ARPN Journal of Engineering and Applied Sciences

A COMPARATIVE STUDY OF TABU SEARCH AND GENETIC ALGORITHMS FOR OPTIMUM GROUNDWATER MANAGEMENT IN AN ARID REGION

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ABSTRACT

A two-dimensional mathematical model was developed to simulate the flow system at the upper aquifer of the quaternary sediments. The proposed conceptual model is fixed to one layer, meaning that the activity of the deeper aquifer is negligible. Twenty soil samples are selected to obtain the soil texture classes. The average values of minimum infiltration rates are interpolated using Kriging techniques in Geostatistical analysis extension of Arc GIS 9.3 to create the soil hydrologic group layer. Based on the covered area by soil hydrologic groups, the initial values of hydraulic conductivity and specific yield are supplied to the numerical program. According to the calibration process, the hydraulic characteristics of the upper aquifer has been identified, the hydraulic conductivity in the study area ranged (1-10) m/day, while the specific yield ranges between (0.1- 0.4). Two management cases (fixed well location and moving well location) were considered by executing the model with adopting calibrated parameters by using tabu search (TS) technique. The choice option for the moving well location in the second case leads to an increase of 9% in the total pumping rates compared to fixed well location. The results were compared with another study conducted for the same study area using genetic algorithms (GA) technique. It can be concluded that TS technique yields better results than GA technique.

Keywords: groundwater modelling, groundwater management, tabu search, genetic algorithms, Iraq.

1. INTRODUCTION

Groundwater in many countries is one of the major water resources. Therefore, it must be carefully managed and protected for most benefit use. With population growth and economic developments, groundwater and surface water resources management have become increasingly important. Recent dry seasons, limited surface water resources and doubling of the projected population over the next fifty years are of great interest in the use of groundwater resources.

The development of both mathematical and conceptual models of groundwater started from the beginning of the 20th century to the current day. Initially, analytical models are frequently used, but there is now greater use of numerical models. Real groundwater problems are often so complex that they can only be analyzed when simple assumptions are introduced. Imaginations and experiences are required to identify the key process which must be included in conceptual models. Development of groundwater resources requires answers to key questions such as:

a) What are the best sites for the installation of wells to develop this resource of groundwater?

b) How much water can be extracted at these sites?

These two questions were answered by developing two models in a studied area. The first model, a numerical model is applied to study groundwater flow patterns. The second model is the approach of combining simulation and optimization to measure water availability in potential locations.

The study area is located in Missan province, in the north and north-east parts. There are two rivers in this area, Teeb river and the Duiraige river. Teeb river crosses the area from north to south and ends in the local marshes. The Duiraige river is located in the south-east of this area.

As a result of the increasing demand for groundwater in recent years, especially after the emergence of drought in the country as well as the expansion of agricultural areas that were associated with water scarcity in Teeb and Duiraig river, there is a great need to estimate groundwater resources within the study area and the optimal management of groundwater in this region. Due to the importance of groundwater as water resources in the study area, different geological and hydrological studies were conducted. Lazim (2002) presented a two-dimensional mathematical model for representing the groundwater flow in both steady and unsteady states. Al-Jaburi (2005) presented hydrogeological and hydrochemical study for Ali Al-Garbi area, according to chemical composition of groundwater, this area can be divided into two major aquifers.

Numerical modeling of groundwater flow is an effective tool for better study of groundwater flow in aquifers and is a good tool for management of groundwater resources. Numerical models are the few tools available that can deal with complex parameters in the aquifer (hydraulic properties, rivers, structure, pumping, recharge and heterogeneities) and the ability of these variables to interact with each other. In the following, some researches dealt with groundwater modeling using finite difference method (Taylor and Luthin 1969; Prickett and Lonquist 1971; McWhorter and Sunada 1977; Vaucline *et al*. 1979; Boonstra and DeRidder 1981; Rushton and Senarath 1983; Al-Badar

VOL. 14, NO. 6, MARCH 2019 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

1987; Rasheeduddin *et al*. 1989; Alwan 1998; Al-Aboodi 2003).

The demand for groundwater in recent decades has led to the development of a variety of groundwater management strategies and effective utilization of underground storage space. This may include conversion of local water sources, changing pumping patterns, reduction of pumping, and artificial recharge (Galloway, *et al* 2003).Nonlinear programming techniques have been used to groundwater management problem since the 1980 (willis 1985).The application of the combined simulation-optimization method has been useful research field in recent years (Gorelick 1983; Ahlfeld *et al* 1988; Wanger and Gorelick 1989; Rizzo and Dougherty 1996; Rana *et al*, 2008; Hamid *et al* 2009).

The overall objective of this study is to develop decision support tools to determine the optimal location of the groundwater wells to meet future requirements in the study area. Also estimate the amount of groundwater available in those locations. The secondary objective is to compare between genetic algorithm and tabu search for optimal management of groundwater flow. These models include a combined simulation-optimization approach to quantify the water availability.

2. STUDY AREA

The study area located at the foot of the Iraqi-Iranian Border Mountains in southern Iraq. It located between latitudinal-line (32°06'-32°30') and longitudinal-line (47°06'-47°36') as shown in Figure-1; with total area of 1860 km^2. The boundaries of the study area extend from the Teeb area near the Iraqi-Iranian border to the Shikh Fars area. The marshes in this region get their waters from the tributaries of Teeb and Duiraige river, and other small streams that generally flow westward and south from Himreen foothills along the Iraqi-Iranian border in southern Iraq. The study area is bounded in the northeast by low anticlinal hills (Hemrin Hills), which rise to a maximum elevation of 250 m (a.s.l) near Teeb area. Generally, the surface is sloping towards the west and southwest to reach elevation about 8 m (a.s.l) along the Tigris River. Jabal Hemrin and its south-eastern extension, Jabal Faugi, which generally follows the Iraqi-Iran border, are the only major anticlinal folds in the studied area. The study area is a part of the alluvial plain which represents its eastern edge. Most of this area is covered with Quaternary sediments, it's generally a river sediments and partly of Aeolian sediments. The thickness of these Quaternary deposits in the Mesopotamian plain exceeds 250 m.

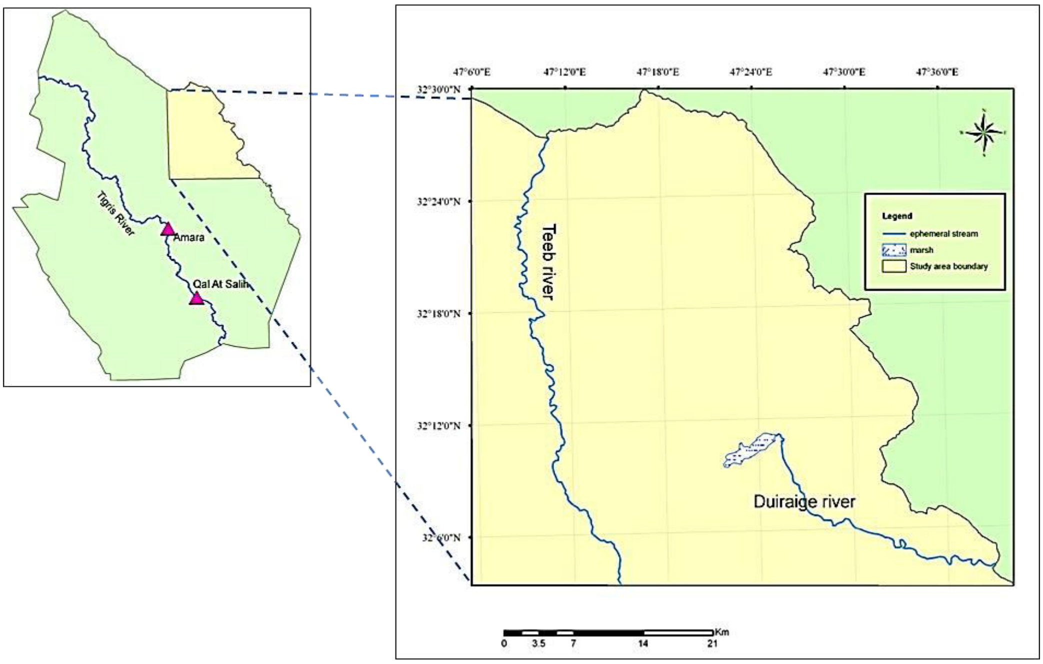

Figure-1. Location of study area in reference to map of Missan province.

3. GROUNDWATER MODELING

A two-dimensional mathematical model was developed to simulate the flow system at the upper aquifer of the quaternary sediments. The proposed conceptual model is fixed to one layer, meaning that the activity of the deeper aquifer is negligible. MODFLOW, "a groundwater flow model" developed by McDonald and Harbaugh (1988) is used in this research. The base of the upper layer of aquitrad is assumed to be an impermeable boundary, there is no moving of groundwater in the vertical direction, in other words, the activity of the deeper aquifer is negligible. There is not enough data for the static water level of the deeper aquifer, which is the main reason for choosing the current conceptual model. The present model consists of (55) column and (50) row. Where the area of cells is equal to (1000 × 1000m). All boundaries in the current model are simulated as head-dependent boundary to allow flow to the model area at a rate proportional to the head difference between the aquifer outside the modeled area and the model boundaries. The

VOL. 14, NO. 6, MARCH 2019 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

upper part of the model was simulated as an unconfined aquifer. Ten monitoring wells distributed over the study area were selected to measure the initial and historical groundwater level for one year. Twenty soil samples are selected to obtain the soil texture classes. Kriging techniques in Geostatistical analysis extension of Arc GIS 9.3 is used for interpolating the average values of minimum infiltration rates to represent the soil hydrologic group layer as illustrated in Figure-2. Based on the covered area by soil hydrologic groups, the initial values of hydraulic conductivity and specific yield are supplied to the numerical program

Groundwater recharge estimating is one of the most difficult issues in water resources research, which depends on many factors and is further complicated by changes in environmental conditions (Maxwell and Kollet, 2008). Scanlon *et al.* (2006) presented a global method for estimating of groundwater recharge in arid (semi-arid) areas and noted that CMB (chloride mass balance) is widely used and is an effective method for estimating recharge of groundwater. In the present model a refined CMB equation is used in this study. The model is calibrated using trial and error procedure in two stages, steady state followed by unsteady state. According to the calibration process, the hydraulic characteristics of the upper aquifer has been identified, the hydraulic conductivity in the study area ranged (1-10) m/day, while the specific yield ranges between (0.1- 0.4).

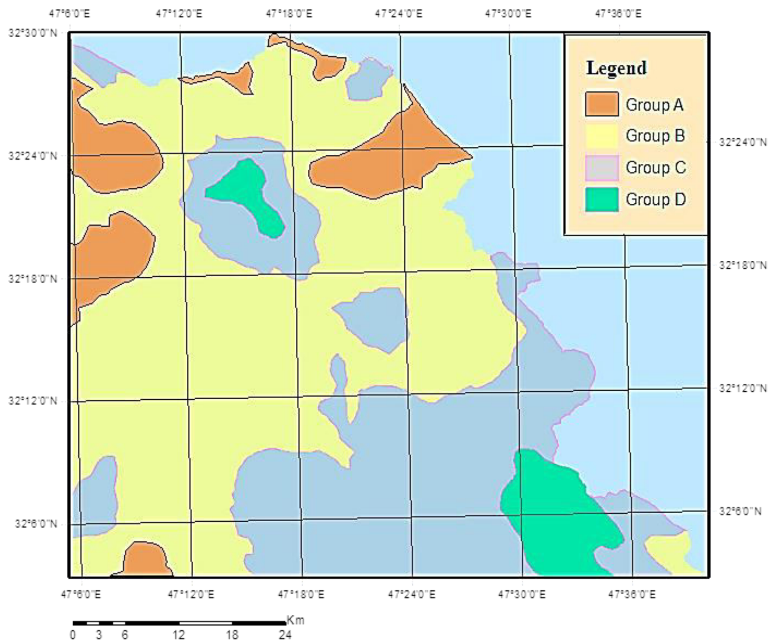

Figure-2. Hydrological soil groups in the study area.

4. OPTIMIZATION TECHNIQUES

In the subject of groundwater management, the variables are divided into two parts: decision variables and state variables, injection and pumping rates of wells are the decision variable. The decision variables are included a well location and the on/off status of a well, while hydraulic head is the state variable. The numerical model updates the status variables, then all optimal values of decision variables determine by the optimization model. This process is performed in a coupled simulation-optimization model. The Modular Groundwater Optimizer (MGO) is one of the most important programs used for general purposes in the development of field-scale applications (Zheng and Wang 2003). MGO is used in this research, three optimization solvers in MGO (genetic algorithms, simulated annealing, and tabu search).The tabu search technique was used in this study for determining the optimal groundwater management. The results were compared with another study conducted by (Al-Aboodi 2011) for the same study area using genetic algorithms technique.

5. TABU SEARCH

Tabu search (TS) is based on the principles presented by Fred Glover (1977, 1986).TS is established according to the principle that solves the optimization problem, so as to qualify intelligent, it must include adaptive memory and quick-response reconnaissance. The Adaptive Memory merit of TS permits for performing actions that are able to search the area of the solution efficiently and economically. Due to information collected during search is used for guiding the local options, TS contrasts with memory-free designs that rely directly on semi-random processes that perform a form of sampling. The assurance on responsive exploration (and therefore objective) in TS, whether in imperative or probabilistic application, is derived from the assumption which a poor strategic option can oftentimes result in more information than good random selection, thus supplies a basis for

ARPN Journal of Engineering and Applied Sciences

gradually amended strategies that benefit from the search history. The basic strategy for TS is a local search scheme and a number of guiding rules such as short-term memory and long-term memory. An effective component is short-term memory, which is named to as a tabu list.

TS starts with a feasible location of well, or configuration, say I_1, and empty tabu memory $L = \Phi$. TS calculates the objective function of well configurationI_1. Next, it evaluates all configurations at neighborsI_1. The vicinity of I_1 means the well configuration which is slightly different from I_1. For example, change one well location from site i to i + 1 or i-1.Select Neighborhood with the lowest objective function value (in minimize objective function) and check the tabu list. If the selected neighbor meets the tabu condition, this action is prohibited. In this case, select the second lowest value of the objective function and check the status again until a move is selected without violating any tabu conditions. At this moment, update the tabu status $L = \{I_1\}$ and start the search. The tabu movement remains active in the tabu list for a number of iterations based on the length of the tabu list L. A new tabu move is added and oldestone of the list is expire and is excluded from the tabu list so that the tabu list remains constant. Basic procedure of tabu search can be summarized in the following steps:

a) The tabu search begins with a feasible well configuration I = {*Ij ,, j = 1, ..., 2NW*}, where *NW* represents the maximum number of wells that can be installed, and *I*j is the coordinate of the well location. The neighborhood set for *I* are constructed and setthe tabu list empty L =φ.

b) The objective function is evaluated for each well in the neighborhood set.

c) The best solution is calculated in the neighborhood excluding the configurations in the tabu list. The tabu list is updated, for example, if the tabu list is not full, the change between the new solution and the old solution is added. Otherwise, the oldest item in the tabu list is deleted and records the change. Then, a new neighborhood set is constructed for the next iteration.

d) The convergence criterion is checked. The program is terminated when stopping criterion is met; otherwise, it goes to step 2.

6. RESULTS AND DISCUSSIONS

Two management cases (fixed well location and moving well location) were considered by executing the model with adopting calibrated parameters. These parameters are obtained according to the calibration process of numerical model as mentioned in section (groundwater modeling). TS technique was used in this paper for determining the optimal solution of groundwater management. The results of two cases were compared with another study conducted by Al-Aboodi (2011) for the same study area using genetic algorithm technique. These cases are presented as following:

A. First case, fixed well location

The formulation of objective function in the first case is illustrated in Equation. (1). The total number of wells in the study area is thirty-five pumping wells (the real number of wells in this region). The objective function and constraints for this case can be formulated as following:

$$\text{Maximize } J = \Delta t \sum_{i=1}^{35} |Q_i| \tag{1}$$

Subject to

$$h_{min} \leq h_m \leq h_{max} \tag{2}$$

$$0 \leq |Q_i| \leq 4000 \tag{3}$$

Where:

Equation (1), the objective function (*J*) is presented by the absolute pumping rates multiplied byΔt, where Δt is the length of the stress period in the groundwater flow model.

Equation (2) is the head limited constraint that requires the hydraulic head to be in any monitoring well location, h_m, must be above h_{min} and below h_{max}, where:

$$h_{min} = h_i - 0.5 \tag{2a}$$

And

$$h_{max} = h_i + 0.5 \tag{2b}$$

h_i is the initial hydraulic head. The number of monitoring wells is equal to 9; these wells are used for detecting the hydraulic heads fluctuations of the groundwater above the study area as illustrated in Figure-3.

In Equations (2a) and (2b), the value of 0.5m is fixed according to the ground water fluctuation in the studied region for the observing period (one year), there is no considerable fluctuation more than this value in the groundwater hydraulic heads during this period, and taking a more realistic value corresponds to the reality fluctuation of groundwater hydraulic heads away from the unjust exploitation of water.

The minimum value of pumping rate is set as zero, where the maximum value is set as (4000 m^3/day) as shown in Equation (3). In general, many testing processes are necessary to determine the suitable value of a maximum pumping rate. In the case for setting value of too high, the optimization solution may be ineffective.

The value of objective function for this case is equal to (0.35438E+08 m^3/year). The distributions of the optimized pumping rates depending on the locations of these wells in the network of MODFLOW program are shown in table (1). The results of this table show that the total value of pumping rate from all pumping wells by using TS is better than the total value of pumping rate by using GA. The percentage ratio of increased pumping rate by using TS compared with using GA is 7.56%.6 cm is the highest drop for hydraulic heads compared to the initial groundwater heads. Pumping rates can be increased to nine times according to the current pumping rates. Also, upper and lower limits are reduced from 0.5 to 0.25; there is no considerable change in the optimization results.

B. Second case, moving well location

Optimum management of wells pumping rates and their locations are carried out through the option of moving well location. In this option, the location of the well is not fixed. The well location has the ability to moves anywhere within a specific area of the selected model grid to achieve optimal location. This case is similar to the first case in terms of the objective function. There is a new constraint in addition to the constraints in first case, where it is required through this constraint that each well-being optimized must be situated within the patterned area (it can be set by a layer, a row, a column for a model cell). For example, the additional constraint can be formulated as:-

$$11 \le I_w \le 48 \quad (4)$$

$$2 \le J_w \le 34 \quad (5)$$

Where:
I_w and $J_w$are the row and column indices in the model grid.

The objective function converges to the maximum value (0.38750E + 08 m^3 / year). For this specific case, the choice of the optimal location of the wells leads to an increase of nine percent of the total pumping rates compared to the fixed well location (first case).This result is attributed to the random distribution of the current sites of wells. The total pumping rate in second case is compared with the total pumping rate which is presented by (Al-Aboodi, 2011) as shown in Table-2. It can be seen that TS technique yields better results than GA technique. Figure-4 shows the distribution of optimal wells locations in the study area of second case according to the study presented by (Al-Aboodi, 2011). Figure-5 shows the distribution of optimal wells locations for this research. It can be seen from Figure-5, the most wells lie in the northern parts, where a good hydraulic characteristics.

VOL. 14, NO. 6, MARCH 2019 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

Table-1. Optimal pumping rates in the study area for the first case.

Well No.	**Raw No.** (I)	**Column No.** (J)	**Pumping rate** (m^3/day) (TS)	**Pumping rate** (m^3/day) (GA) (Al-Aboodi,2011)
1	6	6	-3350	-3200
2	7	7	-3650	-3520
3	8	11	-3650	-3520
4	10	15	-3040	-2720
5	11	11	-3240	-3200
6	11	16	-14.00	0.000
7	12	15	-3650	-3040
8	12	20	-1660	-1440
9	12	26	-3880	-3840
10	13	16	-3750	-3680
11	13	20	-3750	-3680
12	14	17	-880.0	-800
13	15	15	-3520	-3520
14	16	16	-1960	-1760
15	16	19	-2850	-2560
16	17	21	-15.00	0.000
17	19	23	-2720	-2720
18	20	18	-280	-160
19	21	13	-3850	-3520
20	21	20	-3880	-3680
21	22	14	-3600	-2560
22	26	15	-3680	-3680
23	30	33	-3350	-3200
24	32	34	-2720	-2720
25	34	27	-3680	-3360
26	35	28	-2750	-2720
27	35	34	-3740	-3680
28	36	34	-3550	-3360
29	37	31	-2060	-1280
30	37	39	-2960	-2560
31	38	30	-1940	-1920
32	39	32	-3240	-3040
33	39	35	-3680	-3520
34	39	38	-280	-160
35	39	42	-3620	-3200
sum			-98439	-91520

VOL. 14, NO. 6, MARCH 2019 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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Table-2. Optimal pumping rates in the study area for the second case.

Well No.	**Raw No.** (I)	**Column No.** (J)	**Pumping rate** (m^3/day)(GA) (Al-Aboodi,2011)	**Raw No.** (I)	**Column No.** (J)	**Pumping rate** (m^3/day) (TS)
1	21	15	-3360	11	2	3900-
2	15	19	-3360	11	2	3950-
3	16	20	-3200	11	2	3900-
4	30	24	-3040	11	2	3880-
5	12	17	-2720	11	11	3300-
6	34	31	-3040	11	16	2355-
7	15	5	-320	12	15	2650-
8	24	22	-3040	12	20	3080-
9	23	15	-4000	12	26	2560-
10	36	28	-1600	13	16	3950-
11	16	29	-1600	13	20	4000-
12	13	23	-3360	14	17	3565-
13	17	24	-3040	15	15	2833-
14	11	5	-1600	16	16	3080-
15	18	29	-3360	16	19	2758-
16	14	22	-320	17	21	3340-
17	11	23	-3840	19	23	3995-
18	12	32	-3680	20	18	3080-
19	16	13	-3520	21	13	1950-
20	13	4	-3200	21	20	3100-
21	11	4	-2560	22	14	2288-
22	13	8	-2560	26	15	3150-
23	18	9	-3680	30	33	1580-
24	38	11	-3840	32	34	3065-
25	20	11	-4000	34	27	4000-
26	31	16	-320	35	28	3500-
27	24	18	-3360	35	34	3560-
28	29	16	-2720	36	34	3340-
29	16	7	-3200	37	31	3340-
30	19	16	-3040	38	6	800.0-
31	15	21	-3040	38	30	3880-
32	13	31	-1440	39	32	3800-
33	22	9	-2560	40	2	1780-
34	14	18	-3200	40	5	2100-
35	18	26	-4000	40	9	2230-
sum			-98720	-107639		

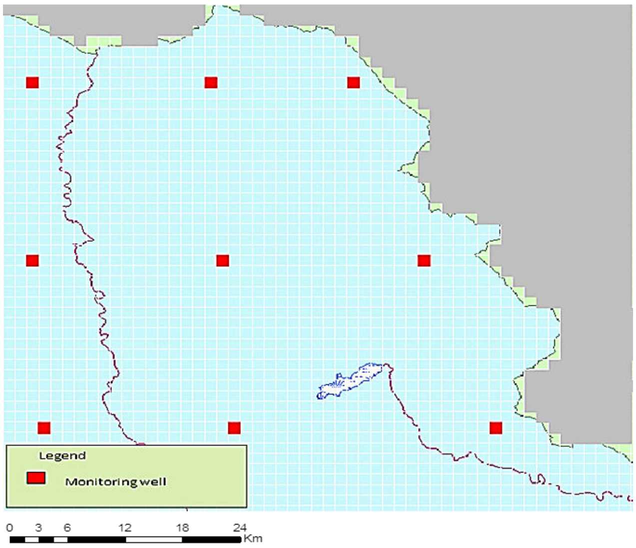

Figure-3. Distribution of monitoring wells locations in the study area.

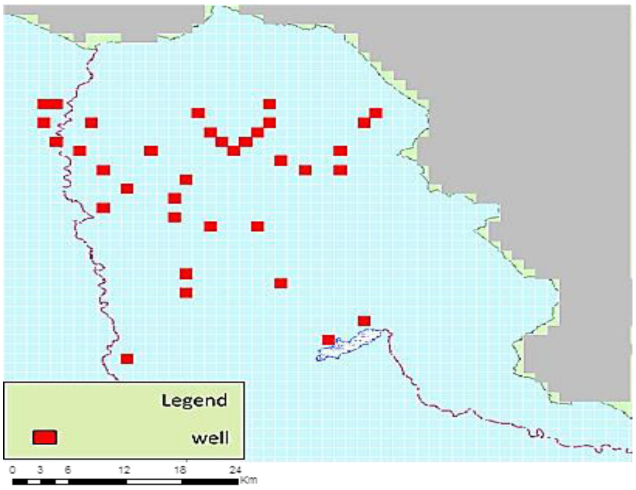

Figure-4. Distribution of optimal wells locations in the study area (Al-Aboodi, 2011).

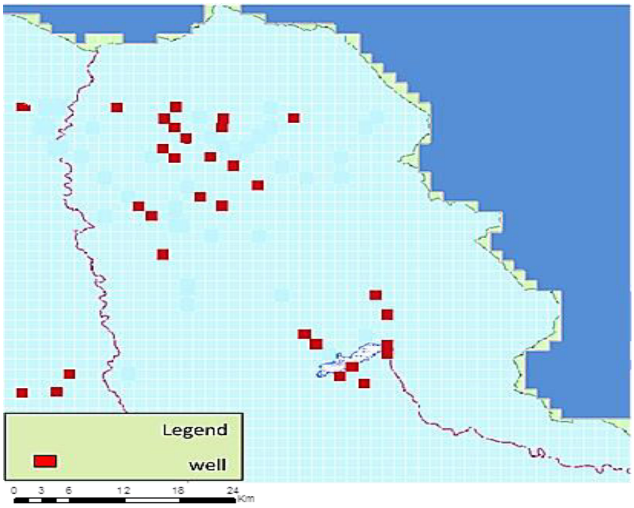

Figure-5. Distribution of optimal wells locations in the study area by using (TS).

7. CONCLUSIONS

The numerical model is calibrated using trial and error procedure in two stages, steady state followed by unsteady state. The hydraulic characteristics of the upper aquifer has been obtained by using calibration process, the hydraulic conductivity is ranged between from 1 to 10 m/day, while the specific yield is ranged from 0.1 to 0.4.Two management cases (fixed well location and moving well location) were considered by executing the model with adopting calibrated parameters. In the first case,the objective function is converged to optimal value of (0.35438E+08 m^3/year).The results of the first case shown that the total value of pumping rate from all pumping wells by using TSis better than the total value of pumping rate by using GA. The percentage ratio of increased pumping rate by using TS compared with the results of GA is 7.56% in the first case. The highest reduction in groundwater hydraulic heads due to optimal pumping rates compared to the initial hydraulic heads is about 6 cm. The objective function in second case converges to the maximum value (0.38750E + 08 m^3 / year).The choice option for the moving wells location in the second case leads to an increase of total pumping rates about 9% compared to the fixed well location. From all obve results; it can be seen that TS technique yields better results than GA technique. According to the optimal location of wells, it could be noted that most of the wells lie in the northern part, where a good hydraulic characteristics.

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