



CYCLIC ALGORITHM APPROACH FOR POLYPHASE SEQUENCES WITH GOOD CORRELATION PROPERTIES AND MERIT FACTOR

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ABSTRACT

The Polyphase Sequences such as P_n $\{n=1, 2, 3, 4, x\}$, Frank, Golomb, and the Chu finds many applications in RADAR, SONAR, and Communication by having good autocorrelation properties. Merit Factor (MF), ISL (Integrated Sidelobe Level) is the performance measures considers for evaluating the goodness of any sequences. In this paper cyclic algorithm approach initialized with these Polyphase sequences for lengths from 10^2 to 10^3 . These cyclic algorithm approaches bring the better merit factor and correlation properties than the standard case. It found that an average of merit factor 40.39 and 92.02 is obtained for length 100 & 1000 respectively. Cyclic algorithm approach Polyphase sequences correlation plots are compared with the standard case. This approach made possible for P2 sequences that improved merit factor for odd integer square length. Four consecutive even and odd integer squared length sequences correlation plots and merit factor values compared. Cyclic algorithmic methodology for these Polyphase sequences for obtaining the design metrics implemented on MATLAB.

Keywords: merit factor, autocorrelation level, polyphase sequence, cyclic algorithmic.

1. INTRODUCTION

The main aim of Radar and Sonar is to determine properties of the target by transmitting specific waveforms towards to the target and analyzing received signal. By measuring the round trip time delay, the distance technically termed as range, and Doppler frequency shift of the received signal is for a speed of the target. Two factors are critical and decide the system performances are the transmitted waveform and receive filter [1]. A suitable and well synthesized transmitted waveform can bring out the accurate parameter estimation and reduce the computational burden in the receiver part [2]. Merit factor is one of the parameters to be improved for a well synthesized transmitted waveform or sequence. Binary sequences, Barker sequences exist to certain length require some search algorithms (Evolutionary search) to obtain two digit merit factor [3-4]. Two widely accepted sequences Chu and Frank in that, Frank sequence is better than the Chu sequence, and additional Polyphase sequences such as P_n $\{n=1,2,3,4\}$ and Golomb[5-7].

P. B. Rapajic and R. A. Kennedy [8] the merit factor values of P1, P3, P4, Px, Golomb, Frank, and Chu found for lengths $N=D^2$ where D is an integer, in that Px sequence perform well. Frank, P1 have the same merit factor. P3, P4, Golomb, and the Chu obtained same merit factor values. R. Frank [9] found that P1 and Frank's sequences are identical to each other. W. Roberts *et al.* [10] proposed the necessary cyclic algorithm approach for random initialization in the calculation of performance parameters. P. Stoica *et al.* [11], implemented the singular value decomposition based cyclic algorithm for obtaining the unimodular sequence.

The objectives of this paper are to compare the performance parameters known as merit factor of Polyphase sequences P1, P2, P3, P4, Px, Golomb, Frank, and the Chu all are exist for square integer length ($N=D^2$). Figure. 1 & 2 i.e. (standard case & cyclic algorithm

approach) the merit factor values for lengths $N=D^2$ here $D=100, 16^2, 17^2, 18^2, 19^2, 1000$ on log scale are shown. Figures. (3)-(8) shows the compassion correlation plots. Table 1 shows the MF values comparison for lengths 100 and 1000. Table 2 & 3 shows the comparison of MF values for 16, 17 & 18, 19 square integer lengths.

2. POLYPHASE SEQUENCES

The Polyphase sequences named Frank, Golomb, Chu, P1, P2, P3, P4, Px possibly exist for square integer length $N=D^2$ (where N is sequence Length, D is an integer) having elements $S_n = (S_1, S_2, S_3, \dots, S_N)$. These Polyphase sequences can be defined as follows. P1, P2, Px, and Frank denoted by $f(n)$ sequences are defined by $S(Dn+k+1)$. P3, P4, Golomb, and Chu defined by $S(k+1)$. The P1, P2, P3, P4, Px, Golomb, Frank and Chu sequences [8, 12-13]. All the sequences have existed for square integer length. The sequences with equal merit factor values exhibit the same correlation. Sequences with large merit factor values have the lower sidelobes in correlation plots. Even though correlation levels are identical but the way in which the sequence representation differs from one another. The Performance parameters analysis, i.e., MF values and plotting the correlation levels are outlined in Figure. 1-A.

2.1 P1, P2, Px sequences

These sequences defined as $S(Dn+k+1) = e^{j\phi_{n,k}}$ for $0 \leq k \leq D$ and $0 \leq n \leq D$, here the phase elements of the sequences [8, 13] defined as shown below.

$$\phi_{n,k} = -(\pi/D)(D-2n-1)(nD+k) \quad (1)$$

$$\phi_{n,k} = +(\pi/D)(D-2n-1)((D-1)/(2-k)) \quad (2)$$

$$\phi_{n,k} = (\pi/D)[(D-1)/2-k](D-2n-1) \quad (3)$$



$$\varphi_{n,k} = (\pi / D)[(D - 2) / 2 - k](D - 2n - 1) \quad (4)$$

Equations (1) and (2) are the phase elements of P1, P2 sequences respectively. Phase elements in Equations (3) and (4) of Px sequence of D even and odd integers.

2.2 P3, P4 sequences

The Polyphase sequence elements S_n for $n=1 \dots N$ of integer length M for P3 and P4 sequence [8] defined as $S(k+1) = e^{i\omega k+1}$ here $0 \leq k \leq D$.

$$\varphi_{k+1} = \pi k^2 \quad (5)$$

$$\varphi_{k+1} = \pi(k - N)k \quad (6)$$

Equations (5) and (6) are phase elements of P3 and P4 sequences respectively. The mathematical equation of these two sequences exhibits the identical MFs and correlation levels.

2.3 Golomb sequence

The Golomb sequence [8, 11] of Polyphase sequence elements S_n for $n=1, \dots, N$ of an integer length D defined as $S(k+1) = e^{i\omega k+1}$ here $0 \leq k \leq D$. the phase elements defined as

$$\varphi_{k+1} = \pi(k+1)k \quad (7)$$

Another way of Golomb sequence [9] represented as $g(n)$ of length N for a positive integer.

$$g(n) = e^{j\pi(n-1)n/N} \quad (8)$$

Equations (7) and (8) shows the Golomb sequence $g(n)$ for $n=1, \dots, N$. these two mathematical representations bring out the same response.

2.4 Frank sequence

These sequences can be defined as $S(Dn+k+1) = e^{i\omega k+1}$ for $0 \leq k \leq D$ and $0 \leq n \leq D$, here the phase elements of the sequence [8] defined as

$$\varphi_{k+1} = 2\pi nk / D \quad (9)$$

Another way of Frank sequence [10] represented as $f(n)$ of length N is given by

$$f(Dn+k+1) = e^{j2\pi nk/D}, k, n = 0, \dots, M-1 \quad (10)$$

Equations (9) and (10) demonstrates the $f(n)$ for $n=1 \dots N$. these two draw out a similar result.

2.5 Chu sequence

The Chu sequence [5, 8], phase elements S_n for $n=1 \dots N$ of a positive integer length N as $S(k+1) = e^{i\omega k+1}$ here $0 \leq k \leq D$

$$\varphi_{k+1} = \pi(k+2q)k \quad (11)$$

$$\varphi_{k+1} = \pi(k+1+2q)k \quad (12)$$

Equation (11) and (12) are the phase elements of Chu sequence for even and odd integers respectively.

3. PERFORMANCE PARAMETERS

The elements of the Polyphase sequence $S_n = S_1, S_2, S_3, \dots, S_N$ that exist for square integer Length ($N=D^2$). Some parameters define the ability for improving the characteristics of the radar system. Some parameters, i.e., Merit Factor increasing in nature defines the performance improvement, and some are integrated sidelobe level decreasing in nature also defines the same. The sequences with good correlation properties and MF values helped in radar and sonar applications.

3.1 Correlation function & correlation level

The autocorrelation $r=\rho(S)$ of a sequence $S_n = (S_1, S_2, S_3, \dots, S_N)$ is a sequence length of $2K-1$ defined as $\rho(S) = v(s, s)$ the main lobe c_k , of the autocorrelation c , is given by $c_k = s s^T$ and complex conjugates and transpose denoted by $(.)^T$ denotes the complex conjugate, conjugate transpose for scalars and vectors, matrices respectively [8].

$$c_k = \sum_{n=k+1}^N s_n s_{n-k}^T = s_{-k}^T, k = 0, \dots, N-1 \quad (13)$$

$$CL = 20 \log_{10} \left| \frac{c_k}{c_0} \right|, k = 1, \dots, N-1 \quad (14)$$

Equation (13) is the autocorrelation function and Equation (14) defines the correlation level in dB, in Equation (13) correlation function is shown.

3.2 Integrated side-lobe level

The integrated sidelobe level from Equation (13) is defined as follows.

$$ISL = \sum_{k=1}^{N-1} |c_k|^2, k = 1, \dots, N-1 \quad (15)$$

Equation (15) defines that $c_0, c_1, c_2, \dots, c_{N-1}$ square modulus.



3.3 Merit factor

The merit factor is an essential measure of the collective smallness of the aperiodic autocorrelations of a sequence length N named by M. J. E Golay [11].

$$MF = \frac{|c_0|^2}{\sum_{\substack{k=-(N-1) \\ k \neq 0}}^{N-1} |c_k|^2} = \frac{N^2}{2ISL} \quad (16)$$

Equation (16) shows the Merit Factor and correlation levels in the denominator consider all except at $k=0$. From Equation. (16) multiplication by 2 in denominator shows the symmetry of the correlation function at zero.

4. CYCLIC ALGORITHMIC APPROACH

This approach in the following works based on singular value decomposition method is accurate in computation and simpler than applying fewer optimization techniques for the Polyphase sequence [14] and [15]. It influences plausible to work with very substantial estimations of N (in some radar and imaging applications we to can pick $N \sim 1000$), P is chosen from reasonable thought and select $P \geq N$ on computational and down to earth activity accounts. Let C be the following block-Toeplitz matrix

$$C * C = \begin{bmatrix} c_0 & c_1 & \cdots & c_{N-1}^* \\ c_1 & c_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_1 \\ c_{P-1} & \cdots & c_1 & c_0 \end{bmatrix}_{P \times P} \quad (17)$$

Minimization criterion of minimizing the autocorrelation terms of $\{|C(k)|_{k=1}\}^{P-1}$ can be achieved is shown below.

$$\|C * C - NI\|^2 \quad (18)$$

To make computationally less complex the minimization problem can be represented as

$$\min_{\{x(n)\}_{n=1}^N, U} \|\tilde{C} - \sqrt{K}U\|^2 \quad (19)$$

Where U is the semi-unitary matrix, i.e., $UU^* = I$ and satisfied the following equation.

$$\tilde{C} = \sqrt{K}U \quad (\text{condition } P \ll \frac{K-1}{N-1}) \quad (20)$$

The minimization criterion in Equation (19) is iteratively minimized by fixing C to compute U , then fixing U to compute C and so on until the stop criterion is satisfied. During this computation, both U and C are

closed form upgrading. This whole process meant for minimizing the ISL metric which intern improve the Merit Factor. This cyclic algorithmic approach is singular value decomposition based but not FFT.

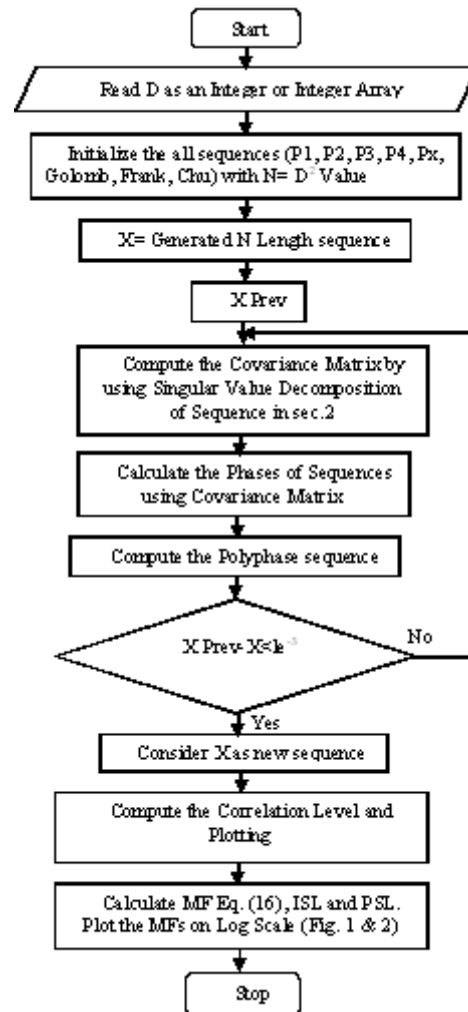


Figure-A-1. Flow chart for MFs and correlation plots.

Cyclic algorithmic approach

Step 1: Initialize the U possibly with the known existing sequence such as (Golomb, Frank, Chu, $P_n \{n=1, 2, 3, 4, x\}$ implies P_1, P_2, P_3, P_4, P_x).

Step 2: Compute the semi-unitary matrix U and minimize the (19) with respect to $\{x(n)_{k=1}\}^N, U$.

Step 3: by setting the $\{x(n)_{k=1}\}^N, U$. to the most recent values computing Equation (20) vice versa. Iteration: repeat step 2 & 3 until a practical convergence criterion is satisfied.

The derivation for the Cyclic Algorithmic approach is relatively uncomplicated can be referred in [16][17]. Even then it does not mean that criterion in (18) is implacable. The cyclic algorithmic approach consider in Equation (17) considers all correlation lags $\{|C(k)|_{k=1}\}^{N-1}$ increases the computational complexity. The duration of the emitted sequence is much larger than interference and arrival times the maximum difference is the prime interest



[18] [19]. In such cases making the $\{|C(k)|_{k=1}^{P-1}\}^{P-1}$ small i.e. $P \geq N$ in place of considering all correlation lags $\{|C(k)|_{k=1}^{N-1}\}^{N-1}$.

The Merit Factors of the Polyphase sequences in session 2 shown in Figure-1 in log scale using performance parameters equations in session 3. Cyclic algorithm initialized with Polyphase sequences denoted as CA(G), CA(F), CA(Chu), CA(P1), CA(P2), CA(P3), CA(P4) and CA(Px) respectively shown in Figure-2. Merit factors of above sequences computed for the lengths $N=100, 16^2, 17^2, 18^2, 19^2$ and 1000 are implemented in

MATLAB [20]. The cyclic algorithmic approach initialized with all above sequences in step 1. Also, standard case correlation levels in dB from Equation (14) shown in Figures 3 to 8 for above said lengths.

Table-1 shows the merit factor values comparison of standard and cyclic algorithm approach for lengths 100 and 1000. For 100 length case, all cyclic algorithm approach sequences exhibit the average merit factor of 40.391. However, for 1000 length case 92.027 average merit factor value is obtained.

Table-1. Merit factor values comparison of polyphase sequences standard and cyclic algorithm approach for lengths $N=100$ & 1000.

Parameter	Merit Factor			
Length	N=100		N= 1000	
Approach	standard	CA	standard	CA
P1	23.099	40.410	78.145	92.839
P2	18.722	40.447	75.041	92.876
P3	15.873	40.365	50.316	91.209
P4	15.873	40.365	50.316	91.209
Px	25.124	40.404	79.241	92.828
Golomb	15.873	40.365	50.316	91.209
Frank	23.099	40.410	78.145	92.839
Chu	15.873	40.365	50.316	91.209

Tables (2) & (3) are MF values comparison between standard and cyclic algorithm approach of Polyphase sequences for lengths $16^2, 17^2, 18^2, 19^2$. P3, P4, CA(Golomb) and CA(Chu) exhibit the merit factor values of 46.095, 45.232, 46.591 and 55.402 for 16, 17, 18, 19 square integer lengths in cyclic algorithm approach. CA(P1), CA(Px) and CA(Frank) exhibit the same merit

factor values of 46.494, 47.100, 47.164 and 56.100 with same consecutive even and odd integer square lengths. For even integer square lengths CA(P2) exhibits the highest merit factor among all cyclic algorithm Polyphase sequences. However, for odd integer square length P1 and Frank exhibit the highest.

Table-2. Merit factor values comparison of polyphase sequences normal and cyclic algorithm approach for lengths $N=16^2$ & 17^2 .

Parameter	Merit Factor			
Length	N=16 ²		N=17 ²	
Approach	Standard	CA	Standard	CA
P1	38.230	46.494	41.077	47.100
P2	34.067	46.499	7.781	35.672
P3	25.235	46.095	26.800	45.232
P4	25.235	46.095	26.800	45.232
Px	39.875	46.494	42.701	47.096
Golomb	25.235	46.095	26.800	45.232
Frank	38.230	46.494	41.077	47.100
Chu	25.235	46.095	26.800	45.232



Table-3. Merit factor values comparison of polyphase sequences normal and cyclic algorithm approach for lengths $n=18^2$ & 19^2 .

Parameter	Merit Factor			
Length	$N=18^2$		$N=19^2$	
Approach	Standard	CA	Standard	CA
P1	43.245	47.164	46.054	56.100
P2	39.225	46.896	8.319	44.423
P3	28.365	46.591	29.932	55.402
P4	28.365	46.591	29.932	55.402
Px	44.791	47.990	47.578	56.062
Golomb	28.365	46.591	29.932	55.402
Frank	43.245	47.164	46.054	56.100
Chu	28.365	46.591	29.932	55.402

Figures (1) and (2) are the MF values for 100, 16^2 , 17^2 , 18^2 , 19^2 and 1000 integer lengths for standard and cyclic approach. It is observed that from Figure-1 P2 sequence has decidedly fewer values say 7.781 & 8.319 for 17^2 & 19^2 lengths respectively. However, from the Figure-2 this cyclic algorithmic P2 sequence exhibits the 35.672 & 44.423. This cyclic algorithm approach improves merit factor for odd integer square lengths.

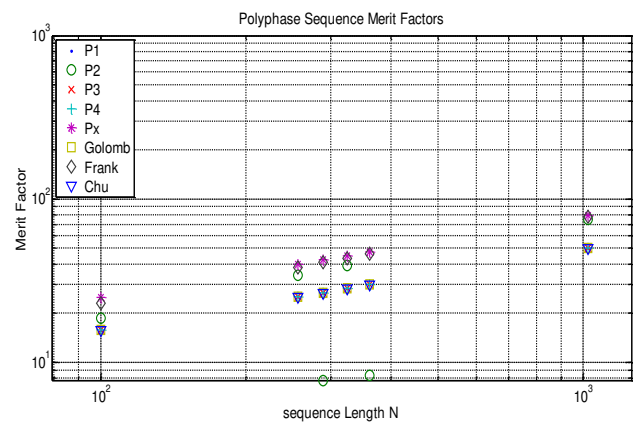


Figure-1. The merit factors of the P1, P2, P3, P4, Px, Golomb, Frank, Chu sequences of lengths 100, 16^2 , 17^2 , 18^2 , 19^2 , 1000.

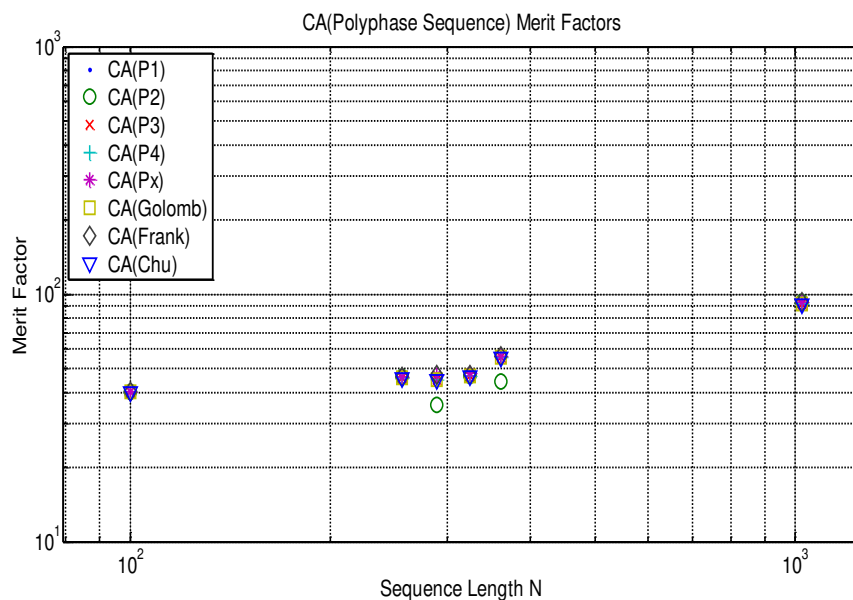




Figure-2. The merit factors of the CA(P1), CA(P2), CA(P3), CA(P4), CA(Px), CA(Golomb), CA(Frank), CA(Chu) sequences of lengths 100, 16^2 , 17^2 , 18^2 , 19^2 , 1000.

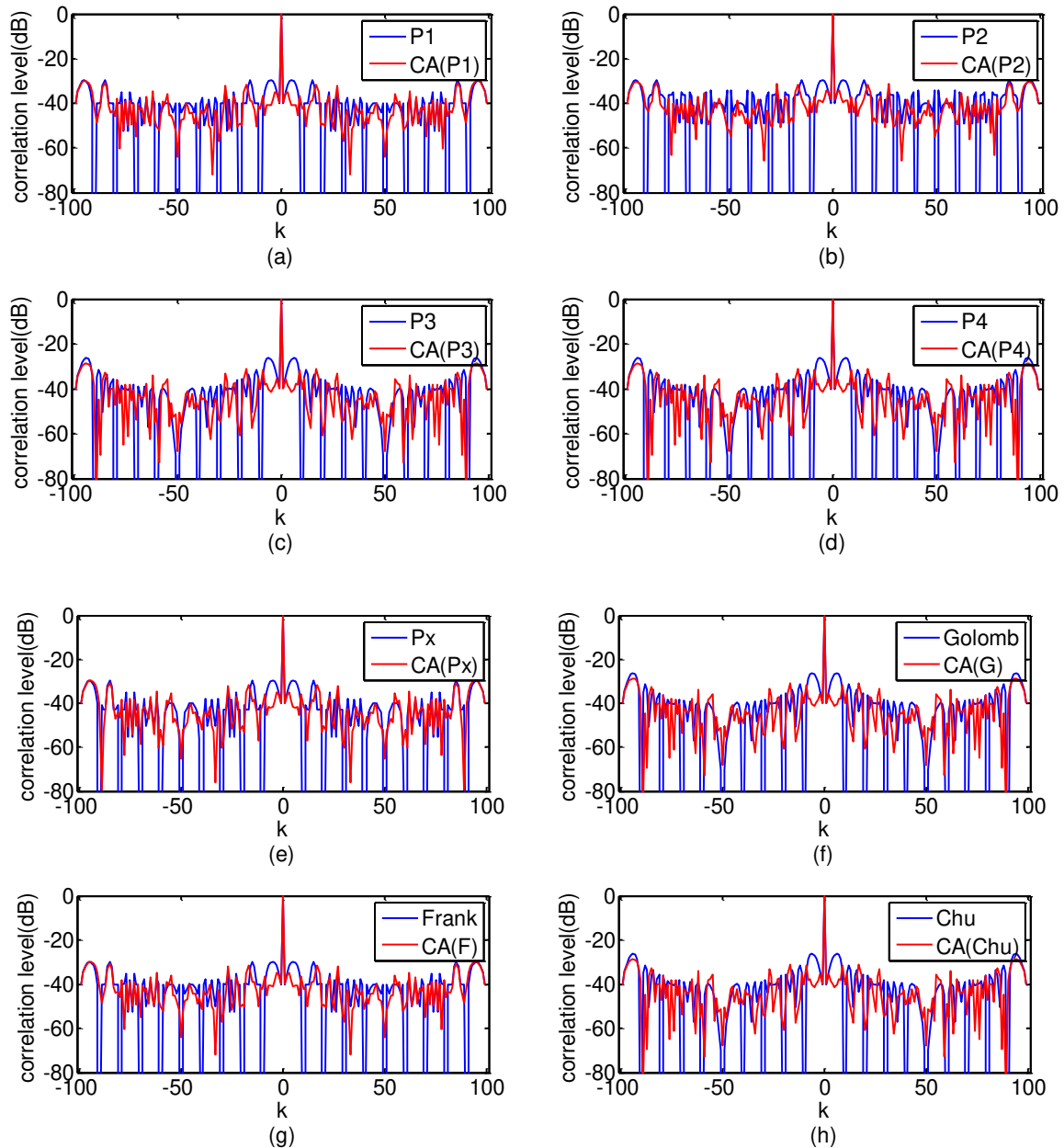


Figure-3. Correlation levels of the of the normal polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=100$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px) (f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu).

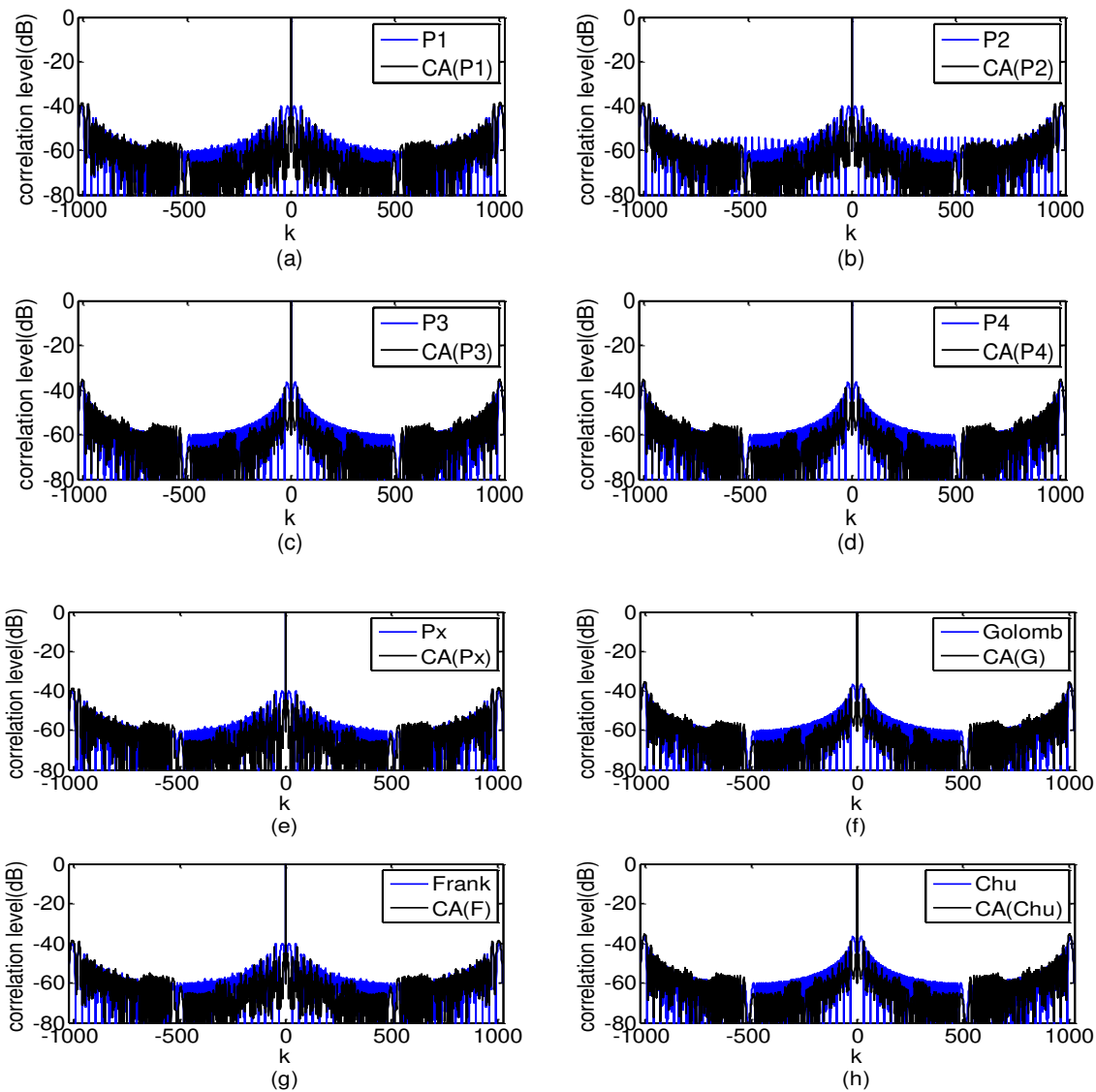


Figure-4. Correlation levels of the of the normal polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=1000$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px) (f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu).

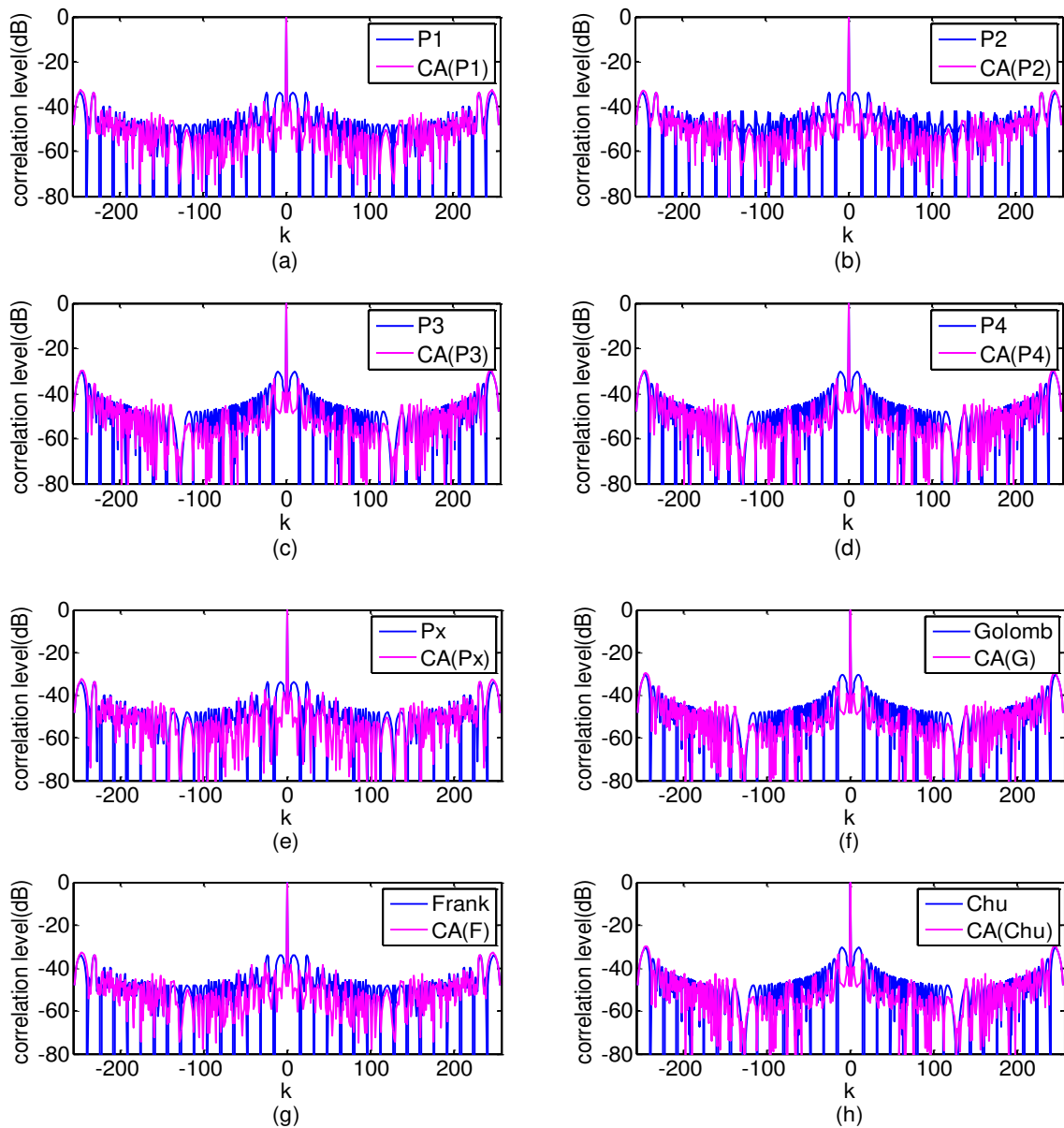


Figure-5. Correlation levels of the of the normal polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=256(16^2)$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px)(f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu).

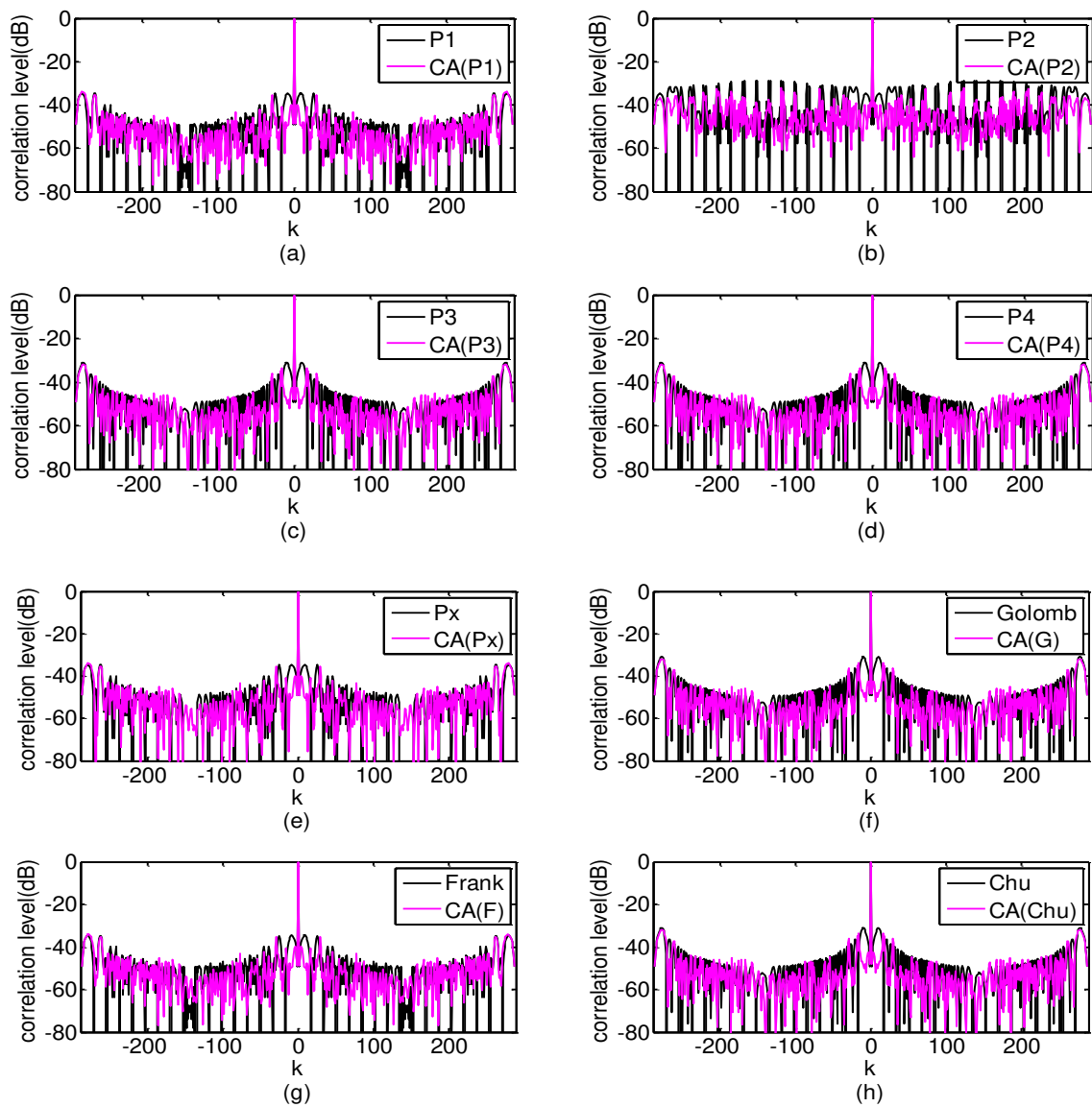


Figure-6. Correlation levels of the of the normal polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=249(17^2)$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px) (f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu).

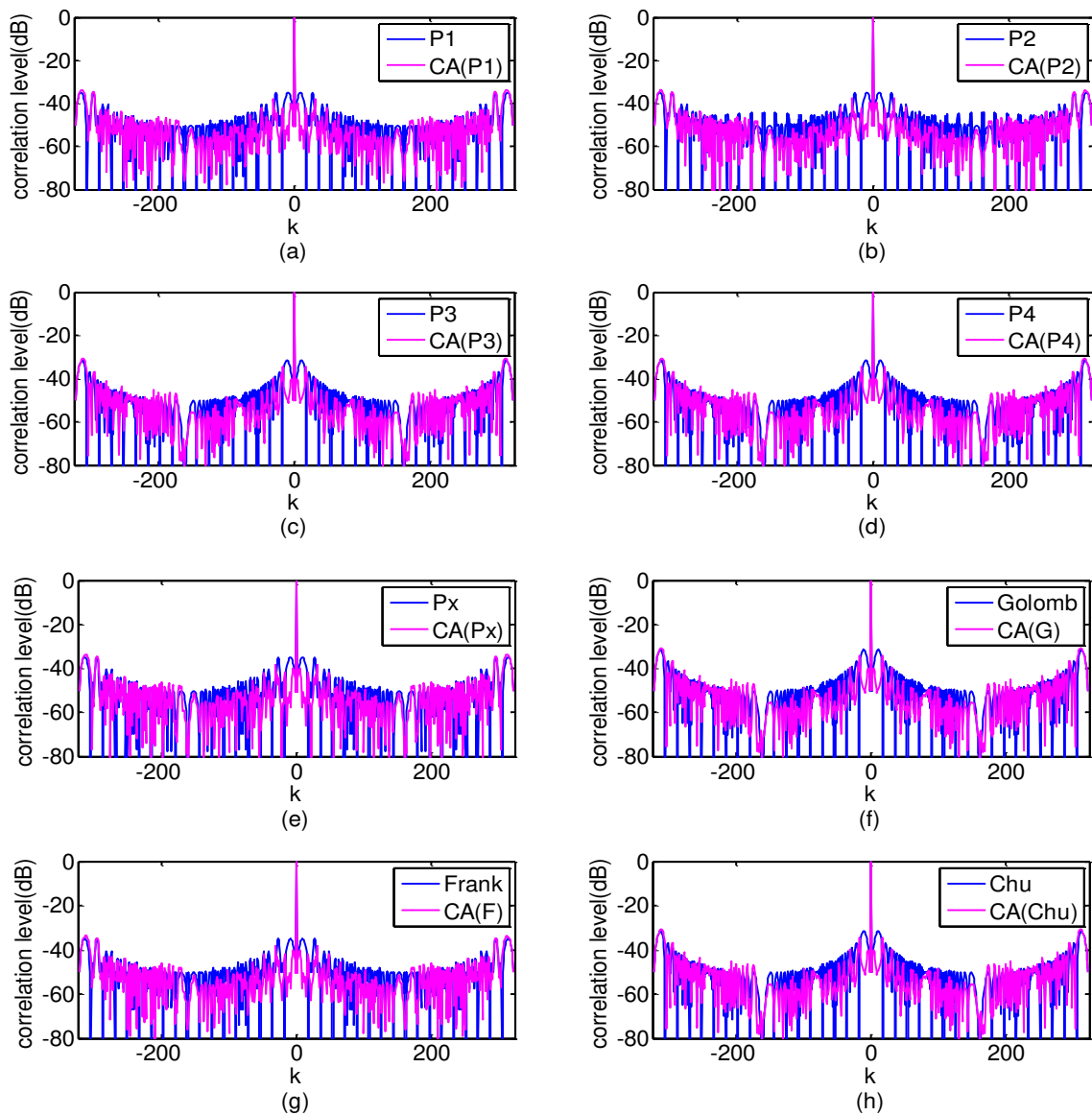


Figure-7. Correlation levels of the of the normal polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=364(18^2)$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px) (f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu) .

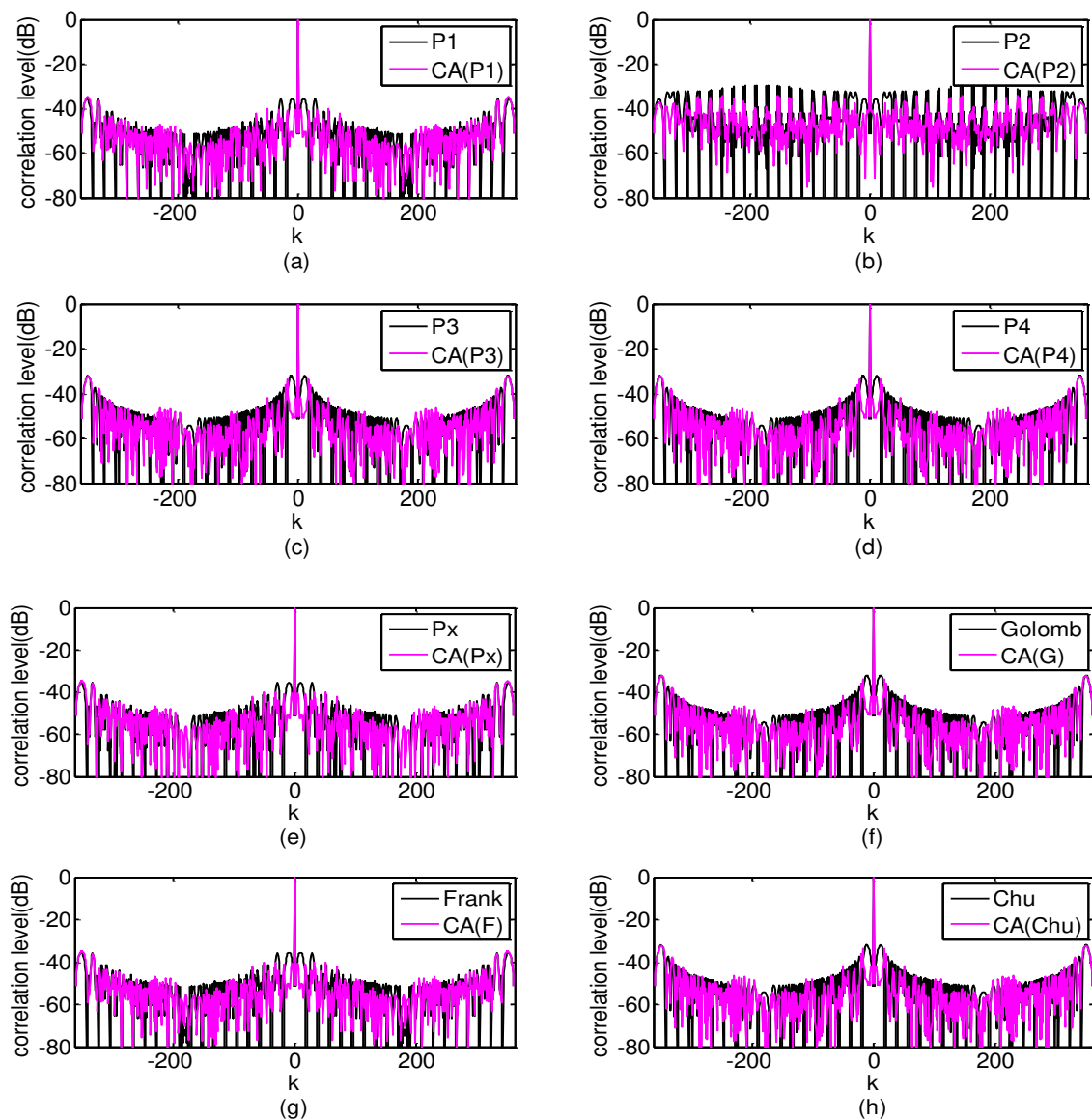


Figure-8. Correlation levels of the of the Normal Polyphase sequences and cyclic algorithm applied polyphase sequences of length $N=391(19^2)$ (a) P1, CA(P1) (b) P2, CA(P2) (c) P3, CA(P3) (d) P4, CA(P4) (e) Px, CA(Px) (f) Golomb, CA(Golomb) (g) Frank, CA(Frank) (h) Chu, CA(Chu).

5. CONCLUSIONS

The effect of Polyphase sequences of square integer length $N=D^2$ from 100 to 1000 case for both 3 is obtained for the standard case and Cyclic algorithmic approach. This approach is on SVD for larger values of $N > 10^3$ computational complexities get increases so that it becomes difficult to run on standard PC. P value can be chosen to obtain the larger merit factor values it also increases the number of multiplications so that complexity gets increases. Some concluding observations are, more than cent percentage of improvement in merit factor achieved for cyclic algorithm approach compared to the standard case for length 100. However, for 1000 length only 30.47 % improvement only achieved. The ascending order of merit factor values are CA (P3, P4, Golomb, Chu) < CA (P1, Frank) < CA (Px) < CA (P2) for both lengths 100 & 1000. The P value must choose to make execution

time should be less. In standard case P2 sequence exhibit very less value for odd integer square length, but for CA approach P2 sequence exhibit good merit factor.

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