



SYNTHESIS OF PRECISE AUTOMATIC CONTROL SYSTEM OF A SECOND ORDER OBJECT IN UNCERTAINTY CONDITIONS

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ABSTRACT

A high-precision control law of a second-order nonlinear object was synthesized based on the method of minimizing local functionals. It provides control under uncertainty conditions without using information about the derivatives of the controlled variable. Criteria for the selection of control law parameters are developed. The system of automatic control of the vertical movement of a remotely-operated underwater vehicle was synthesized. Computer simulation results showed high dynamic control accuracy.

Keywords: automatic control system, method of minimization of local functionals, uncertainties, underwater vehicle.

1. INTRODUCTION

Synthesis of automatic control systems (ACS) of nonlinear objects is a known scientific problem. Its solution usually requires taking into account the features of the nonlinear controlled object and is reduced to the solution of the synthesis problem for each particular case. A well-known class of nonlinear objects is an underwater complex with flexible tethers. They consist of mobile objects (underwater vehicles, ships, etc.), directly or indirectly connected with flexible tethers (tether-cables, tow-cables, etc.) and intended for the joint implementation in a particular underwater mission [1]. In spite of the fact that their mathematical models are well developed, in practice ACSs of such complexes motion operate under uncertainty conditions. Firstly, it is not always possible to determine all the parameters of the underwater complex. Secondly, the parameters of the underwater complex can dynamically change during the operation. For example, capturing and lifting up the underwater artifact changes the total mass of the underwater vehicle [2]. And, thirdly, there are many perturbing factors, the measurement of which is essentially complicated: the flow chart (distribution of flow velocity via depth), the vector of tension force of the flexible tether, acceleration of an underwater vehicle, etc. Even if the ACS is equipped with all the necessary sensors, the errors in measuring the parameters of the object and the conditions of its operation in total will lead to significant uncertain influences on the motion of underwater complex elements.

2. LITERATURE REVIEW

Many scientific publications are devoted to control automation of nonlinear mobile objects, in particular of underwater vehicles (UV). However, usually the proposed systems implement control of the UV motion in limited by vehicle velocity parameters modes, or do not take into account the influence of external disturbances and uncertainties of its parameters.

In [3] a multidimensional regulator of the motion velocity of a moving object in sliding mode is proposed. The parameters for adjusting the regulator were calculated by decomposing the matrix of the inertia characteristics of

the control object, which allowed compensating the influence of the water added masses on the motion velocity parameters. The authors note that this regulator should be used only at low object velocity in the absence of external disturbances.

In [4] the UV trajectory motion ACS is proposed, which operates in sliding mode. The effectiveness of ACS is confirmed by the simulation of a UV with four controlled degrees of freedom. The coherence of the operation of the propulsive devices is ensured by the choice of such parameters of the reference model, in which the UV will be able to move along the reference trajectory. When the UV leaves the reference trajectory, the desired values of control inputs may exceed the permissible limits.

In [5] a UV spatial motion ACS on the basis of neuro-fuzzy regulator is proposed. To eliminate the influence of nonlinearity of propulsion devices such as the "zone of insensitivity", the authors proposed to use a fuzzy compensator. Each ACS contour manages one degree of UV freedom, which greatly complicates the procedure of setting up such system.

In [6] a multidimensional ACS of the UV spatial motion on the basis of the PID-regulator is proposed. The compensation of external disturbances is performed by a nonlinear observer with a high gain. The quality of the control processes that the ACS implements depends on the stability of feedback data samples. Also, the disadvantage of a system is the need to hold a UV in a static state during the regulator initializing procedure.

The ACS with the use of PID-regulators for controlling UV spatial motion is proposed in [7]. The authors propose to compensate the hydrodynamic effect by entering into the control law the coefficients obtained by interval linearization. The influence of external disturbances on the ACS operation is not considered.

The analysis of literature review shows that the issue of nonlinear mobile objects ACS synthesis in conditions of uncertainty (in particular, UVs) is not sufficiently developed. In addition, for high-precision control, known ACSs need information about derivatives



of controlled parameters. This determines the relevance of this study.

3. THE PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of this paper is to synthesize the law of control of a one-dimensional nonlinear object of the second order by minimizing local functional and to develop criteria for selecting parameters of the control law that will make it possible to synthesize automatic control systems of high dynamic precision under conditions of uncertainty without using information about the derivatives of the controlled variable.

To achieve the research purpose, the following tasks must be solved:

- synthesis of the control law of high dynamic accuracy of nonlinear object of the second order for operation in conditions of uncertainty;
- developing the criteria for selecting the parameters of the synthesized control law;
- developing a mathematical model of a UV for the study of a one-dimensional motion ACS;
- synthesis of a UV's one-dimensional motion ACS;
- studying the operation of the synthesized ACS in conditions of uncertainty.

4. MATERIALS AND METHODS OF NONLINEAR OBJECT HIGH DYNAMIC PRECISION AUTOMATIC CONTROL SYSTEM SYNTHESIS

4.1 The method of ACS synthesis based on minimization of local functionals

One of the known methods of nonlinear objects ACS synthesis is the inverse dynamics method of the inverse control concept. It is successfully used for the synthesis of underwater vehicles ACSs and other marine mobile objects [8, 9]. The inverse dynamics task is known from theoretical mechanics as a task of determining the forces acting on the body, if the trajectory of its motion is known. In the theory of automatic control, the method of inverse dynamics was initiated as a method of structural synthesis of nonlinear ACSs [10]. The essence of the method is to synthesize the reference model of the ACS (a differential equation with the desired dynamic characteristics of the ACS) based on the object equation and the requirements for the quality of the control system. Further from the equation of the reference model, the highest derivative is expressed, and the control law is found by substitution of the obtained expression instead of the highest derivative in the object's equation [11].

On the basis of the concept of inverse dynamics it is possible to synthesize ACSs of one-dimensional and multidimensional systems of high dynamic precision. However, ACSs, synthesized by the classical method of inverse dynamics, are sensitive to changes in the parameters of controlled objects, and under influence of uncertain disturbances the control quality deteriorates. In order to provide robust properties and possibility to work

in conditions of uncertainty, it is necessary to apply specialized methods for eliminating these drawbacks.

One such method is the method of ACS synthesis, which is based on the concept of inverse dynamics in conjunction with the minimization of local functional characterizing the motion energy in the vicinity of the phase trajectories of reference models [12].

4.2 Synthesis of the differential control law of a second order nonlinear object

When controlling a one-dimensional second order object the functional $G(u)$ is to be minimized. It is the normalized energy of the second derivative of a controlled parameter [12]:

$$G(u) = \frac{1}{2} [\ddot{y}_d(y_g, t) - \ddot{y}(t, u)]^2, \quad t \geq 0; \quad (1)$$

where y - controlled parameter; y_d - desired value of the controlled parameter obtained from the ACS's reference model; y_g - given value of the controlled parameter (control task); u - control signal (controlling parameter); t - time. Derivatives by time t are marked by dots.

Based on functional minimization $\min_u G(u) \rightarrow 0$ a differential control law that allows lowering the ACS's order is received. In order to exclude derivatives \ddot{y} and \dot{y} and leave only the controlled parameter y in the equation of the control law, we synthesize it by minimizing local functionals using the gradient search method of the second order [12]:

$$\frac{d^2 u}{dt^2} + h \frac{du}{dt} = -\lambda \frac{\partial G(u)}{\partial u}; \quad h, \lambda = \text{const} > 0,$$

where h and λ - gradient search parameters. Functional $G(u)$ is determined by (1).

On its basis the differential control law is formed:

$$\ddot{u} + h\dot{u} = \sigma k (\ddot{y}_d - \ddot{y}); \quad k > 0; \quad (2)$$

$$\sigma = \text{sign} \left(\frac{\partial \ddot{y}}{\partial u} \right), \quad (3)$$

where σ - parameter that defines the direction of the gradient search.

The obtained control law allows the reduction of order to u that is two orders lower. The stability of the contour of the control function as a necessary condition for the stability of the process of minimizing the functional $G(u)$ is provided by the rule of signs, which for nonlinear systems is taken into account by the parameter σ .

In [12] it is shown that for nonlinear ACSs, synthesized on the basis of differential control law (2), one can always specify such limited values of k , at which stationary state of $y = y_g$ is asymptotically stable. With an unlimited increase in the gain, the asymptotic stability of



the stationary state $y = y_g$ is preserved. Thus with $k \rightarrow \infty$ the process $y(t) \rightarrow y_g$ asymptotically approximates to the corresponding reference process. This means that theoretically by increasing the coefficient k any given level of process $y(t)$ approximation to the corresponding reference process can be achieved with any $t > 0$. In this regard, the parameters of the reference model must meet the required quality of the ACS.

4.3 Development of criteria for selecting the parameters of the nonlinear ACS's regulator

4.3.1 The basic condition for selecting the regulator's parameters

The parameter k of the control law (2) is chosen from the condition according to which the performance of the contour of the control function u should be much higher than the performance of the reference model [12]:

$$\frac{T_r}{T_u} = c \gg 1, \quad (4)$$

where T_r - time constant of the reference model, T_u - time constant of the contour of the control function; c - parameter that defines the correlation between T_r and T_u .

4.3.2 Formation of the stability condition of the numerical solution process of the equations of the control law

Theoretically, it is possible to achieve any degree of approximation of the accuracy of the ACS transitional process to the reference model through an unlimited increase of k . On the other hand, it should be taken into account that the equations of the control law (2) are solved by numerical methods and significant increase of the coefficient k can lead to stability loss of numerical solution process of differential equations systems of the control law. To ensure the stability of the functional $G(u)$ minimization process it is proposed to ensure the stability of the numerical solution process by fulfilling the following condition:

$$\frac{T_u}{T_\Delta} = n \gg 1, \quad (5)$$

where T_Δ - period of quantization by time in the computer realization of the control law; n - parameter that defines the correlation between T_Δ and T_u .

4.3.3 Linearization of the mathematical model of a nonlinear one-dimensional second-order control object

In general case, the concept of time constant for the contour of the control function of a nonlinear object is meaningless, since this parameter can dynamically change depending on the current phase coordinates of the object. But it can be estimated in the vicinity of some operating

point of the phase space. To do this, it is necessary to linearize the nonlinear equation of the control object.

Let the control object be described by the following dependence, which has all the continuous derivatives of all arguments:

$$\ddot{y} = f(u, \dot{y}, y).$$

y - controlled parameter; u - control signal (controlling parameter). Derivatives by time t are marked by dots.

Let's put it in the Taylor series, taking into account partial derivatives of only the first order:

$$\begin{aligned} \ddot{y} &= F + \frac{\partial F}{\partial u}(u - u_0) + \frac{\partial F}{\partial \dot{y}}(\dot{y} - \dot{y}_0) + \frac{\partial F}{\partial y}(y - y_0) = \\ &= \frac{\partial F}{\partial u}u + \frac{\partial F}{\partial \dot{y}}\dot{y} + \frac{\partial F}{\partial y}y + F - \frac{\partial F}{\partial u}u_0 - \frac{\partial F}{\partial \dot{y}}\dot{y}_0 - \frac{\partial F}{\partial y}y_0; \\ &F = f(u_0, \dot{y}_0, y_0), \end{aligned}$$

where u_0, \dot{y}_0, y_0 - coordinates, in the vicinity of which the object is linearized; F - the function \ddot{y} at a coordinate point u_0, \dot{y}_0, y_0 .

Let us give the resulting equation to a form suitable for the formation of an inverse model of an object:

$$\begin{aligned} \ddot{y} &= \frac{1}{\alpha_2}(u - \alpha_1\dot{y} - \alpha_0y + \beta); \quad (6) \\ \frac{1}{\alpha_2} &= \frac{\partial F}{\partial u}; \quad \alpha_1 = \alpha_2 \frac{-\partial F}{\partial \dot{y}}; \quad \alpha_0 = \alpha_2 \frac{-\partial F}{\partial y}; \\ \beta &= \alpha_2 \left[F - \frac{\partial F}{\partial u}u_0 - \frac{\partial F}{\partial \dot{y}}\dot{y}_0 - \frac{\partial F}{\partial y}y_0 \right]; \quad F = f(u_0, \dot{y}_0, y_0). \end{aligned}$$

where $\alpha_{0,1,2}$ and β - linearized parameters of a mathematical model of a nonlinear object.

The equation (6) can be used to study the nonlinear object in the vicinity of the point u_0, \dot{y}_0, y_0 .

4.3.4 Definition of the time constant of the control function contour

Let's find a time constant of the control function contour T_u for the control object at the point u_0, \dot{y}_0, y_0 . For this purpose, in the differential control law (2) we substitute the linearized equation (6):

$$\sigma(\ddot{u} + h\dot{u}) = k\ddot{y}_d - \frac{k}{\alpha_2}(u - \alpha_1\dot{y} - \alpha_0y + \beta).$$

Let's move the control signal and its derivatives to the left side of the equation:

$$\sigma \frac{\alpha_2}{k}(\ddot{u} + h\dot{u}) + u = \alpha_2\ddot{y}_d + \alpha_1\dot{y} + \alpha_0y - \beta. \quad (7)$$



According to the classical method of inverse dynamics, the control signal for an object, which is described by equation (6), is determined from its inverse model:

$$u_{opt} = \alpha_2 \ddot{y}_d + \alpha_1 \dot{y} + \alpha_0 y - \beta.$$

It provides an absolute minimum of functional (1) $\min_u G(u) = 0$ (if model parameters are known). Let's substitute u_{opt} instead of the right part in the expression (7) and obtain the equation of the dynamics of the transition process in the contour of the control function:

$$\frac{|\alpha_2|}{k} (\ddot{u} + h\dot{u}) + u = u_{opt}; \quad k > 0. \quad (8)$$

The input signal in (8) is parameter u_{opt} , the output - parameter u . The sign of parameter α_2 corresponds to the rule of signs (3), so it is reduced and the absolute value of α_2 remains in the equation.

Let's bring (8) to the type of the standard chain of the second order:

$$\begin{aligned} \frac{|\alpha_2|}{k} \ddot{u} &= u_{opt} - \frac{|\alpha_2|}{k} h \dot{u} - u; \\ T_u^2 \ddot{u} &= K u_{opt} - 2\zeta T_u \dot{u} - u; \\ T_u^2 &= \frac{|\alpha_2|}{k}; \quad 2\zeta T_u = \frac{|\alpha_2|}{k} h; \quad K = 1; \quad \zeta \geq 1, \end{aligned} \quad (9)$$

where K - gain factor; ζ - damping factor, it should be selected from the range $[1, +\infty)$, so that the link was aperiodic.

Thus in (9) we obtained the expression for the time constant of the contour of the control function of the second order in the vicinity of the point u_0, \dot{y}_0, y_0 .

4.3.5 Formation of criteria for choosing the parameters of the control law

When choosing the coefficient k it is required to set values of parameters c and n , which determine the correlation between T_r and T_u and between T_Δ and T_u . Conditions (4) and (5) impose a rigid correlation between c and n . To avoid this, we will replace the signs of equality in (4) and (5) with signs «more or equal»:

$$\frac{T_r}{T_u} \geq c; \quad \frac{T_u}{T_\Delta} \geq n; \quad c \gg 1; \quad n \gg 1.$$

Let's solve the inequality with respect to the parameter T_u^{-1} and obtain the main criterion for choosing the parameters of the control law of the nonlinear ACS:

$$\frac{c}{T_r} \leq \frac{1}{T_u} \leq \frac{1}{nT_\Delta}; \quad c \gg 1; \quad n \gg 1. \quad (10)$$

It is the basis for forming a range of valid values for the parameter k of the control law (2).

Using the time constant T_u from (9), we rewrite the condition (10) and obtain an expression for the calculated parameter (coefficient k) of the nonlinear object control law:

$$\left[(\forall u \in U) (\forall \dot{y} \in Y') (\forall y \in Y) \right] \left[\frac{c}{T_r} \leq \sqrt{\frac{k}{|\alpha_2|}} \leq \frac{1}{nT_\Delta} \right]. \quad (11)$$

$$c \gg 1; \quad n \gg 1; \quad \alpha_2 = \left(\frac{\partial F}{\partial u} \right)^{-1}; \quad F = f(u, \dot{y}, y),$$

where U - the set of permissible control signals, Y' - set of permissible object's accelerations, Y - set of permissible object's controlled parameter, F - the value of function \ddot{y} in current coordinates: u, \dot{y}, y .

To determine the criterion for choosing the coefficient k the resulting inequality must be squared. If the members of the inequality are positive, then with their rising to the same positive power the meaning of inequality does not change [13]. Let's square the members of the inequality and transfer $|\alpha_2|$ to the left and right sides of the inequality:

$$\frac{c^2}{T_r^2} \max |\alpha_2| \leq k \leq \frac{1}{n^2 T_\Delta^2} \min |\alpha_2|; \quad (12)$$

$$\alpha_2 = \left(\frac{\partial F}{\partial u} \right)^{-1}; \quad F = f(u, \dot{y}, y).$$

When transferring the parameter $|\alpha_2|$ to the left side its maximum value is selected, and when transferring to the right part - the minimum value, because α_2 nonlinearly depends on u, \dot{y} , and y .

Let's substitute instead of the parameter α_2 its value and obtain the criterion for choosing the parameter k in the resulting form:

$$\frac{c^2}{T_r^2 \min \left| \frac{\partial \ddot{y}(u, \dot{y}, y)}{\partial u} \right|} \leq k \leq \frac{1}{n^2 T_\Delta^2 \max \left| \frac{\partial \ddot{y}(u, \dot{y}, y)}{\partial u} \right|}; \quad (13)$$

$$c \gg 1; \quad n \gg 1; \quad u \in U; \quad \dot{y} \in Y'; \quad y \in Y.$$

The criteria for selecting the parameter h will be formed on the basis of (9) taking into account that the contour of the control function must have an aperiodic appearance:

$$h = 2\zeta \sqrt{k \max \left| \frac{\partial \ddot{y}(u, \dot{y}, y)}{\partial u} \right|}; \quad (14)$$

$$\zeta \geq 1; \quad u \in U; \quad \dot{y} \in Y'; \quad y \in Y.$$



In this case, the function of choosing the maximum value of a partial derivative provides an aperiodic form of the contour of the control function with all allowable values of the parameters u , \dot{y} , та y .

Expressions (13) and (14) form the criteria for choosing parameters of the control law with gradient search of the second order. If the conditions are met, then the performance of the control function contour will be sufficient to ensure the process of minimizing the functional (1), and the process of numerical solution of the differential equations of the control law will be stable.

4.4 Synthesis of a high dynamic precision automatic control system of a nonlinear object without usage of information about derivatives of a controlled parameter

Let's develop a high dynamic precision controller of a second order nonlinear object. For this purpose, the reference model is chosen in the following form:

$$T_r^2 \ddot{e} = Ke_g - 2\zeta_r T_r \dot{e} - e;$$

$$e = y_g - y; \quad K = 1; \quad \zeta_r = 1; \quad e_g = 0,$$

where e - control error, e_g - given value of a control error; y_g - given value of a controlled parameter; T_r - reference model time constant, K - reference model gain coefficient, ζ_r - reference model damping coefficient.

This reference model contains derivatives not only from the managed parameter y , but also from the control task y_g . It ensures precise tracking of y_g by y when smooth changing of y_g . This eliminates the delay of y and after eliminating the control error e the controlled parameter y precisely follows the given parameter y_g [10].

The parameter e_g must be zero to meet the given control task. The value of the parameter K in the reference model is usually chosen equal to one, but in our case it is not essential, since the Ke_g member is still equal to zero. The damping coefficient ζ is selected equal to one to provide the aperiodic type of the reference model chain. The T_r parameter is selected on the basis of dynamics analysis of the control object and transitional processes of the ACS.

Let's separate the highest derivative of the controlled parameter from the reference model; it will represent the desired acceleration of the controlled value \ddot{y}_d :

$$\ddot{y}_d = \ddot{y}_g + \frac{2}{T_r} \dot{e} + \frac{1}{T_r^2} e. \quad (15)$$

We write down the differential control law of a second order nonlinear object (2), replacing it with a reference model (15):

$$\ddot{u} + h\dot{u} = \sigma k \left(\ddot{y}_g - \ddot{y} + \frac{2}{T_r} \dot{e} + \frac{1}{T_r^2} e \right); \quad k > 0. \quad (16)$$

It provides high dynamic control accuracy of a nonlinear object due to the chosen form of the reference model, which includes the first and second derivatives of a given value of the controlled parameter y_g .

The control function (16) requires information about the first and second derivatives of the controlled parameter. Let's lower the order of control function - we integrate it twice and get the control law in the following form:

$$u = k(y_g - y) + \frac{2k}{T_r} \int_0^t edt + \frac{k}{T_r^2} \int_0^t \int_0^t edtdt - h \int_0^t udt + \int_0^t C_1 dt + C_2, \quad (17)$$

or in this form:

$$u = k \left(y_g - y + \frac{2}{T_r} \int_0^t edt + \frac{1}{T_r^2} \beta_1 \right) - h\beta_2 + C_2;$$

$$\beta_1 = \int_0^t \left(\int_0^t edt + \frac{T_r^2}{k} C_1 \right) dt; \quad \beta_2 = \int_0^t udt.$$

If to use the control law in this form, then, after eliminating the control error, the absolute values of β_1 and β_2 members will increase infinitely, compensating each other. Theoretically, this will not affect the performance of the ACS, but in practice, for large t , incorrect computational results are possible. This is due to the fact that in computer realization, the numbers of finite accuracy are used.

In order to avoid the effect of increasing the members of the control law, it is necessary to adjust its structure. We denote the subintegral expressions with the variables e_i and χ and obtain the control law in the form of the system:

$$\left. \begin{aligned} u &= \sigma k \left(e + \frac{2}{T_r} e_i \right) + \int_0^t \chi dt + C_2; \\ e_i &= \int_0^t edt + C_1; \quad \chi = \frac{\sigma k}{T_r^2} e_i - hu; \quad k > 0; \quad \sigma = \text{sign} \left(\frac{\partial \dot{y}}{\partial u} \right). \end{aligned} \right\} \quad (18)$$

In this form, members of the control law will not increase infinitely after eliminating the control error, so it is suitable for use in real systems. To implement such a control law, information about the derivatives of the control object is not needed. In this case, the dynamic characteristics of the ACS, constructed on the basis of the laws (16) and (18), are identical (under zero initial conditions).

Usually, controlled objects are designed in such a way that increasing in the control signal causes an appropriate increasing in the controlled value (by increasing its highest derivative). Therefore, usually $\sigma > 0$. The parameter σ is estimated at the stage of ACS synthesis and in the process of its operation remains unchanged.

The controller input must receive information only about the control task and the controlled parameter.



Measuring the derivatives of the controlled parameter is not required.

4.5 Removal of ACS's integral saturation by the method of integration by condition

Underwater vehicles usually have limited control signals. Restrictions for u result in limitation of driving forces and moments from propulsion and/or steering and bearing surfaces (for self-propelled and/or towed UVs). If the UV is autonomous, then limited control forces and/or moments can accelerate it to limited translational and/or rotating velocities. If the UV is tethered, the presence of a flexible tether (tether-cable or tow-cable) also limits its working area. But moving it beyond limits is possible only with the exceeding of some limiting force value - the breaking force of the flexible tether, which is an emergency. Therefore, in simulators of underwater vehicles it is enough to take into account limits of control signals, which, due to the corresponding nonlinear dependencies, will limit the kinematic parameters of the underwater vehicle. Control signals are convenient to set in the range $u \in [-1, 1]$. The limitation of control influences leads to the integral saturation.

As a basis for solving the integral saturation problem, we use integration by condition, since this method is sufficiently effective and easy to implement in the computer implementation of the regulator [14]. The block diagram of the regulator, constructed on the basis of the control law (18), takes the form, represented in Figure-1.

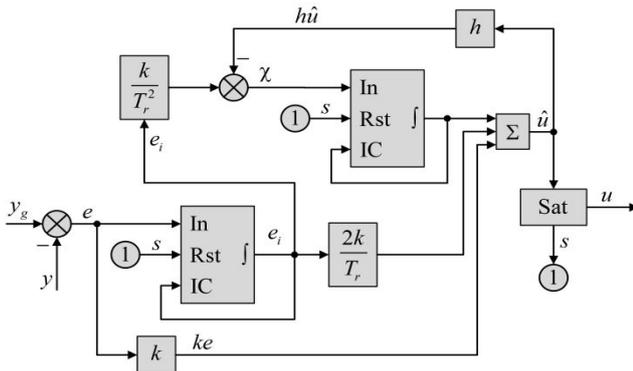


Figure-1. The regulator block diagram.

Let's describe the features of the integrator, which is part of the regulator (Figure-1). It contains three inputs: the input "In" - is for the parameter that is integrated; the input "Rst" - is to reset the output of the integrator; he "IC" input - is to set the initial value of the integral. The integrator works as follows: if the logical value "false" is given to the "Rst" input, then the integrator works normally, if "true" is received, then the output value of the integrator corresponds to the value of the "IC" input. The feedback in the structure of the integrator provides holding of the last value that was at integrator's output before reset. The operation of the integrator we will describe by the following recurrence equation:

$$I_x(x, s, C, t) := \begin{cases} 0, & \text{if } t = 0; \\ \int_{t_{s=0}}^{\max(t) < t_{s=1}} x dt + C_{t_{s=0}}, & \text{if } s = 0; t > 0; \\ C, & \text{if } s = 1; t > 0, \end{cases} \quad (19)$$

where I_x - integrator output; x - parameter that is given to the "In" input of the integrator; s - the flag of ACS's integral saturation, this parameter is given to the integrator's "Rst" input; C - parameter that specifies the initial integration condition and is fed to the integrator's "IC" input; $t_{s=0}$ - the moment of the last exit of the ACS from the saturation zone; $t_{s=1}$ - the moment of entering of the ACS into the saturation zone.

The actual integration occurs within the limits $[t_{s=0}, t_{s=1})$ each time the ACS goes out of the saturation zone. The parameter $C_{t_{s=0}}$ corresponds to the parameter C , which was fed to the integrator's input at the time $t_{s=0}$.

According to the structure of the controller (Figure-1), the output of the integrator is connected to its "IC" input, so the value of parameter I_x from the previous computational iteration is given as an argument C in equation (19): $I_x(x, s, I_x, t)$.

The "Sat" block in the controller's structure (Figure-1) calculates the parameter s and the limited control signal u , which is fed to the object, based on its unlimited value \hat{u} :

$$s(\hat{u}) = \begin{cases} \text{false,} & \text{if } \hat{u} \in [-1, 1]; \\ \text{true,} & \text{else;} \end{cases} \quad u(\hat{u}) = \begin{cases} \hat{u}, & \text{if } \hat{u} \in [-1, 1]; \\ \text{sign}(\hat{u}), & \text{else.} \end{cases}$$

Using the recurrence equation (19) that describes the integrator, we write the control law in the recurrent form:

$$\left. \begin{aligned} \hat{u} &:= \sigma k \left(e + \frac{k}{T_r} e_i \right) + \chi_i; & \chi_i &:= I_x(\chi, s, I_x, t); \\ e_i &:= I_e(e, s, I_e, t); & \chi &:= \frac{\sigma k}{T_r} e_i - hu; & k > 0; & \sigma = \text{sign} \left(\frac{\partial \hat{y}}{\partial u} \right). \end{aligned} \right\} \quad (20)$$

where e_i and χ_i - results of integration of parameters e and χ respectively by the formula (19).

The form (20) of the control law is the basis for the computer implementation of the ACS's regulator.

4.6 Synthesis of the automatic control system of the remotely-operated underwater vehicle

4.6.1 Development of one-dimensional remotely-operated vehicle dynamics mathematical model

The most widespread classes of tethered underwater vehicles are self-propelled remotely operated vehicles (ROV) and towed underwater vehicles [1]. Both classes can be controlled by translational and rotational degrees of freedom. The dynamics of such motions can be described by a second-order differential equation, which



contains the coordinate, velocity and acceleration of the object.

As a basis for the ACS study, we will use the mathematical model (MM) of an ROV's one-dimensional motion dynamics, given in [15]. The controlled movement of the ROV is provided by a propulsion device, which is a DC-motor with a gearbox connected by a gear-shaft with a propeller. The interior space of the electric motor is filled with a liquid dielectric. The mathematical model of the propulsion device consists of the electromotor MM and the propeller MM. Kinematic parameters of ROV's motion are calculated in the MM of ROV's body dynamics, which takes into account the hydrodynamic resistive force. The main disturbing force is the tension force of the tether-cable (TC), which is calculated in the MM of its dynamics [16].

The ROV mathematical model [15] is of the third order, since it takes into account the transient process in the propulsion device. However, it is assumed that such parameters of ROV elements will be chosen, with which the dynamics of propulsion device becomes insignificantly small. In this regard, the mathematical model of one-dimensional vertical motion of the ROV takes the form of:

$$\left. \begin{aligned} m\ddot{y} &= F_p(\dot{y}, u) + F_h(\dot{y}) + F_c(y, \dot{y}); \\ F_p &= f(\dot{y}, u); \quad F_h = -0,5\rho k_h S |\dot{y}| \dot{y}; \\ F_c &= f(y, v_c, \rho, L_c, D_c, C_n, C_t); \\ \dot{y} &= v_{rv}; \quad v_c = v_{wg} - v_{vg}, \end{aligned} \right\} \quad (21)$$

where m - the ROV's mass with added masses of water; \ddot{y} - ROV's vertical motion acceleration; \dot{y} - ROV's vertical motion velocity relative to water; y - vertical position of the ROV relative to the vessel-carrier; F_p - propulsion force; F_h - hydrodynamic resistance of the ROV's body; F_c - disturbing force of the TC; k_u - gain coefficient of the electromotor driver; u - control signal; ρ - water density; k_h - hydrodynamic coefficient of the ROV's body; S - characteristic area of the ROV; L_c - length of the released part of the TC; D_c - diameter of TC; C_n, C_t - normal and tangential hydrodynamic coefficients of TC respectively; v_c - horizontal velocity of the TC relative to water; v_{rv} - vertical velocity of the ROV relative to the vessel-carrier; v_{vg} - horizontal velocity of the vessel-carrier relative to the ground; v_{wg} - water flow velocity.

At any given time, the state of the ROV is characterized by three phase coordinates: y, \dot{y} та u .

The coefficient of hydrodynamic impact is determined by performing experiments in hydrodynamic tubes or by means of computational hydrodynamics [17]. Simulation of the controlled motion process will be performed for the ROV, which parameters are summarized in Table-1.

Table-1. Parameters of the ROV vertical motion model.

Parameter	Value
Body characteristic area	0.5 m²
Coefficient of hydrodynamic resistance	1.5
Mass	100 kg
Gain coefficient of propulsion device driver	310 V
The range of propulsion device permissible control signals	[-1, 1]
Number of vertical propulsion devices	1 unit
Range of forces of the propulsion device	[-126, 205] N
Tether-cable length	50 m
Tether-cable diameter cross-section	20 mm
The CT hydrodynamic resistance normal component coefficient	1
The CT hydrodynamic resistance tangential component coefficient	0.1

Such an ROV can develop the immersion velocity up to 1 m/s.

4.6.2 Selecting the controller parameters

The minimum and maximum values of the α_2 parameter were determined by analyzing the stationary states of the mathematical model of the propulsion device [18]:

$$\alpha_2 = m \left(\frac{\partial F_p(\dot{y}, u)}{\partial u} \right)^{-1}; \quad u \in [-1, -0.1] \cap [0.1, 1]; \quad \dot{y} \in [-1, 1] \text{ m/s.}$$

The dependence of the propulsion device force from the control signal is a power function. This means,



that when $u \rightarrow 0$ the parameter $|\alpha_2| \rightarrow \infty$. In practice, for the ROV's propulsion devices, the zone of insensitivity is characteristic. Therefore, when determining α_2 , the values u belonging to the range $(-0.1, 0.1)$ were excluded from the calculation. As a result of the analysis, the minimum and maximum values of the α_2 parameter were determined as $\alpha_2 \in [0.22, 0.98] \text{ s}^2/\text{m}$.

The reference model time constant was chosen as for a typical small-scaled ROV's ACS [8]: $T_r = 2 \text{ s}$. Let's choose parameters $c = 10$, $n = 10$, $T_\Delta = 0.001 \text{ s}$. From (4) we determine $T_u = 0.2 \text{ s}$. Based on (13) we calculate the range $k \in [24, 76, 2200] \text{ m}^{-1}$. Let's choose $k = 24.8 \text{ m}^{-1}$. From (14), having accepted $\zeta = 1$, we get $h = 21.2$. We substitute the calculated parameters in (20) and get a high-precision regulator of ROV's vertical motion ACS.

5. RESULTS OF THE STUDY OF TRANSIENT PROCESSES DYNAMICS OF THE ACS OF HIGH DYNAMIC ACCURACY

The dynamics of ACS transitional processes were studied by the computer simulation method. One-dimensional ROV's motion was carried out along the vertical axis (that is depth) at a distance of 45 m from the vessel-carrier stern. The vessel-carrier's velocity relative to ground was 1.5 m/s, the velocity of the counter flow was 1 m/s. Thus the whole system (vessel-carrier, ROV and TC) was moving with velocity $v_{vw} = 2.5 \text{ m/s}$ relative to the water.

The system's response to the task, which varied according to the step-like law, was analyzed:

$$y_g(t) = y_{g0} + A \cdot \text{sign}(\sin(2\pi ft + \varphi)), \tag{22}$$

where y_{g0} - task parameter offset; A - task parameter amplitude; f - task parameter frequency; φ - task parameter phase; $\text{sign}(\cdot)$ - function, that determines its argument's sign; t - simulating time.

As parameters of the signal (22) the following values were set: $y_{g0} = -10 \text{ m}$, $A = 2.5 \text{ m}$, $f = 0.02 \text{ s}^{-1}$, $\varphi = 0$. The graph of the transition process is given in Figure-2, a.

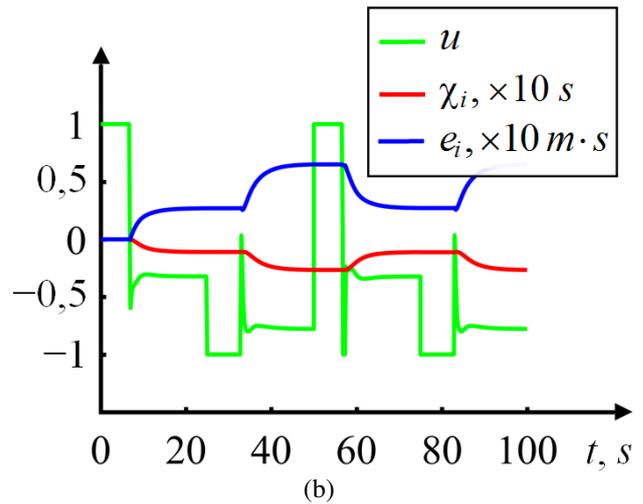
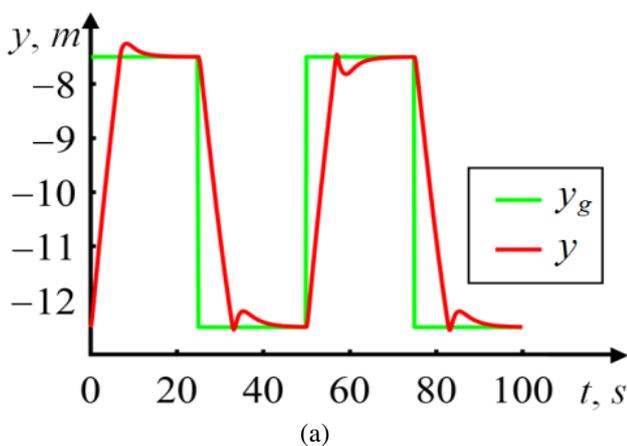


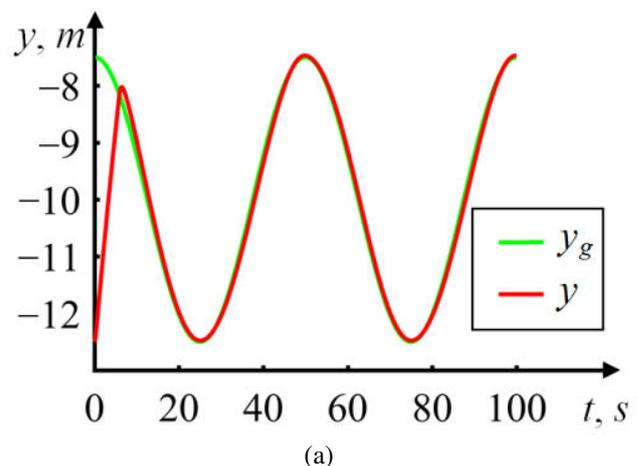
Figure-2. Dynamics of the ACS with a step-like change in the control task: a - given and actual values of the controlled parameter; b - control signal and integrators outputs.

The ACS eliminates the error in the controlled parameter. However, at the end of the transition process there is overregulation (up to 0.4 m). It arises because during the process of reducing the error the ACS is being mostly in saturation mode and the regulator's integrators are in the mode of integration restricting (Figure-2, b). It is worth noting that after exiting the saturation mode, the ACS quickly eliminates the error and keeps the controlled object in a steady state until the task y_g changes.

Let's study the ACS's transitional process dynamics for the task y_g , which varies according to the harmonic law:

$$y_g(t) = y_{g0} + A \cdot \sin(2\pi ft + \varphi); \quad \varphi = 0,5\pi f \text{ s}. \tag{23}$$

Parameters of the reference model, task and initial conditions will be left unchanged. The graph of the transition process is depicted in Figure-3, a.



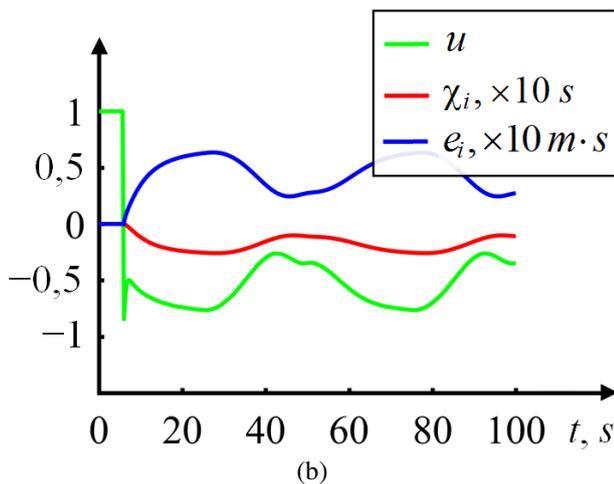


Figure-3. Dynamics of the ACS with harmonic change in the control task: a - given and actual values of the controlled parameter; b - control signal and integrators outputs.

In this case, the transient error elimination process is also accompanied by a slight overregulation. The reason for its occurrence is the same as in the computer experiment for the stepped input signal. This is clearly seen from the control signal graph (Figure-3, b).

It is worth noting that the amount of overregulation depends on the parameters of the tethered UV, the initial conditions of the simulation and the range of change of the control task. Its value for some regimes may be unacceptable, which forms the problem to solve in further research. However, after eliminating the control error the ACS shows high dynamic control accuracy under uncertainty conditions.

6. DISCUSSION OF THE SYNTHESIS RESULTS OF THE NONLINEAR OBJECT AUTOMATIC CONTROL SYSTEM OF HIGH DYNAMIC ACCURACY

In this paper, the second order nonlinear object control law of high dynamic accuracy is synthesized and the criteria for choosing its parameters are developed. To ensure the stability of the ACS, synthesized on the basis of method of local functional minimizing, the following conditions must be ensured:

- stability of the reference model;
- the performance of the contour of the control function should be much higher than the performance of the reference model (it is provided by an increase in the gain in the contour of the control function);
- the stability of the process of functional $G(u)$ minimizing, that is the stability of the contour of the control function:
 - the rule of signs must be satisfied (3);

- the stability of control law equations solving numerical process must be provided.

The stability of the reference model is ensured by the choice of its structure and parameters that correspond to the aperiodic chain of the second order.

The conditions of the performance and stability of the control function contour will be met if the control law parameters are in accordance with the proposed criterion (10). For the ACS synthesized on the basis of the control law (20), the criterion (10) takes the form of inequality (13) and equation (14).

The simulation results show high dynamic accuracy of following the given parameter by the controlled parameter after the control error has been reduced. At the same time, the important feature of the synthesized control law is that its implementation does not require information about the derivatives of the controlled parameter. The regulator's input must receive information only about the control task and the controllable value, i.e. the control error e .

When the given value of the controlled parameter changes step-like, overregulation is observed. Its elimination is a subject of further research.

The obtained results can be applied not only for self-propelled UVs but also for towed underwater vehicles and other nonlinear objects whose dynamics can be described with differential laws of the second order.

7. CONCLUSIONS

- The control law of high dynamic precision is synthesized on the basis of the methods of local functional minimization and integration on the condition. It gives an opportunity to develop automatic control systems of nonlinear objects of the second order with limited control signals under conditions of uncertainty without using the information about the derivatives of the controlled parameter. In particular such objects are tethered underwater vehicles.
- The criteria for selecting the parameters of the control law synthesized by the method of minimizing local functional were developed on the basis of the formed control law equations numerical solution stability condition and the known condition of increasing the performance of the contour of the control function with respect to performance of the reference model contour. They form the theoretical basis for the synthesis of nonlinear automatic control systems of high dynamic precision.
- A mathematical model of one-dimensional motion of a remotely operated underwater vehicle as a second-order object is developed on the basis of known tether-cable model and underwater vehicle elements



models for the study of automatic control systems of its movement.

- d) The automatic control system of the vertical motion of a remotely operated underwater vehicle is synthesized on the basis of the synthesized control law, developed criteria for selecting its parameters and analysis of the dynamics of changes in its acceleration, depending on the change of control signal. It provides high dynamic control accuracy in conditions of uncertainty of parameters of the underwater vehicle and disturbing influence of a tether-cable.
- e) The synthesized automatic control system of vertical motion of a tethered underwater vehicle in conditions of uncertainty was studied by the method of computer simulation. The simulation results show high dynamic accuracy after the control error has been eliminated. With a step-like change in the given value of the controlled parameter in some operating modes of the underwater vehicle the overregulation may increase. Its elimination is a subject of further research.

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