}(0) = \frac{1}{\mathrm{E}}. \tag{3}$$

$$f(\infty) = \frac{1}{E} + \sum_{i=1}^{i=n} \frac{1}{E_i}. \tag{4}$$

2.2 The Carson-Laplace transform process for the determination of the relaxation function

The Carson Laplace transform of the Creep function is shown in Equation (5),

$$f^*(p) = \frac{1}{E} + \sum_{i=1}^{i=n} \frac{1}{E_i + \eta_i p} \tag{5}$$

Equation (5) can be written as shown in Equation (6),

$$f^*(p) = \frac{1}{E} + \sum_{i=1}^{i=n} \left(\frac{1}{\eta_i} \frac{1}{p + \frac{1}{[\tau_f]_i}}\right) \tag{6}$$

The Carson-Laplace transform of the relaxation function $g^*(p)$ is obtained from the Carson-Laplace transform of the Creep function, as shown in Equation (7)

$$g^*(p) = \frac{1}{f^*(p)} \tag{7}$$

Equation (7) can be written as shown in Equation (8),

$$g^*(p) = \frac{1}{\frac{1}{E} + \sum_{i=1}^{i=n} \left(\frac{1}{n_i} \frac{1}{\left(p + \frac{E_i}{\eta_i}\right)}\right)} \tag{8}$$

The relaxation function and the characteristic relaxation time spectrum are obtained from the inverse Carson-Laplace transform of the function g*(p) as shown in the Equation (9),

$$g^*(p) \overset{inverse}{\rightarrow} g(t), \tau_g \tag{9}$$

On the other hand, Creep deformation is calculated as Equation (10),

$$\varepsilon_{Cr}(t) = f(t)\sigma \tag{10}$$

2.3 Algorithms to calculate the creep function, the relaxation function and the characteristic times using MAXIMA and MAPLE®

The process of calculating the Creep function is performed using the pseudocode described in Figure-4.

```
Begin
- Assign the number n of the Kelvin's elements
- Assign the Modulus value [E] of the spring configured in series
- Assign the Modulus value [Ei] of the Kelvin elements springs
- Assign the value of the parameters [ηi] of the Kelvin elements dashpots
- Calculate the spectrum of characteristic times in Creep
- Calculate the Creep function f (t) symbolically
- Calculate f(0) and f(∞)
- Calculate f'(t) symbolically
- Calculate f'(0)
- Calculate the characteristic time in Creep as: τf = [f(∞)-f(0)]/f´(0)
End
```

Figure-4. Pseudocode for calculating the creep function and its characteristic time.

After implementing the pseudocode of Figure-4 in the MAXIMA-Computational Algebra System, a calculation process of the relaxation function and its characteristic time is programmed following the pseudocode of Figure-5.

```
Begin
- Calculate the Carson-Laplace transform of the relaxation function g*(p) from Equation (8), symbolically
- Simplify the function g*(p) obtained in the previous step, symbolically
- Factor the function g*(p) obtained in the previous step, symbolically
- Calculate the partial fractions of the function g*(p) obtained in the previous step
- Calculate the relaxation function g(t) using the inverse transform of Carson-Laplace, symbolically
- Calculate g(0) y f(∞)
- Calculate symbolically g'(t)
- Calculate g'(0)
- Calculate the characteristic time in relaxation as: τg= [g(∞)-g(0)]/g´(0)
End
```

Figure-5. Pseudocode for calculating the relaxation function and its characteristic time.

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All the steps described in the pseudo code of Figure-5 have been implemented in the MAXIMA-Computational Algebra System, except the step that calculates the partial fractions which was programmed in the MAPLE® software.

2.4 Experimental data

This study is part of an investigation of Creep in asphalt mixtures so the starting data are the same as the Flow Time Test obtained from the Kenlayer program [9]. Kenlayer is a program used to pavements design by multi-layer theory and elastic, viscoelastic and non-linear constitutive models for each of the layers. The sample tested has dimensions of 100 mm in diameter and 66 mm in height.

Table-1. Data of creep and deformation function in time. Source: [9]

Point	t (s)	σ (kPa)	f_l (t) (1/kPa)	ε (mm/mm)
1	1	100	2.669E-05	2.67E-03
2	3	100	3.863E-05	3.86E-03
3	5	100	4.172E-05	4.17E-03
4	10	100	4.450E-05	4.45E-03
5	15	100	4.612E-05	4.61E-03
6	20	100	4.728E-05	4.73E-03
7	30	100	4.874E-05	4.87E-03
8	40	100	4.983E-05	4.98E-03
9	50	100	5.068E-05	5.07E-03
10	60	100	5.129E-05	5.13E-03
11	70	100	5.191E-05	5.19E-03
12	80	100	5.238E-05	5.24E-03
13	90	100	5.303E-05	5.30E-03
14	100	100	5.342E-05	5.34E-03

A model with calibrated parameters was used. The calibration comes from an optimization process presented in a previous work of the authors [9]. Table-2 shows the calibrated values of the generalized Kelvin model parameters when the number of elements is four, n = 4.

Table-2. Calibrated parameters of the formal creep function.

Modulus [E]	Value	Coefficient [η]	Value
E1	32270,742	η1	32270,742
E2	209626,416	η2	1068191,396
E3	220348,377	η3	2695069,634
E4	90992,278	η4	8293885,953
E0	173301,708	---	---

3. RESULTS AND DISCUSSIONS

3.1 Formal Creep function of the generalized Kelvin model and calculation of creep characteristic time

The creep function found by MAXIMA is presented in Equation (11),

$$f(t) = 1.098994356422201 * 10^{-5}\,(1 - e^{-0.010971006656667\,t}) + 4.5382680474134809 * 10^{-6}\,(1 - e^{-0.081759808548234\,t}) + 4.770391156017657 * 10^{-6}\,(1 - e^{-0.19624424666308\,t}) + 3.0987821378398558 * 10^{-5}\,(1 - e^{-1.0\,t}) + 5.7702835829928941 * 10^{-6} \tag{11}$$

Figure-3 shows the similarity between the Creep function of equation (11) and the experimental data, when both are represented in MAXIMA.

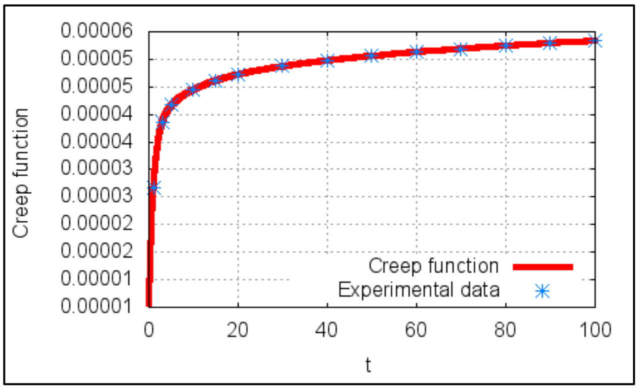

Figure-6. Creep function and experimental data.

The tangent line to the Creep function at t = 0 has been obtained in MAXIMA by equation (12),

$$y1 = 3.2415601867796013 * 10^{-5}\,t + 5.7702835829928941 * 10^{-6} \tag{12}$$

The characteristic Creep time $\tau_f=[f(\infty)-f(0)]/f'(0)$, according to MAXIMA is 1,582152457178447 seconds. This characteristic time in Figure-4 is at the cut-off point between the tangent line to the Creep function and the horizontal asymptotic line, at t = 0.

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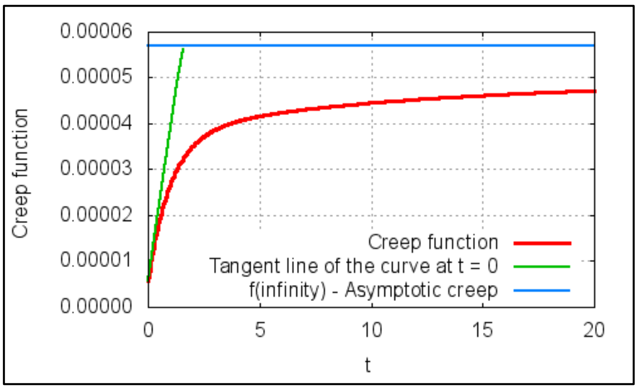

Figure-7. Creep function, asymptotic creep and characteristic creep time.

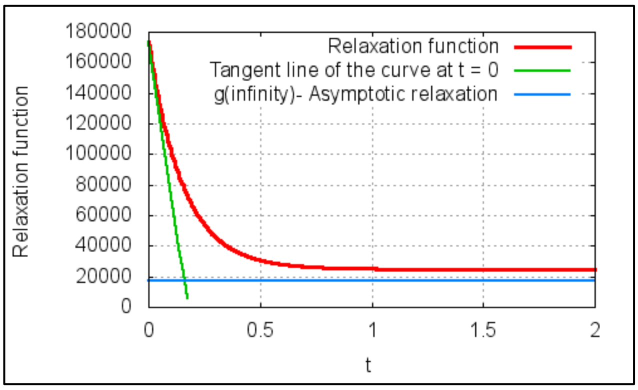

Figure-8. Relaxation function, asymptotic relaxation and characteristic relaxation time.

3.2 Relaxation function and characteristic relaxation time

The Carson-Laplace transform expressed in partial fractions has been obtained from MAPLE® and it is represented by equation (13),

$$g^*(p) = -\frac{972829.2567602861}{p+6.58504551508725} - \frac{507.892340004661}{p+0.21846760842275} - \frac{163.729748639632}{p+0.089644543442664} - \frac{52.5143986699071}{p+0.013496561369991} + 173301.7078 \quad (13)$$

The relaxation function has been obtained from MAXIMA using the Carson-Laplace inverse transform as shown in equation (14),

$$g(t) = 3890.946532994015\, e^{-0.013496561369991\, t} + 1826.43295790058\, e^{-0.089644543442664\, t} + 2324.794708338881\, e^{-0.21846760842275\, t} + 147733.1105049162\, e^{-6.585045515087249\, t} + 17526.42309585033 \quad (14)$$

The tangent line to the relaxation function at t = 0 has been obtained in MAXIMA by equation (15),

$$y2 = 173301.7078 - 973553.3932476015\, t \quad (15)$$

The characteristic relaxation time $\tau_g=[g(\infty)-g(0)]/g'(0)$, according to MAXIMA is 0,16000692492531 seconds. This characteristic time in Figure-5 is at the cut-off point between the tangent line to the relaxation function and the horizontal asymptotic line, at t = 0.

The characteristic relaxation time is less than the characteristic creep time, so relaxation is faster than creep

3.3 Approximate analytical model of Schapery to calculate the relaxation function (1965)

The Carson-Laplace transform of the relaxation function g*(p) is used with the substitution p=α/t, where, $\alpha=e^{-C}$ *and C is the* same Euler's constant (e). The value obtained from the substitution is 0.56. From the substitution made in MAXIMA using equation (13) we obtained the equation (16),

$$g(t) = -\frac{972829.2567602861}{\frac{0.56}{t}+6.58504551508725} - \frac{507.892340004661}{\frac{0.56}{t}+0.21846760842275} - \frac{163.729748639632}{\frac{0.56}{t}+0.089644543442664} - \frac{52.5143986699071}{\frac{0.56}{t}+0.013496561369991} + 173301.7078 \quad (16)$$

Figure-9 shows a comparison between the formal relaxation model and the approximate Schapery model.

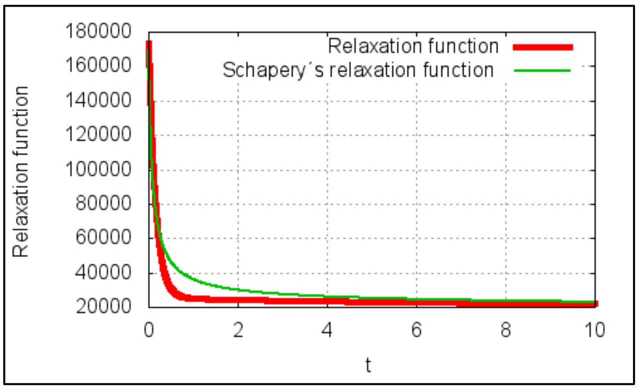

Figure-9. Formal relaxation function and approximate Schapery model.

In Figure-9 the best fit of the Shapery approximate model is at the extremes of the relaxation function.

4. CONCLUSIONS

The formal mathematical models of Creep and relaxation were implemented in the MAXIMA

Computational Algebra System. Due to the complexity of the models and the number of terms in the series, there were difficulties in calculating partial fractions of the Carson-Laplace transformation of the relaxation function that were solved with the MAPLE Computational Algebra System. The calculation of the Carson-Laplace Inverse transform was verified using Maple.

To demonstrate the possible existence of error in the use of approximate analytical methods, the approximate analytical model of Schapery (1965) was implemented. This model uses the formal expression of relaxation of the reverse Carson-Laplace transform with a simple substitution reported in this article, however, the adjustment of this solution against the formal solution of the relaxation function is only acceptable at the extremes because the differences are significant for the time values that are in the middle.

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